Primary Market Design:
Direct Mechanisms And Markets

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Abstract

We develop a model that allows for the coexistence of bookbuilding and when-issued trading. We show how information gathering through bookbuilding can enable informative when-issued trading. Furthermore, we show that if bookbuilding is a prerequisite for the when-issued market to function, then the existence of a liquid when-issued market will, in most cases, not interfere with information gathering through bookbuilding.

Key words: Initial public offerings; information gathering mechanisms; when-issued trading
JEL classification: G32
1 Introduction

Primary markets differ in the procedures that are used to price securities and to allocate them to investors. In Treasury auctions, investors submit bids, and then securities are priced and allocated according to explicit rules. Other primary markets feature pricing and allocation decisions that are made in a more discretionary way. One such example is the method of bookbuilding which is commonly used in initial public offerings of shares (IPOs). In bookbuilt IPOs, underwriters collect investors’ indications of interest, and then exercise discretion in the pricing and allocation of the securities.

Many of the pricing and allocation procedures observed in primary markets have been interpreted as direct mechanisms. Inspired by the U.S. Treasury’s experiments with uniform-price auctions for selling Treasury notes, Back and Zender (1993) compare different auction formats with a direct mechanism. Biais, Bossaerts and Rochet (2002) derive an optimal mechanism for pricing IPO shares and find that uniform pricing is optimal.

In this paper, we investigate uniform-price mechanisms for selling securities in primary markets. We analyze how the structure of this mechanism is affected by a striking variation across primary markets: the existence of a market for when-issued trading of some unseasoned securities, but not of others. For example, while there is an active market for when-issued trading of U.S. Treasury securities, no such market exists for IPO shares in the U.S.

The fact that there is no when-issued trading of IPO shares in the U.S. stands in contrast to other countries. Germany, in particular, stands out as a country with a very active when-issued market for IPO shares. This market operates concurrently with a bookbuilding process in which underwriters collect indications of interest from investors. While bookbuilding is well-recognized as a potential source of information for IPO pricing, practitioners may also view the when-issued market as an indicator for how IPOs should be priced. To quote one of the largest market makers in the German when-issued market, this market affects IPO pricing in that: “By observing when-issued trading, the underwriter can gauge the market’s interest in an IPO.” Aussenegg et al. (2003) analyze a German IPO market and

1This quote was taken from the website of Schnigge AG, http://www.schnigge.de/info/service/pre-ipo-trading.html. The
find evidence consistent with this quote. However, they also find that when-issued trading does not supplant bookbuilding as a source of information. Instead, underwriters seem to conduct bookbuilding to gather information before they post price ranges. When-issued trading commences after the ranges have been posted; this trading indicates how IPOs should be priced relative to the price ranges.

We compare IPO pricing, using only bookbuilding as a source of relevant information, and IPO pricing in the presence of a market for when-issued trading. We argue that when-issued trading can be beneficial since bookbuilding may be limited as a means of reducing IPO underpricing. We then analyze the interaction between bookbuilding and when-issued trading. This analysis provides a rationale for the findings of Aussenegg et al. (2003). We show that when-issued trading may fail to open unless informational asymmetries (about the value of IPO shares) have first been alleviated. This can be done if the underwriter conducts bookbuilding and then publicly releases information that has been learned. Consistent with this result, when-issued trading never opens in Germany before the underwriters post price ranges that give investors an indication of how they plan to price IPO shares. Finally, we analyze how the presence of a when-issued market affects the form of the optimal direct mechanism to elicit information from investors. We find that the presence of a when-issued market can greatly change the nature of the optimal mechanism, if this market can open irrespective of whether bookbuilding is conducted first. However, if the when-issued market cannot open on its own, then the direct mechanism is less affected by this market. In this case, we can show that the presence of a liquid when-issued market may increase the cost of conducting bookbuilding, but only in very special circumstances, i.e. when a very small number of investors have access to some piece of relevant information. If some pricing-relevant information is dispersed among investors, then when-issued trading provides a strictly positive benefit in pricing IPOs. Thus, we argue that it is only in the case that information is very closely held that a liquid when-issued market may be harmful to issuers. Otherwise, allowing for such a market should benefit issuers.

Our paper extends the existing literature on the pricing of unseasoned securities. We

original quote was in German: “Der Emissionsführer kann auf Grund der Handelstätigkeit im Handel per Erscheinen das Interesse des Marktes an der Neuemission messen.”
build on the models of Rock (1986) and Benveniste and Spindt (1989). This paper is also related to the literature on when-issued markets. Bikchandani and Huang (1993) and Nyborg and Sundaresan (1996) examine the when-issued market for U.S. Treasury securities. Löffler, Panther and Theissen (2002) examine the when-issued market for Neuer Markt IPOs and find that the final prices in this market are unbiased predictors of opening prices in the secondary market. Dorn (2002) examines this same when-issued market to investigate whether sentiment drives retail participation. Ezzel, Miles and Mulherin (2002) examine when-issued trading of shares of publicly traded subsidiaries prior to full divestiture.

The paper is organized as follows. In the next section, we provide a brief description of relevant institutional aspects of several primary markets. In the third section, we present the rationale for when-issued trading of IPO shares in a simplified analysis that fails to address several possible caveats. These caveats are dealt with in the remainder of the paper. In particular, in Section 5 we examine the opening of the when-issued market. In Section 6 we model the direct mechanism for eliciting information from investors, both with and without when-issued trading.

2 A selective survey of institutional features of primary markets

In this section, we briefly survey the structure of some primary markets. This survey highlights a difference between the institutional framework of U.S. Treasury markets and that of U.S. IPOs: the existence of a market for when-issued (forward) trading of Treasuries but not for IPO shares. We point out that some European markets feature when-issued trading of IPOs, and we describe one notable example, the German Neuer Markt.

When-issued trading of Treasury securities: Bikchandani and Huang (1993) and Nyborg and Sundaresan (1996) provide detailed descriptions of institutional features of the primary market for U.S. Treasury securities, including the market for when-issued trading of Treasuries. The when-issued market is a forward market for trading in not yet issued securities. Trading starts on the date of the announcement of a Treasury auction and continues after the auction takes place (up until the issue date). The forward contracts represent commitments
to trade when, and if, the security is issued. The contracts specify physical delivery of the underlying security on the date at which this security is issued.

Nyborg and Sundaresan (1996) give an assessment of the role of when-issued trading in the process of selling Treasuries. They argue that the when-issued market plays a price discovery role in that this market generates and aggregates information about the expected depth of a Treasury auction and the diversity of auction participants.

**Initial public offerings of shares:** Jenkinson and Ljungqvist (2001), Ritter and Welch (2002), and Ritter (2002) provide recent surveys of the institutional structure of IPO markets and the extensive literature on IPOs. Lungqvist, Jenkinson and Wilhelm (2003) point out that the U.S. method of IPO pricing through bookbuilding has become increasingly popular outside the U.S. We therefore choose to focus on this method for IPO pricing in our analysis below. Price discovery through bookbuilding differs from price discovery through when-issued trading in that, in bookbuilding information is gathered directly from investors. According to Benveniste and Spindt (1989), in order to provide incentives for investors to truthfully reveal positive information about an issue, underwriters only partially adjust the IPO prices in response to such information. The underwriters then allocate underpriced shares to those investors who provided the positive information. Hence, the investors who hold positive information earn informational rents.

**When-issued trading of IPO shares:** In the United States, Treasury issues and IPOs differ in that there is no market for when-issued trading of IPO shares. Such when-issued trading is restricted by securities laws;\(^2\) the stated reason for the restriction is: “Such short sales could result in a lower offering price and reduce an issuer’s proceeds.”\(^3\)

In contrast to IPO markets in the U.S., those in many European countries feature when-issued trading of IPO shares. Since many of these markets also employ bookbuilding methods to price IPOs, this implies that there are potentially two sources of information for IPO

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\(^2\) Regulation M, Rule 105 prohibits the covering of short positions in IPO shares that were created within the last five days before pricing, with allocations received in the IPO. In addition to this rule, there are also restrictions on trading in unregistered shares.

pricing. Price discovery may take place both through the market for when-issued trading (by analogy to Nyborg and Sundaresan (1996)), and through a direct mechanism in bookbuilding (as suggested by Benveniste and Spindt (1989)).

Aussenegg et al. (2003) investigate one notable example of an IPO market with bookbuilding and when-issued trading of IPO shares, the German Neuer Markt IPO market. They provide a detailed description of the institutional framework of this market that can be summarized by Figure 1. This figure provides a stylized timeline to illustrate the timing of bookbuilding and when-issued trading on the Neuer Markt. The timeline has three stages, the period before the opening of when-issued trading (Stage 1), the period during which when-issued trading occurs (Stage 2), and the period after when-issued trading (Stage 3). During Stage 1, the underwriters may gather information to use in setting price ranges. When-issued trading opens at time $t_W$, after these ranges are set. As in U.S. Treasury markets, such trading is in forward contracts that specify physical delivery. When-issued trading continues beyond the time $t_P$ at which the underwriters set the IPO offer prices, up to the evening before the first day of trading in the secondary market.

$^4$The price ranges are binding in that underwriters do not set offer prices above posted ranges. See Ljungqvist and Wilhelm (2002) and Aussenegg et al (2003).
3 The rationale for when-issued trading: a simple model

There have been numerous explanations as to why underpricing occurs in IPOs. Many of the theories use as their starting point the problem of adverse selection in primary markets due to asymmetries of information across investors, as modeled by Rock (1986). Benveniste and Spindt (1989) showed how the process of eliciting information through bookbuilding can lead to IPO underpricing. Maksimovic and Pichler (2002) showed how these two ideas are directly linked in that the optimal amount of information gathering through bookbuilding depends on the adverse selection risk, as well as on how the bookbuilding is done. Loughran and Ritter (2002a) and (2002b) take an agency theoretic perspective. They argue that underwriters benefit from underpricing IPOs in the form of quid pro quos. In Loughran and Ritter (2002a) they argue further that issuers may be willing to accept large amounts of underpricing if the issue price is much higher than what they had originally expected.

In our analysis we will focus on informational reasons, rather than agency reasons, for IPO underpricing. Our starting point is the adverse selection problem modeled by Rock. Since this problem is due to informational asymmetries across investors, it can be mitigated if the IPO offer price can be set based on information that is held by the more informed investors. This seems to be what underwriters can accomplish through bookbuilding. However, for several reasons bookbuilding may be of limited effectiveness. First, some of the information that is relevant for IPO pricing may not reside with the investors who participate in bookbuilding. Second, investors’ information may be noisy, making it impossible to fully resolve informational asymmetries across investors if only a limited number of them participate in bookbuilding. As such, there will remain residual adverse selection risk due to the presence of investors whose information was not obtained through bookbuilding. Finally, bookbuilding may itself be a reason for IPO underpricing. As modeled by Benveniste and Spindt (1989), underpricing may be required in order to pay informed investors rents for revealing their information directly to the underwriter.

We present in this section a very simple model of IPO pricing that incorporates all three of the limits of bookbuilding mentioned above. The objective of this section is to illustrate, in the simplest manner possible, the potential value of a market for when-issued trading of
IPO shares, as a source of information for IPO pricing. The model presented here provides the foundation for the analysis of the following sections in which we will explore in depth the potential pros and cons of allowing when-issued trading.

We next present the elements of our model. We first specify how the value of IPO shares depends on two kinds of information, only one of which is held by informed investors who might participate in bookbuilding. We then define the objective function that the underwriter maximizes in pricing the IPO. We will use an objective function that is similar to that proposed by Hughes and Thakor (1992). As will be discussed below, we choose this objective function in order to introduce a reason why the underwriter cares about information that cannot be obtained through bookbuilding. The reason is that the underwriter is averse to overpricing the IPO. This reasoning is inspired in part by Hughes and Thakor (1992) and by Nanda and Yun (1997). Nanda and Yun find that, for IPOs that are significantly overpriced, the lead underwriter suffers a negative impact on equity market value. This impact is greater than what can be attributed to the cost of price stabilization. Thus, overpricing is costly.\footnote{Overpricing at the IPO can also be costly for issuers, in that it may be taken as a negative signal about the firm. For entrepreneurs who retain large share ownership, or for firms that engage in follow up offerings, the cost of such a signal can easily outweigh the benefit of higher IPO proceeds. In addition, some issuers conduct IPOs with the objective of advertising their firms. See for example Demers and Lewellen (2002).}

**The value of IPO shares:** As our focus is on valuation, we will take as exogenous the number of shares that are sold in the IPO; only the offer price will be endogenous. We will also assume that the offering is uniform price, in that all investors at the IPO pay the same price. As a simplification, we will normalize to one the number of shares issued at the IPO. $\tilde{V}$ is the unknown secondary market value of this share.\footnote{More specifically, in order to match existing empirical literature we will think of $\tilde{V}$ as the value based on the first day closing price in the secondary market.}

Prior to the IPO there are two sources of uncertainty about the realization of $\tilde{V}$:

$$\tilde{V} = v_0 + \tilde{s} w + \tilde{d},$$

where $v_0$ is the prior expected value of $\tilde{V}$, $w$ is a positive parameter that is strictly less than $v_0$, $\tilde{s}$ is a random variable that can take on the realizations $1$ or $-1$, and $\tilde{d}$ is a random variable.
that is uniformly distributed on the interval \([-\delta, \delta]\).\(^7\) (We assume that \(v_0 - w - \delta \geq 0\).) The model of equation (1) and the parameters \(v_0\), \(w\), and \(\delta\) are all common knowledge. A key difference between the two random components of the share value \(\tilde{V}\) is that one of these variables, \(\tilde{s}\), has been observed, albeit noisily, by a number of informed investors, whereas no individual investors have private information about the other variable, \(\tilde{d}\). Therefore, the component \(\tilde{s}w\) of the value is due to information that the underwriter can obtain from informed investors who participate in bookbuilding. By contrast, we interpret the variable \(\tilde{d}\) as dispersed information that the underwriter cannot obtain from a limited number of investors.

The prior distribution on \(\tilde{s}\) is that each outcome (1 or -1) will occur with a probability of one-half. Thus, for an uninformed investor, the prior expected value of \(\tilde{s}\) is zero. A number of informed investors have observed signals of \(\tilde{s}\); the signal of investor \(i\) is a random variable \(\tilde{\varsigma}_i\) which can take on one of two realizations, \(\varsigma_i \in \{-1, 1\}\). Conditional on the realization of \(\tilde{s}\), the signals \(\tilde{\varsigma}_i\) and \(\tilde{\varsigma}_j\) of any two informed investors \(i\) and \(j\) are independent of each other. Moreover, these signals are identically distributed: with probability \(q > 1/2\), any given informed investor has correctly observed the realization of \(\tilde{s}\). For an investor who sees a positive signal, the probability that \(s = 1\) is \(q\) and the probability that \(s = -1\) is \(1 - q\), so that the expected value of \(\tilde{s}\) is given by \(q - (1 - q) = 2q - 1 > 0\). For an investor who sees a negative signal, the expected value of \(\tilde{s}\) is \(1 - 2q < 0\). We will assume that a fraction \(\alpha\) (\(0 < \alpha < 1\)) of all potential investors are informed. On average, a fraction \(q\alpha\) of investors will have correctly observed the realization of \(\tilde{s}\) and \((1 - q)\alpha\) will have observed \(-\tilde{s}\).

**Adverse selection risk, bookbuilding, and IPO pricing:** The presence of informed investors implies that uninformed investors face an adverse selection risk when investing in IPO shares. The reason is as follows: Suppose that the underwriter prices the IPO at the prior expected value of \(v_0\). Because \(q > 1/2\), fewer informed investors invest in IPO shares when the issue is overpriced \((s = -1)\) than when it is underpriced \((s = 1)\). Thus, uninformed investors will be allocated more shares when the issue is overpriced. In order to induce uninformed investors to buy IPO shares, the offering must be priced so that it is a “fair bet” for them.

\(^7\)A list of variables with their definitions is given at the beginning of the Appendix.
This is done by pricing the issue at $v_0 - u_{AS}$, where $u_{AS} \geq 0$ is expected underpricing due to adverse selection risk. As long as there are some investors who are strictly better informed than others, $u_{AS}$ will be strictly positive.

The underwriter can, through bookbuilding, mitigate the problem of adverse selection risk. To discuss this in the simplest manner possible, we abstain (for now) from modeling the bookbuilding process itself. Instead, we just assume that, during this process, the underwriter uses a direct mechanism to induce a number of informed investors to truthfully report the realizations of their signals. We will represent the number of positive signals that are reported in bookbuilding minus the number of negative signals with the symbol $z$. Underpricing due to residual adverse selection risk is a function of $z$, and so can be written as $u_{AS}(z)$. For any finite value of $z$, underpricing due to adverse selection risk is positive: $u_{AS}(z) > 0$. However, it is shown in the next section that for all $|z| \geq 1$, $u_{AS}(z) < u_{AS}(0)$ and $u_{AS}(z)$ is decreasing in $|z|$. Thus, gathering information through bookbuilding ($|z| \geq 1$) lowers underpricing due to adverse selection risk. It may be necessary, however, to offer underpriced shares to informed investors in order to induce them to truthfully report their signals. We will let $u_B (u_B \geq 0)$ denote the expected level of IPO underpricing required to obtain such reports in bookbuilding. The level of expected underpricing that both induces investors to report their information and that encourages uninformed investors to participate in the offering is $\max[u_{AS}(z), u_B]$. Both of these values are precisely modeled in later sections. For what follows in this section we need only know that these values place lower bounds on underpricing, and that the bounds may be strictly positive.

**The underwriter’s objective function in IPO pricing:** We assume that the underwriter is risk neutral and wishes to maximize IPO proceeds, minus the expected cost of overpricing. More formally, the underwriter’s objective function is given by:

$$\max_{p_I} \Pi = p_I - c(p_I - E[\tilde{V}|p_I > \tilde{V}, z]) \prob\{p_I > \tilde{V}|z\},$$

where $p_I$ is the offer price and $c(p_I - \tilde{V})$ is the penalty for overpricing. We assume that this pricing problem is constrained due to a requirement that the issue be priced so that retail

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8We use the term *residual* adverse selection risk to refer to the adverse selection risk that remains due to the presence of informed investors whose information is not contained in $z$. 

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investors will be willing to participate. As discussed above, expected underpricing must be at least as large as \( \max[u_{AS}(z), u_B] \). Therefore, the underwriter must satisfy the following constraint when solving (2):

\[
p_I \leq E[\tilde{V}|z] - \max[u_{AS}(z), u_B]. \tag{3}
\]

The objective function (2) is somewhat similar to that modeled in Hughes and Thakor (1992). The main difference is that, in their model the underwriter knows strictly more than the investors, whereas our underwriter is not better informed. Instead, we assume that there is some information that the underwriter can learn from investors and some information, \( \tilde{d} \), which the underwriter cannot obtain through bookbuilding. In their model, for litigation to occur (i.e., a cost to be imposed on the underwriter), there must be a belief that the underwriter has purposely overpriced the issue. In our model the underwriter does not misrepresent information; overpricing happens by chance. However, the underwriter can lower the odds, and the expected extent, of overpricing both by collecting information to reduce uncertainty and by underpricing in expected value.

Even without explicitly solving the problem (2), we can see that the optimal issue price will either be the price that solves (2) as an unconstrained problem, or it will be the price that satisfies (3) with equality. The price that solves (2) can be represented as \( E[\tilde{V}|z] - u_M \), where \( u_M \) is derived below and represents the expected underpricing that occurs due to the underwriter’s concern about overpricing (making a mistake). Thus, expected underpricing will be the maximum needed due to each of three possible reasons for IPO underpricing: the cost of avoiding overpricing \((u_M)\), the cost of adverse selection risk \((u_{AS}(z))\) and the expected cost of bookbuilding \((u_B)\).

As discussed above, bookbuilding can decrease the expected level of IPO underpricing due to adverse selection risk, but at a cost represented by \( u_B \). In fact, information gathering through bookbuilding should optimally be done up to the point that \( u_{AS}(z) = u_B \) in expectation.\(^{10}\) Bookbuilding can also decrease IPO underpricing due to the underwriter’s concern about committing an overpricing mistake. However, this is only possible to a limited extent.

\(^9\)The underwriter may choose not to conduct bookbuilding, in which case \( z = u_B = 0 \).

\(^{10}\)See Maksimovic and Pichler (2002).
We show in the next paragraph that $u_M$ may remain strictly positive even if the underwriter obtains perfect information about the realization of $\tilde{s}$. This is because bookbuilding is not suited for gathering dispersed information about the value of IPO shares. That is, bookbuilding cannot eliminate uncertainty about $\tilde{d}$.

**Determination of $u_M$:** If there are two sources of uncertainty about the value of IPO shares, $\tilde{s}$ and $\tilde{d}$, then the derivation of the offer price that maximizes the objective function (2) is complicated by the fact that the functional form of the second term of (2) varies across the parameter space. The full derivation of this problem is given in the Appendix. We derive here a simpler version of the problem in which we assume that all uncertainty about $\tilde{s}$ has been resolved through bookbuilding, so that $\bar{V} = v_0 + sw + \tilde{d}$ and $E[\bar{V}|z] = v_0 + sw$. In this case,

$$E[p_I - \bar{V}|p_I > \bar{V}, z] \text{prob}\{p_I > \bar{V}|z\} = \frac{(p_I - v_0 - sw + \delta)^2}{4\delta}$$

and the first order condition of problem (2) is given by:

$$\frac{\partial \Pi}{\partial p_I} = 1 - \frac{c}{2\delta} (p_I - v_0 - sw + \delta) = 0 \implies p_I = \frac{2\delta}{c} + v_0 + sw - \delta = E[\bar{V}|z] - \delta \left(1 - \frac{2}{c}\right)$$

The expected underpricing due to the cost of an overpricing mistake is thus:

$$u_M = \max \left[0, \delta \left(1 - \frac{2}{c}\right)\right] \quad (4)$$

Concern about overpricing results in underpricing only if the cost of overpricing is high enough ($c > 2$). The reason for this is that the underwriter faces a tradeoff between avoiding overpricing and maximizing IPO proceeds.

The more general solution, in which there is residual uncertainty about $\tilde{s}$ even after bookbuilding, is given by equation (35) in the Appendix. As in equation (4), this solution has the characteristic that expected IPO underpricing due to uncertainty about $\tilde{d}$ is strictly positive only if $c$ is large enough, in which case the expected underpricing is increasing in the variance of $\tilde{d}$ (increasing in $\delta$).
The rationale for when-issued trading: By aggregating information that is widely dispersed across investors, a liquid when-issued trading market can reveal information that the underwriter cannot obtain through bookbuilding. This tends to decrease IPO underpricing. In addition, a market for when-issued trading can aggregate information that is privately held by informed traders, i.e., information that could potentially be obtained through bookbuilding, but that the underwriter missed. Thus, the market can reduce any residual adverse selection risk that remains after bookbuilding. Finally, any information released through when-issued trading is available for free. Hence, less IPO underpricing may be required in order to obtain information that is held by informed investors.

Open questions: We cannot just assume, however, based on the arguments given above, that when-issued trading will generally be beneficial for issuers. There are two remaining open questions. The first question is: Under what conditions will when-issued trading open? As modeled by Glosten and Milgrom (1985), a market may fail to open in the presence of severe informational asymmetries across traders. If this happens, then when-issued trading cannot supplant bookbuilding as a source of information for IPO pricing. We will show in Section 5 that the opening of when-issued trading may be facilitated by using a direct mechanism (bookbuilding) to collect information held by some of the investors, and then making the information publicly available. This indeed seems to be what happens before when-issued trading starts in Germany. There, the when-issued market opens only after the underwriter posts a price range within which the IPO will be priced. Before this price range is set, the underwriter appears to gather information from investors; the range then reveals information the underwriter has received.\footnote{See Aussenegg, Pichler and Stomper (2003).}

The second question is: Does when-issued trading increase the cost of gathering information by means of a direct mechanism? We proposed, as one rationale for when-issued trading, the notion that because information is provided for free in the when-issued market, such trading can decrease the cost of gathering information. However, we also need to consider the possibility that when-issued trading can interfere with bookbuilding, and thus make information gathering more expensive. The presence of a when-issued market
may contribute to investors’ reluctance to directly reveal private information, since such revelation will deprive them of the opportunity to trade on this information. As a result, it could be more expensive to collect information through bookbuilding in the presence of a when-issued market, than without when-issued trading. We will show in Section 6 that this can indeed happen. However, when-issued trading interferes with bookbuilding only if one of the following conditions holds: i) bookbuilding is not needed for the when-issued market to open; or ii) bookbuilding is needed, but the when-issued market is very illiquid; or iii) bookbuilding is needed and information is held by a very small number of investors. In the first case, the presence of a when-issued market should not make issuers worse off because bookbuilding is not needed. In the second and third cases it is possible, but by no means certain, that when-issued trading will make an issuer worse off. In what follows we will show that, apart from these two special cases, a when-issued market will make issuers better off, in expected value.

In the following section we model underpricing due to adverse selection risk. This section provides further foundation for our analysis of when-issued trading and bookbuilding. In Sections 5 and 6 we model the opening of when-issued trading and the bookbuilding mechanism. It is in these latter two sections that we derive the results described above.

4 Underpricing due to Adverse Selection Risk

The purpose of this section is to model IPO underpricing due to adverse selection risk. As the benchmark case, we first consider the required level of underpricing if uninformed investors have no access to information beyond the prior public information. Then, we will show that the required level of underpricing decreases if uninformed investors can observe information about the value of IPO shares that is contained in the prices of transactions in when-issued trading. This result constitutes a potential rationale for when-issued trading, beyond what was presented in the previous section.

The model presented here is a simplified version of Rock (1986). It is assumed that investors arrive randomly in the primary market and decide whether to buy a small fraction of the issue; these allocations are made on a first-come first-served basis until the entire issue
is sold. Thus, if all of the informed investors participate, on average a fraction $\alpha$ of the issue will go to informed investors. If an uninformed investor participates in an offering, then he has a higher probability of receiving an allocation when informed investors don’t participate. This is the source of the adverse selection risk.

According to expression (1), investors face uncertainty about two components of the IPO value, $\tilde{s}$ and $\tilde{d}$. Some investors have information about $\tilde{s}$, and so any uncertainty about the value of $\tilde{s}$ will contribute to adverse selection risk. No investors have private information about $\tilde{d}$. In addition, the expected value of $\tilde{d}$ is zero and all investors are risk neutral. Thus, $\tilde{d}$ does not affect the adverse selection risk and we can ignore $\tilde{d}$ in the analysis of this section.

**Underpricing due to adverse selection risk:** Through bookbuilding, the underwriter realizes a value $z$ that has been defined above as the difference between the number of positive and negative signals reported by investors who participate in bookbuilding. This variable is a sufficient statistic for the information contained in the investors’ reports. When-issued trading reveals two pieces of information of relevance for IPO pricing. These are the total number of trades in the when-issued market prior to the time of IPO pricing, denoted by $T$, and the number of buy orders minus the number of sell orders, denoted by $y_T$ ($y_T \in \{-T, -(T-1), ..., 0, ..., T-1, T\}$). As discussed above, when-issued trading may reveal information both about $\tilde{s}$ and $\tilde{d}$, but the latter is not relevant to the analysis of this section. We will denote by $\bar{d}_T$ the expected value of $\tilde{d}$, conditioned on $\{z, T, y_T\}$.

Let $\pi_T(z, y_T)$ denote the probability that $s = 1$, given the information reported in bookbuilding and revealed through when-issued trading. For shorthand we will write $\pi_T(z, y_T)$ simply as $\pi_T$. Conditional on all publicly available information, the expected value of the IPO is given by:

$$E[\hat{V}|z, T, y_T] = v_0 + (2\pi_T - 1)w + \bar{d}_T.$$ (5)

We next derive the expected value of IPO shares from the perspective of an informed investor $i$. Such an investor sees a signal of $\tilde{s}$: $\tilde{\varsigma}_i \in \{-1, 1\}$. We derive in the Appendix the following posterior probabilities:

$$prob\{\tilde{s} = 1|z, T, y_T, \tilde{\varsigma}_i = 1\} = \frac{q\pi_T}{q\pi_T + (1 - q)(1 - \pi_T)},$$
Realization of $\tilde{s}$ & $s = -1$ & $s = 1$ \\ 
Probability of this realization & $1 - \pi_T$ & $\pi_T$ \\ 
Expected secondary market value $\tilde{V}$ & $v_0 - w + \tilde{d}_T$ & $v_0 + w + \tilde{d}_T$ \\ 
Allocation to informed investors & $(1 - q)\alpha$ & $q\alpha$ \\ 
Allocation to uninformed investors & $\frac{1 - \alpha}{1 - q\alpha}$ & $\frac{1 - \alpha}{(1 - q)\alpha}$ \\
Table 1: Expected IPO Value and Allocations

$$prob\{\tilde{s} = 1|z, T, y, \varsigma_i = -1\} = \frac{(1 - q)\pi_T}{(1 - q)\pi_T + q(1 - \pi_T)}.$$  

Given these probabilities, an informed investor values the IPO shares as follows:

$$E[\tilde{V}|z, T, y, \varsigma_i = 1] = v_0 + \frac{q\pi_T - (1 - q)(1 - \pi_T)}{q\pi_T + (1 - q)(1 - \pi_T)}w,$$  

$$(6)$$

$$E[\tilde{V}|z, T, y, \varsigma_i = -1] = v_0 + \frac{(1 - q)\pi_T - q(1 - \pi_T)}{(1 - q)\pi_T + q(1 - \pi_T)}w.$$  

$$(7)$$

An informed investor who has observed a negative signal will refrain from participating in the issue if the IPO price $p_I$ is such that $p_I > E[\tilde{V}|z, T, y, \varsigma_i = -1]$. Table 1 presents the expected issue value and the expected allocations to each group of investors (informed and uninformed), for each possible realization of $\tilde{s}$. The expected secondary market value is conditioned on all information that has been gathered and on the realization of $\tilde{s}$. The allocations are written as fractions of the total issue. This table is written assuming that $p_I > E[\tilde{V}|z, T, y, \varsigma_i = -1]$, so that investors who have observed negative signals do not participate.$^{12}$ The table shows that the exposure of uninformed investors to adverse selection risk depends on the probability $q$ with which informed investors correctly observe the realization of $\tilde{s}$. If $q = 1/2$, then there is no adverse selection risk because the uninformed investors expect to receive the same number of shares, regardless of the realized value of $\tilde{s}$. But, if $q > 1/2$, then the uninformed will on average receive more shares if the value of these shares is low ($s = -1$).

In order to induce uninformed investors to participate in the offering, the expected return

$^{12}$The informed participation given in Table 1 is the participation, conditioned on the realization of $\tilde{s}$. We assume that the number of investors who have revealed their information is small relative to the total number of informed investors who may participate in the offering. Thus, the relative level of informed participation is not affected by bookbuilding or when-issued trading. The effect of this assumption is that the expected underpricing calculated here is really an upper bound on underpricing.
to these investors must be nonnegative. When underpricing is minimized, this expected return will be zero:

\[ 0 = (1 - \pi_T) \left( v_0 - w + \tilde{d}_T - p_I \right) \frac{1 - \alpha}{1 - q \alpha} + \pi_T \left( v_0 + w + \tilde{d}_T - p_I \right) \frac{1 - \alpha}{1 - (1 - q) \alpha}, \]  

(8)

where the first (second) term is the product of the expected return to the uninformed investors if \( s = -1 \) (\( s = 1 \)), and the fraction of the offering allocated to them. Each term is multiplied by the probability with which \( \tilde{s} \) takes on that realization. Solving equation (8) for \( p_I \) yields:

\[ p_I = v_0 + \tilde{d}_T + \left( \frac{2 \pi_T - 1 - (\pi_T + q - 1) \alpha}{1 - ((2 \pi_T - 1)q + 1 - \pi_T) \alpha} \right) w. \]  

(9)

If \( \alpha = 0 \), then the expression in (9) equals \( E[\tilde{V} | \pi_T] \), as given in equation (5). As \( \alpha \to 1 \), the expression in (9) \( \to E[\tilde{V} | z, T, y_T, s_i = -1] \), as given in equation (7). This result is quite intuitive: as \( \alpha \to 1 \), uninformed investors face so severe an adverse selection problem that they are willing to participate in the IPO only at a price that approaches the lowest possible valuation that can be assigned by an informed investor.

The expected underpricing is:

\[ u_{AS}(z, T, y_T) = E[\tilde{V} | z, T, y_T] - p_I = \frac{q - 2 \pi_T (1 - \pi_T) - (2 \pi_T - 1)^2 q}{1 - ((2 \pi_T - 1)q + 1 - \pi_T) \alpha} \alpha w, \]  

(10)

If \( \pi_T = 0 \) or \( \pi_T = 1 \), so that perfect information about \( \tilde{s} \) is available, then the above is zero. If no information has been gathered so that \( \pi_T = 1/2 \), then the above is:

\[ u_{AS} \bigg| \text{no information gathering} = \frac{(2q - 1) \alpha w}{2 - \alpha}. \]  

(11)

Expression (10) contains as a special case the expected level of IPO underpricing due to adverse selection risk, in the absence of when-issued trading. In Section 3, this was denoted as \( u_{AS}(z) \), given by: \( u_{AS}(z) = u_{AS}(z, 0, 0) \). Lemma 1 characterizes how this expected level of underpricing depends upon the informativeness of bookbuilding, as measured by \(|z|\). As we claimed in Section 3, \( u_{AS}(z) \) decreases as bookbuilding becomes more informative (\(|z|\) becomes larger).

\[ ^{13} \text{Consistent with the assumption behind Table 1, } p_I > E[\tilde{V} | z, T, y_T, s_i = -1]. \]
Lemma 1. Underpricing due to adverse selection risk. Expected underpricing due to adverse selection risk is: i) strictly decreasing in $|\pi_T - 1/2|$, for all values of $\pi_T \leq 1/2$ and $\pi_T \geq 1/2 + \alpha(q - 1/2)$, and ii) strictly lower for $\pi_T = 1/2 + \alpha(q - 1/2)$, than for $\pi_T = 1/2$. Thus, with only bookbuilding (no when-issued trading), expected underpricing due to adverse selection risk is strictly decreasing in the informativeness of bookbuilding, as measured by $|z|$.

The implication of Lemma 1 is that bookbuilding by itself will never increase underpricing due to adverse selection risk, and will strictly decrease such underpricing if bookbuilding is informative ($|z| \geq 1$). If when-issued trading occurs in conjunction with bookbuilding (or following bookbuilding), then it is possible that when-issued trading will worsen the underpricing due to adverse selection risk. This can happen if $z \times y_T < 0$, i.e., if when-issued trading reveals information that contradicts with the information learned in bookbuilding. However, Lemma 1 enables us to show that, in expected value (with the expectation calculated prior to the onset of when-issued trading), when-issued trading will decrease the underpricing due to adverse selection risk. This is presented in our first proposition.

Proposition 1. The expected benefit from when-issued trading, as measured by the decrease in expected underpricing due to adverse selection risk, is strictly positive.

Proposition 1 verifies one of the claims made in Section 3 as part of the rationale for when-issued trading. In expected value, when-issued trading decreases underpricing due to adverse selection risk. We still need to address the two questions posed at the end of Section 3. The first of these (Under what conditions will when-issued trading open?), is answered in the next section.

5 When-issued trading

In this section, we present a model of when-issued trading that is similar to the model of Glosten and Milgrom (1985). We will use this model to analyze whether the when-issued market can open, and whether both informed and uninformed investors will participate in trading. We will first derive a condition for this market to open, given that uninformed
investors have access only to information that is contained in expression (1). If this condition is violated, when-issued trading fails to open. We will next determine whether this problem of market failure can be resolved by collecting, and then making publicly available, information that is held by some of the informed investors. Finally, we will analyze problems of market breakdown after the opening of when-issued trading.

5.1 The Model of When-issued Trading

Players: The players are the same as we have modeled so far, with the following exceptions: i) the underwriter does not participate in when-issued trading\(^{14}\); ii) the market is facilitated by purely competitive, risk neutral market makers. As before, a fraction \(\alpha\) of the investors (traders) are informed, with their information structure being the same as described in Section 3. The market makers are uninformed. All of the players are risk neutral. The market makers have no inventory costs, or costs of trading.

Time-line: First, the market makers post competitive bid and ask prices. Traders arrive sequentially. Each arrival either buys one unit at the ask price, or sells one unit at the bid price. Market makers update their bid and ask prices after every trade. All bid, ask and transaction prices are publicly observed. Because the market makers have identical information and no inventory costs, they post identical prices at all times. In what follows, we will thus refer to a single bid and a single ask price at each point in time.

Information structure: The information structure is similar to that in Glosten and Milgrom (1985) in that there is a pricing-relevant random variable that has not been observed by the market makers, and that can take on one of only two values: \(\tilde{s} \in \{-1, 1\}\). In both models some fraction \(\alpha\) of the potential traders are informed in that they have observed a signal of this random variable. Our information structure differs from that of Glosten and Milgrom in that there is another source of uncertainty, captured by the random variable \(\tilde{d}\). As discussed above, we interpret this variable as dispersed information that underwriters cannot obtain directly from investors because they cannot identify investors with private information about

\(^{14}\)This assumption is consistent with common practice on the German Neuer Markt. See Aussenegg et al. (2003).
the realization of $d$. Correspondingly, we assume that this information is neither available to any individual traders nor to any market makers in the when-issued market. Instead, we will extend the model of Glosten and Milgrom to allow $d$ to affect the arrival rates of liquidity buyers and sellers. While this extension gives rise to another reason for market failure, it creates a further potential for when-issued trading to serve as a source of information for IPO pricing.

The trader who arrives at time $t$ in the when-issued market values the traded asset at 

$$Z_t = E[\tilde{V}|F_t] + \rho_t,$$

where $F_t$ is the time $t$ trader’s information set. $\rho_t$ is the valuation parameter of the time $t$ arrival. We will assume that $\rho_t = 0$ for all informed traders. (We also assume that the market makers have a valuation parameter of zero.) For uninformed traders, $\rho_t \in \{-\rho, \rho\}$, $\rho \geq 0$. An uninformed trader with a valuation parameter of $\rho$ is thus a potential buyer, while an uninformed trader with a valuation parameter of $-\rho$ is a potential seller.\(^{15}\)

The trading rule is the same as in Glosten and Milgrom (1985): A trader will buy if $Z_t > A_t = \text{the ask price}$, and sell if $Z_t < B_t = \text{the bid price}$. If all $Z_t \in [B_t, A_t]$, then no trade occurs. Because the market makers are competitive and risk neutral they will post bid and ask prices such that:

$$A_t = \text{ask price} = E[\tilde{V}|H_t, \text{time } t \text{ arrival is a buyer}]$$

$$B_t = \text{bid price} = E[\tilde{V}|H_t, \text{time } t \text{ arrival is a seller}]$$

where $H_t$ is the market makers’ information set, just before the $t$th arrival. $H_t$ includes all past bid, ask and transaction prices, as well as any information that has been revealed prior to the start of when-issued trading. If an informed trader arrives at time $t$, the trader has an information set that includes $H_t$ and the trader’s signal of $s$. We assume that if an uninformed trader arrives at time $t$, the trader’s information set includes only the current posted bid

\(^{15}\)Another difference between our model and that of Glosten and Milgrom (1985) is that our valuation parameter is additive, rather than multiplicative. A multiplicative valuation parameter has the effect that liquidity increases when the spread decreases and/or when the expected value of the asset increases. This means that obtaining positive information through bookbuilding will always increase the liquidity of the when-issued market, but obtaining negative information may worsen the liquidity because it decreases the expected value. While this may be realistic, it adds a level of complexity to our analysis that is not central to our work. For this reason we choose an additive form for our valuation parameter. The effect of this additive form is that liquidity depends only on the absolute value of the spread.
and ask prices, $A_t$ and $B_t$. To simplify the analysis, we will assume that uninformed traders' expectations are formed in a very simple manner: $E^U[\tilde{V}|A_t, B_t] = (A_t + B_t)/2$.\footnote{Due to asymmetries resulting from our information structure, if uninformed traders knew as much as the market makers, their valuation would not be exactly the center point of the quotes. However, the simplification given here does not affect any of our qualitative results.}

**When-issued trading and the aggregation of dispersed information, $\tilde{d}$:** We now discuss how we extend the model of Glosten and Milgrom (1985) to allow for correlation between dispersed information about the value of IPO shares, $\tilde{d}$, and the arrival rates of liquidity buyers and sellers.

The probability that any given uninformed arrival is a buyer ($\rho_t = \rho$) is $\bar{\lambda}$, where

$$\bar{\lambda} = \frac{1}{2} \left( 1 + \frac{\tilde{d}}{\delta} \right). \tag{12}$$

The probability that any given uninformed arrival is a seller is $1 - \bar{\lambda}$. The prior distribution on $\bar{\lambda}$ is uniform on $[0, 1]$. This is a beta distribution with a prior mean of $\bar{\lambda}_0 = 1/2$. Each uninformed arrival is a random draw from a Bernoulli distribution with parameter $\bar{\lambda}$. If $t_u$ is the number of uninformed arrivals and $y_u$ is the number of uninformed buyers minus the number of uninformed sellers, then the updated mean value of $\bar{\lambda}$ is:\footnote{This expression for the mean is just a slight transformation of the standard equation for updating a beta distribution: $\bar{x}_t = a_t/(a_t + b_t)$, where $a_t = 1$ + the number of successful trials and $b_t = 1$ + the total number of trials – the number of successful trials. In our notation above, $t_u = \text{the total number of trials}$ and $y_u = \text{the number of successful trials minus the number of unsuccessful trials.}$}

$$\bar{\lambda}(t_u, y_u) = \frac{1 + (t_u + y_u)/2}{2 + t_u} \tag{13}$$

Putting together equations (12) and (13), the expected value of $\tilde{d}$, conditioned on $t_u$ uninformed arrivals is:

$$\tilde{d}(t_u, y_u) = \frac{y_u \delta}{2 + t_u} \tag{14}$$

### 5.2 Market failure in the opening of when-issued trading

Glosten and Milgrom (1985) pointed out that if there is asymmetric information in the market (causing an adverse selection risk), then the ask price will be strictly greater than the bid price. Thus, a necessary condition for uninformed traders to participate is $\rho > 0$. If uninformed traders do not participate, the market will breakdown because the market
makers will set the spread so wide that informed traders will also not participate. In our model there are two reasons, associated with the two sources of uncertainty, \( \tilde{s} \) and \( \tilde{d} \), why the market may breakdown. In what follows we will determine both necessary and sufficient conditions such that the when-issued market will open, and we will show how these conditions relate to the two sources of uncertainty, \( \tilde{s} \) and \( \tilde{d} \). We will then determine conditions such that problems of market failure in the when-issued market can be resolved if a direct mechanism (bookbuilding) is used to obtain information from some informed traders, and if this information is revealed to all traders. This analysis shows that bookbuilding can only resolve problems of market failure due to uncertainty about \( \tilde{s} \), since it is only information about \( \tilde{s} \) that underwriters obtain in bookbuilding.

**Conditions for when-issued trading to open:** We begin by assuming that no information is gathered through a direct mechanism. Thus, prior to the start of when-issued trading, the expected value of \( \tilde{s} \) is zero. The probability that an informed trader arrives equals \( \alpha \). In this event, it is just as likely that the trader is a seller, as it is that the trader is a buyer. If both uninformed and informed traders participate in when-issued trading, then the quotes at the open are given by:

\[
A_1 = v_0 + E[\tilde{s}|Z_1 > A_1]w + E[\tilde{d}|Z_1 > A_1] = v_0 + \alpha(2q - 1)w + \frac{(1 - \alpha)\delta}{3},
\]

\[
B_1 = v_0 + E[\tilde{s}|Z_1 < B_1]w + E[\tilde{d}|Z_1 < B_1] = v_0 - \alpha(2q - 1)w - \frac{(1 - \alpha)\delta}{3}.
\]

If uninformed traders do not participate in when-issued trading, then no such trading occurs. To see the reason for this classic result about market failure, note that the opening quotes would be \( A_1 = v_0 + (2q - 1)w \) and \( B_1 = v_0 - (2q - 1)w \). At these quotes no informed traders have any incentives to buy or sell since the quotes are set equal to their valuation of IPO shares. Hence, none of the traders participate in the market.

We will now derive conditions for when-issued trading to open. This analysis proceeds in three steps. First, we determine conditions that must be satisfied for uninformed traders to participate in when-issued trading, if informed traders also participate. Next, we derive a condition for informed traders to participate, given that uninformed traders participate.\(^\text{18}\)

\(^{18}\)In contrast to Glosten and Milgrom (1985), because of uncertainty about \( \tilde{d} \), informed traders may abstain from trading for
As our final step, we determine the general condition for this to happen.

We start with the market participation of the uninformed traders. An uninformed trader will be willing to buy at the open if \( v_0 + \rho > A_1 \) and sell if \( v_0 - \rho < B_1 \). Thus, a necessary and sufficient condition for uninformed traders of both types to participate at the open, if informed traders also participate, is:

\[
\rho > (A_1 - B_1)/2 \equiv S_1/2 = \alpha(2q - 1)w + (1 - \alpha)\delta/3. \tag{17}
\]

We next consider the market participation of the informed traders. At the open, a potential informed buyer will value the issue at \( v_0 + (2q - 1)w \); a potential informed seller will value the issue at \( v_0 - (2q - 1)w \). By examining equations (15) and (16) we can see that, if uninformed traders participate in trading at the open, then a necessary and sufficient condition for informed traders to also participate is: \( (2q - 1)w > \delta/3 \). If this condition does not hold, then only uninformed traders will participate at the open, so that \( A_1 = v_0 + \delta/3 \) and \( B_1 = v_0 - \delta/3 \). The market will open with only uninformed traders as long as \( \rho > \delta/3 \). If the market opens with only uninformed traders, then after some periods of trading, enough uncertainty about \( \tilde{d} \) will be resolved so that informed traders will also participate.

We can now state the condition for when-issued trading to open. The above-stated conditions for market participation of the two groups of traders imply that the when-issued market can open, without any prior information gathering, if the following inequality is satisfied:

\[
\rho > S_1/2 = (1 - \alpha)\delta/3 + \alpha \max \left[(2q - 1)w, \frac{\delta}{3}\right]. \tag{18}
\]

If this condition does not hold, then there is market breakdown: trading will not open in the when-issued market. We will next examine whether this problem can be resolved by using a direct mechanism (bookbuilding) to obtain information directly from some informed investors, and by revealing this information to all traders.

**Resolving problems of market failure:** As before, we let \( z \) denote the number of investors who report to the underwriter that they have observed positive signals minus the number some period of time after the market opens. The necessary requirement for the market to open is that uninformed traders are willing to participate at the open.
who report negative signals. We assume that investors who are polled in bookbuilding report truthfully and we analyze whether problems of market failure can be resolved by making the value of $z$ publicly available. (We will model the bookbuilding mechanism that achieves this truthtelling in Section 6.)

At the opening of the when-issued market, the spread $A_1 - B_1$ has two components. Part of the spread is due to uncertainty about dispersed information $\tilde{d}$, and part is due to asymmetric information about $\tilde{s}w$. If condition (18) fails to hold due to uncertainty about dispersed information (high value of $\delta$), then the when-issued market will not open even if all traders learn the realization of $\tilde{s}$, so that there is no asymmetric information. In this case, conducting bookbuilding will not enable when-issued trading. However, if condition (18) fails to hold due to asymmetric information about $\tilde{s}$ (high value of $(2q-1)w$), then the opening of when-issued trading may be enabled by gathering information through bookbuilding, and then publicizing the value of $z$.

The expected value of the IPO, conditioned on information learned in bookbuilding is:

$$E[\tilde{V}|z] = v_0 + (2\pi(z) - 1)w$$

The above expression is strictly increasing in $z$. If $z = 0$, then investors’ reports have effectively canceled each other out. We will say that bookbuilding is informative if $|z| \geq 1$.

If $z$ is strictly positive and both informed and uninformed traders participate in when-issued trading, then the opening quotes are given by:

$$A_1 \bigg|_{z \geq 1} = \alpha^+(z)E[\tilde{V}|z+1] + (1 - \alpha^+(z))E[\tilde{V}|z] + \frac{(1-\alpha^+(z))\delta}{3}$$

$$B_1 \bigg|_{z \geq 1} = \alpha^-(z)E[\tilde{V}|z-1] + (1 - \alpha^-(z))E[\tilde{V}|z] - \frac{(1-\alpha^-(z))\delta}{3}$$

where $\alpha^+(z)$ ($\alpha^-(z)$) is the probability that the first trader in the market is informed, given that a buyer (seller) has arrived at the open.\footnote{The expressions for $\alpha^+(z)$ and $\alpha^-(z)$ are derived in the proof of Proposition 2 in the Appendix where it is shown that for $z \geq 1$: $\alpha^+(z) = \alpha^-(-z) > \alpha > \alpha^-(z) = \alpha^+(-z)$.}

As demonstrated above, uninformed traders participate at the open if and only if:

$$\rho > (A_1 - B_1)/2 \equiv S_1/2.$$  

Thus, the parameter range in which the when-issued market will open is increased if the spread is narrowed ($S_1$ decreases). It is shown in the proof of
Proposition 2 that the problem is symmetric in that the opening spread is the same regardless of whether $z$ is positive or negative; it is merely necessary to replace $z$ with $|z|$. Thus, as long as something has been learned in bookbuilding, so that $|z| \geq 1$, the opening spread ($S_1 \equiv A_1 - B_1$) will be:

$$S_1 \bigg|_{|z| \geq 1} = \frac{(2 - \alpha^+ (z) - \alpha^- (z))\delta}{3} + \max \left[ R(q, z) w, \frac{(\alpha^+ (z) + \alpha^- (z))\delta}{3} \right], \quad (21)$$

where $R(q, z)$ is defined in the Appendix. As above, the maximum is taken because the spread equals $2\delta/3$ if when-issued trading opens with only the uninformed traders participating. It is shown in the proof of the following proposition that, as long as bookbuilding is informative ($|z| \geq 1$), the expression given in (21) is strictly less than the spread with no direct information gathering, as given in (17).

**Proposition 2. Opening of when-issued trading.**

1. **There exist parameter values such that when-issued trading will commence without any prior direct information gathering, and parameter values such that this cannot occur.**

2. **Direct information gathering, and public revelation of the information, increases the parameter range such that when-issued trading can commence.**

The second part of the above proposition claims a weak, rather than strict, increase in the parameter range such that when-issued trading can open. There are two reasons for this. First, gathering information by means of bookbuilding may fail ($z = 0$). Second, the when-issued market may fail to open due to uncertainty about information $\tilde{d}$ that is too dispersed to be gathered from investors by means of bookbuilding. In this case, condition (18) fails to hold due to uncertainty about $\tilde{d}$, rather than due to uncertainty about $\tilde{sw}$. Since bookbuilding can only mitigate the second kind of uncertainty, the when-issued market cannot open, regardless of the informativeness of the direct mechanism. Apart from these two cases, the opening spread will be strictly narrower if information is directly gathered from informed investors, and the information is publicly revealed prior to the opening of when-issued trading. As a consequence, there exist parameter values such that prior information gathering is necessary, and also with high probability sufficient, for the when-issued market to open.
The likelihood that prior information gathering will be both necessary and sufficient depends on the extent to which the opening spread can be reduced by such information gathering. Thus, it depends in part on the informativeness of bookbuilding as measured by \( \pi(z) = \text{prob}\{\tilde{s} = 1|z\} \). We show in the Appendix that \( \pi(z) \) moves away from one-half (uncertainty about \( \tilde{s} \) decreases) both as \(|z|\) increases and as \( q \), the quality of each private signal, increases. Moreover, as \( q \) increases, the probability that bookbuilding will be informative \((z \neq 0)\) increases. This is because, as \( q \) increases, the probability of disagreement between investors who participate in bookbuilding decreases. Not only do these two effects tend to raise the informativeness of bookbuilding, but as indicated by (18), as \( q \) increases, the likelihood that the when-issued market will open without prior information gathering decreases. We thus have the following Corollary to Proposition 2.

**Corollary 1.** Prior information gathering is both more necessary and more beneficial for enabling when-issued trading when the information held by informed investors is more accurate (higher value of \( q \)).

### 5.3 Market failure after when-issued trading opens

Up to this point we have examined only the opening of when-issued trading, and whether it is necessary to collect information in order to enable the opening. In what follows we examine whether the when-issued market breaks down after the opening.

Let \( y_t \) be the difference between the number of buy orders and the number of sell orders in the when-issued market, up to and including the trade at time \( t \). We had earlier defined \( \pi(z) \) as the probability that \( \tilde{s} = 1 \), given information from bookbuilding. We now define \( \pi_t(z, y_t) \) as the probability that \( \tilde{s} = 1 \), given also that \( t \) trades have taken place in the when-issued market, and \(-t \leq y_t \leq t\).\(^{20}\) The calculation of \( \pi_t(z, y_t) \) is complicated by the fact that the updating is nonlinear: \( \pi(z + 1) + \pi(z - 1) \neq 2\pi(z) \), unless \( z = 0 \). Thus, even though the actual path (to arrive at \( y_t \)) does not matter, both \( t \) (the number of trades) and \( y_t \) matter.

It is possible that the when-issued market breaks down after it opens. This can happen if information revealed through trading effectively negates information that was learned prior

\(^{20}\) If \( t \) is odd, then \( y_t \) can take on any odd value in this range. If \( t \) is even, then \( y_t \) can take on any even value in this range.
to the start of trading. For example, suppose that positive information has been reported in bookbuilding. Then, a high enough number of sell orders in the when-issued market will cause the market makers to question the correctness of the information learned in bookbuilding. As a result, the adverse selection risk faced by the market makers will be higher than at the open. If, due to adverse selection risk, informative bookbuilding was a prerequisite for when-issued trading to open, then an increase in such risk can cause the market to break down.

The severity of this problem, however, can be decreased by engaging in more extensive information gathering prior to the opening of the when-issued market. This is because the higher the value of $z$, the less likely it is that informed traders post sell orders. Similarly, the more negative the value of $z$, the less likely it is that informed traders post buy orders. Thus, the likelihood of market breakdown after the open is lower if more information is learned prior to the opening of when-issued trading.

**Proposition 3. Market breakdown after the open.** *The probability of market breakdown, due to adverse selection risk in the when-issued market, is decreasing in the quality of information learned in bookbuilding, as represented by $|z|$.*

If the when-issued market breaks down after trading begins, then underpricing due to adverse selection risk will be worse than without when-issued trading. Thus, there is no guarantee that when-issued trading will be beneficial. What the above proposition tells us is that, if the when-issued market can open only after some information has been publicly revealed, then the more information that is revealed, the less likely it is that when-issued trading will cause an increase in underpricing due to adverse selection risk.

### 6 Direct Mechanisms

In this section we model the process of bookbuilding as a direct mechanism for eliciting information from investors, possibly in exchange for (implicit) promises of allocations of underpriced IPO shares. As discussed in Section 3, when-issued trading of IPO shares may affect the cost of using a direct mechanism to induce investors to reveal their private
information. The reason for this is that these investors have incentives to conceal their information in order to trade on it. In the following subsections we will first model the bookbuilding process in the absence of when-issued trading, then we will model the process in the presence of when-issued trading.

6.1 Bookbuilding as a direct mechanism

To keep the model simple we will assume that only two investors participate in bookbuilding. These investors are referred to as the “polled investors”. There are thus three possible outcomes of bookbuilding: either both report positive information, both report negative information, or one reports positive information and the other negative. We will represent the outcome of bookbuilding with the pair \((a, b)\) \(\in \{(+, +), (+, -), (-, -)\}\). This representation corresponds to our earlier notation of \(z \in \{2, 0, -2\}\). We add the notation with “+” and “−” so as to be able to differentiate between investors who report different outcomes of their signals. As above, \(u_B\) represents the expected level of underpricing due to the bookbuilding process, with the expectation taken before any information has been gathered. We denote by \(u_{ab}\) the expected level of underpricing, conditioned on a given outcome \((a, b)\) of bookbuilding.

To maximize expected IPO proceeds, the underwriter must minimize the expected underpricing:\(^{21}\)

\[
\min u_B \equiv \left(\frac{q^2}{2} + \frac{(1-q)^2}{2}\right) u_{++} + 2q(1-q)u_{+-} + \left(\frac{q^2}{2} + \frac{(1-q)^2}{2}\right) u_{-+}
\]

where the weights on the \(u_{ab}\)'s are the probabilities of each outcome \((a, b)\) of bookbuilding. The objective function may also be written as follows:

\[
\min u_B \equiv Er^R + Er^+ + Er^-
\]

where \(Er^+ = \) expected return to a polled investor who sees and reports +

\[
= \left(\frac{q^2}{2} + \frac{(1-q)^2}{2}\right) u_{++} + 2q(1-q)u_{+-}
\]

\(Er^- = \) expected return to a polled investor who sees and reports −

\[
= \left(\frac{q^2}{2} + \frac{(1-q)^2}{2}\right) u_{-+} + 2q(1-q)u_{-+}
\]

\(Er^R = \) expected return to retail investors (nonpolled investors)

\(^{21}\)We are assuming that the issue size is exogenously given.
\[
\frac{q^2}{2} + \frac{(1 - q)^2}{2} u^{++}(1 - 2h^{++}) + \left(\frac{q^2}{2} + \frac{(1 - q)^2}{2}\right) u^{--}(1 - 2h^{--}) \\
+ 2q(1 - q)u^{+-}(1 - h^{+-} - h^{--})
\]

(25)

and where \(h^{ab}\) is the fraction of the offering that is allocated to a polled investor who reports \(a\) while the other reports \(b\). The objective function is minimized subject to a number of constraints that are stated below.

**Participation constraints:** We will assume that investors who participate in bookbuilding cannot be compelled to purchase overpriced shares. Thus, we have the following participation constraint:\(^{22}\)

\[u^{ab} \geq 0 \quad \forall u^{ab} \in \{u^{++}, u^{+-}, u^{--}\} \quad (PC - I)\]

**Incentive compatibility constraints:** To induce truthful revelation, the underwriter must take into account the amount that investors can gain from “lying”. We first derive the expected value of IPO shares, given the three possible realizations of bookbuilding, \(z \in \{2, 0, -2\} \):

\[
E[\tilde{V}|z = 2] = v_0 + \frac{(2q - 1)w}{q^2 + (1 - q)^2}, \quad E[\tilde{V}|z = 0] = v_0, \quad E[\tilde{V}|z = -2] = v_0 - \frac{(2q - 1)w}{q^2 + (1 - q)^2}.
\]

An investor who lies increases or decreases the perceived value of \(z\) by 2. An investor who sees positive information, but reports a negative signal thus causes the expected value of \(\tilde{V}\), conditioned on bookbuilding information, to be lower by an amount \(w_L\):

\[
w_L = \text{expected impact of lie} = \frac{(2q - 1)w}{q^2 + (1 - q)^2} \quad (26)
\]

Similarly, an investor who sees negative information and reports positive information will cause the issue price to be higher by an amount \(w_L\). Investors will truthfully report their information, as long as the following incentive compatibility constraints are satisfied:

\[
Er^+ \geq \left(\frac{q^2}{2} + \frac{(1 - q)^2}{2}\right) \left(u^{+-} + w_L\right) h^{++} + 2q(1 - q) \left(u^{--} + w_L\right) h^{--} \quad (IC^+)
\]

\[
Er^- \geq \left(\frac{q^2}{2} + \frac{(1 - q)^2}{2}\right) \left(u^{++} - w_L\right) h^{++} + 2q(1 - q) \left(u^{+-} - w_L\right) h^{+-} \quad (IC^-)
\]

\(^{22}\)This constraint means that the underwriter does not overprice in expected value, conditioned on the information that is learned in bookbuilding. Actual overpricing may occur, if there remains uncertainty about \(\tilde{V}\) after bookbuilding. We could also introduce an a priori strictly positive participation constraint: \((Er^+ + Er^-)/2 \geq \gamma > 0\), but adding this extra complexity will not affect our results. (Note: If \(\gamma = 0\), then the a priori constraint is subsumed by \((PC - I)\).

28
where $E_r^+$ and $E_r^-$ are as given in (23) and (24). The constraint \((IC^-)\) is written based on the notion that an investor cannot refuse an allocation, without revealing that she has lied. For this reason, providing false positive information is generally not beneficial to investors, and \((IC^-)\) will typically be nonbinding. This is one aspect of the mechanism design problem that may change below when we examine bookbuilding in the presence of when-issued trading.

**Allocation constraints:** We assume that the underwriter must allocate a fraction $h_R$ of the IPO to retail investors.

$$h^{ab} \geq 0 \quad h^{--}, h^{++} \leq \frac{1 - h_R}{2} \quad h^{-+} + h^{++} \leq 1 - h_R$$

**Underpricing due to residual adverse selection risk:** We assume that retail investors are willing to participate as long as they expect to break even. Because the underwriter cannot distinguish between informed and uninformed retail investors, the retail investors face a residual adverse selection risk. (The term “residual” is used because this is adverse selection risk that still remains after collecting information through bookbuilding.) Investors who participate in bookbuilding do not face an adverse selection risk, because their allocations are based only on the reported information, not on any information that may still reside with other investors. For this reason, the constraint \((PC - I)\), which requires only nonnegative underpricing, is sufficient to ensure the participation of the polled investors. For the retail investors to participate the following conditions must be satisfied:

$$u^{++} \geq u_{AS}(2), \quad u^{-+} \geq u_{AS}(0), \quad u^{--} \geq u_{AS}(-2), \quad (PC - R)$$

where $u_{AS}(z)$ (first introduced in Section 3) represents the underpricing needed due to residual adverse selection risk. This underpricing is calculated based on the number of positive reports in bookbuilding minus the number of negative reports = $z \in \{2, 0, -2\}$. Each constraint in \((PC - R)\) needs to be satisfied only if shares are allocated to retail investors in that state. If $h_R$ is strictly positive, so that some part of the IPO must always be allocated to retail investors, then all of the constraints in \((PC - R)\) must be satisfied.
The optimal direct mechanism: In structuring the mechanism design problem we have assumed that informed investors do not demand strictly positive expected returns in order to participate in the bookbuilding process. We do this so that we can focus on the incentive compatibility constraints. In particular, we want to focus on any effect that the presence of a when-issued market may have on investors’ incentives to truthfully reveal information. We begin by presenting the optimal direct mechanism, without when-issued trading.

As shown by Maksimovic and Pichler (2002), if the underwriter is able to gather all information, so that there is no residual adverse selection risk, then expected underpricing will be zero. In general, underpricing is needed to induce truth-telling only if there are allocation restrictions that require that shares be allocated to polled investors, or if the residual adverse selection risk is so high that it is optimal to allocate all shares to the polled investors, even when they report negative information. *In the standard mechanism design problem, without when-issued trading: If the optimal solution allows the underwriter to give no allocations to polled investors when they provide negative information, then underpricing is not needed in order to induce truth-telling.*  

In our model, we do not assume that the underwriter is required to always allocate shares to the polled investors. In addition, in the Appendix we show that, even though the underwriter cannot gather all available information, so that there is a strictly positive residual adverse selection risk, the optimal solution does call for the entire issue to be allocated to the retail market in the case that both polled investors report negative information. If the underwriter is not required to always allocate shares to the retail market \( h_R = 0 \), then the entire issue is allocated to polled investors who report positive information. In this case, positive expected underpricing after bookbuilding occurs only due to residual adverse selection risk and only after bad information is revealed:

\[
 u_B \bigg|_{h_R=0, \text{no when-issued trading}} = \left( \frac{q^2 + (1 - q)^2}{2} \right) u_{AS}(-2)
\]

An important aspect of this optimal mechanism is that *informed investors do not earn rents for their information.* Strictly positive underpricing occurs only in a state in which the

---

23 This result is an essential part of our analysis. We do not place it inside of a proposition, because it is not new to us. See Maksimovic and Pichler (2002).
polled investors receive no shares.

In contrast, polled investors do earn positive rents if there are allocation restrictions. If the underwriter is required to always allocate shares to the retail market \((h_R > 0)\), then underpricing due to residual adverse selection risk occurs in all states:

\[
\left. u_B \right|_{h_R > 0, \text{no when-issued trading}} = \left( \frac{q^2 + (1-q)^2}{2} \right) u_{AS}(-2) + 2q(1-q)u_{AS}(0) + \left( \frac{q^2 + (1-q)^2}{2} \right) u_{AS}(2) \tag{27}
\]

The optimal direct mechanism still calls for polled investors to receive the maximum possible allocations when they report positive information. Because of the need to underprice in all states, due to residual adverse selection risk, the polled investors earn strictly positive expected returns for reporting positive information. Thus, without when-issued trading, polled investors earn rents for their information only if there are allocation restrictions imposed on the design of the mechanism. We next extend the mechanism design problem to account for the effect of a when-issued market, and check if this result is changed.

### 6.2 Bookbuilding in the presence of when-issued trading

The optimal mechanism described above may fail to induce truthful reporting if informed investors can earn strictly positive expected profits by trading on their information, instead of reporting it to the underwriter. In examining the effect of when-issued trading on book-building, we will assume that the underwriter is faced with one allocation restriction: the underwriter must always allocate shares to the retail market \((h_R > 0)\).\(^{24}\) Thus, we will use the underpricing given in equation (27) as our base case for comparison for determining whether when-issued trading increases the cost of bookbuilding.

When-issued trading changes the mechanism design problem by affecting the incentive compatibility constraints in two ways: i) a polled investor who lies has insider trading opportunities;\(^{25}\) ii) a mistake in pricing may be corrected – i.e., \(w_L\) may be driven to zero.

\(^{24}\)By ignoring any other possible restrictions, such as a restriction to always allocate some shares to polled investors, we are biasing the results against when-issued trading. This is because by ignoring such possible constraints we make bookbuilding without when-issued trading appear less expensive than it might actually be.

\(^{25}\)Conceptually, such opportunities also exist in the secondary market, after the IPO has been priced and issued. However, there may be less competition to trade on such information in the when-issued market. It may also be much easier to sell short in the when-issued market than in the secondary market.
The significance of this second point is that lying by providing false positive information may no longer be costly to the investor. As a result, the incentive compatibility constraint for investors who have observed negative information, which was nonbinding without when-issued trading, may become binding. In addition, it may become necessary, in the presence of when-issued trading, for the underwriter to pay strictly positive rents to investors in order to induce truth-telling.

In what follows we will model the direct mechanism for two different cases. First, we consider the case in which when-issued trading can open without any prior information gathering. This is the case such that the condition (18) is satisfied. The significance of this case is that a lie on the part of a polled investor does not affect the opening of the when-issued market. In the second case, a necessary condition for when-issued trading to open is that bookbuilding elicits non-contradictory reports from the polled investors. In this case, a lie on the part of a polled investor will affect the probability that the when-issued market opens. That is, a lie will affect the probability that the investor can profit by trading on that lie.\footnote{Note, if we were to model the mechanism with a large number of polled investors, then a single investor would affect the opening of the when-issued market only if that investor had rather valuable and independent information. At the same time, in such a model the investor would make significant profits by trading in the when-issued market only if she had valuable and independent information. By modeling the mechanism with only two polled investors we capture this effect without an undue level of modeling complexity.}

6.2.1 The case without market failure of when-issued trading

The new incentive compatibility constraints will differ from \((IC^+)\) and \((IC^-)\) in two ways. First, the expected impact of a lie on the offer price will be lower because when-issued trading will reveal information that in expected value will counteract the effect of the lie. We will denote this lower impact by \(w_{LT}\), where \(w_{LT} < w_L\). Second, the right-hand side of each constraint will include the expected profit that informed investors can earn by trading on their information. We use the symbol \(\psi_{L}^{+b}\) to denote the expected trading profit for an investor who sees + but reports −, while the other polled investor reports \(b\), and \(\psi_{L}^{-b}\) as the expected trading profit for an investor who sees − but reports +, while the other polled
The incentive compatibility constraints are now:

\[\begin{align*}
Er^+ &\geq \left(q^2 + (1-q)^2\right) \left((u^{+-} + w_{LT})h^{+-} + \psi^{+-}_L\right) + 2q(1-q)\left((u^{--} + w_{LT})h^{--} + \psi^{--}_L\right) (IC^+_T) \\
Er^- &\geq \left(q^2 + (1-q)^2\right) \left((u^{+-} - w_{LT})h^{+-} + \psi^{+-}_L\right) + 2q(1-q)\left((u^{--} - w_{LT})h^{--} + \psi^{--}_L\right) (IC^-_T)
\end{align*}\]

where, as in \((IC^+)\) and \((IC^-)\), \(Er^+\) and \(Er^-\) are as given in (23) and (24).

**Underpricing due to residual adverse selection risk:** As discussed above, when-issued trading can mitigate informational asymmetries across investors. As a result, uninformed investors face less severe problems of adverse selection, and less underpricing is required to induce uninformed investors to buy IPO shares. If when-issued trading **fully** reveals all relevant information, then underpricing due to residual adverse selection risk will go to zero: \(u_{AS}(2), u_{AS}(0)\) and \(u_{AS}(-2) \rightarrow 0\).

**The optimal direct mechanism:** If expected insider trading profits are strictly positive, then the optimal mechanism described above, for bookbuilding without when-issued trading, will not satisfy the new incentive compatibility constraints. The precise form of the optimal mechanism depends on whether the underwriter is able to condition allocations on the outcome of when-issued trading.\(^{28}\) It is shown in the proof of Proposition 4 that if the underwriter cannot condition allocations on the results of when-issued trading, then the optimal mechanism will call for the underwriter to allocate nothing to the polled investors when they disagree with each other, but to give them underpriced allocations when they agree with each other.

The polled investors will receive underpriced allocations both when they agree on positive information and when they agree on negative information.\(^{29}\) If, instead, the allocations can be conditioned on the outcome of the when-issued market, then the optimal mechanism will call for the underwriter to allocate nothing to a polled investor whose report is later contradicted by information released by the when-issued market. Underpriced shares will be

\(^{27}\)The participation constraints will also change in that the right-hand-side will be strictly positive, instead of zero. \(Er^+\) and \(Er^-\) must both be at least as large as expected trading profits, given that the investor doesn’t participate. The new participation constraints will be satisfied as long as the new incentive compatibility constraints are satisfied.

\(^{28}\)If polled investors expect to receive commitments of allocations prior to the onset of when-issued trading, then this cannot be done.

\(^{29}\)Note, we do not consider the possibility of collusion on the part of polled investors. This is reasonable in that the underwriter can prevent collusion simply by not informing one polled investor of the identity of the other.
allocated to each polled investor who is not contradicted by when-issued trading, regardless of whether the report was positive or negative.

Regardless of whether allocations can, or cannot, be conditioned on the outcome of when-issued trading, the optimal mechanism exhibits two key characteristics that are different from the optimal mechanism without when-issued trading. First, the incentive compatibility constraints are strictly binding. That is, investors must receive strictly positive rents in order to induce them to truthfully report their signals. Second, these positive rents must be paid to investors who observe negative signals, as well as to those who observe positive signals. In fact, if the when-issued market is expected to fully reveal all information, then investors who truthfully report positive signals and investors who truthfully report negative signals receive identical expected returns. This is in contrast to the optimal mechanism without when-issued trading, in which those who reported negative signals received no IPO allocations, and thus no rents for their information. The solution to the mechanism design problem is thus very much changed due to the presence of the when-issued market. The following proposition summarizes these changes.

**Proposition 4. Bookbuilding with when-issued trading – Part I.** If the parameter values are such that informative bookbuilding is not a prerequisite for the opening of when-issued trading, then

1. The bookbuilding mechanism is qualitatively changed due to the possibility of polled investors profiting both from providing false negative and false positive reports.

2. The when-issued market can make bookbuilding more expensive.

The when-issued market will, in expected value, decrease expected underpricing due to residual adverse selection risk. It will also increase the underpricing that is needed in order to induce polled investors to truthfully report their signals. If the latter effect dominates, then when-issued trading will cause an increase in underpricing. That is, when-issued trading can increase the cost of doing bookbuilding. However, at this point we can only say that this possibility exists in the case that bookbuilding is not needed for the when-issued market to function. In what follows we will examine the effect that when-issued trading has on
bookbuilding, in the case that informative bookbuilding is a prerequisite for when-issued trading to open.

6.2.2 The case with market failure of when-issued trading

We now consider the case in which when-issued trading cannot open on its own. Market makers, and other uninformed traders, must first receive some additional information about the value of IPO shares, such as information that is reported by investors who participate in bookbuilding.\(^{30}\) As a consequence, when-issued trading cannot open unless bookbuilding elicits non-contradictory reports: \(z \neq 0\). We investigate below how this affects the incentives of the investors to truthfully report their signals.

**Incentive compatibility constraints:** An informed investor can expect to profit by trading on private information in the when-issued market, only if this market opens. If an informed investor, by refusing to truthfully reveal information, causes bookbuilding to be uninformative, then the market will not open. There is thus a direct link between truthful reporting and the likelihood that the when-issued market opens. This link changes the way in which we write the incentive compatibility constraints.

\[
Er^+ \geq \left( q^2 + (1 - q)^2 \right) (w_L + u^+ - h^- + 2q(1 - q))(w'_{LT} + u^- - h^- + \psi^+_L) \quad (IC_T^+) \\
Er^- \geq \left( q^2 + (1 - q)^2 \right) (u^+ - w_L)h^+ + 2q(1 - q)((u^+ - w'_{LT})h^+ + \psi^-_L) \quad (IC_T^-)
\]

These constraints are a cross between the constraints \((IC_T^+)\) and \((IC_T^-)\), and those without when-issued trading, \((IC^+)\) and \((IC^-)\). \(Er^+\) and \(Er^-\) are the same in all three sets of constraints. The differences are in the impact of a lie, and the expected trading profits after lying. If the polled investors report contradictory signals \((+-)\), then the market does not open. This market failure means that a lie is not revealed when the polled investors report different things. In this event, the impact of the lie on IPO pricing is given by \(w_L > w_{LT}\). If the when-issued market opens in spite of a lie, then some or all of the lie’s impact will be corrected. In expectation, the impact of a lie will be \(w'_{LT}\), for \(w_L > w'_{LT} \geq w_{LT}\) since

\(^{30}\)This information can be made public if the underwriter posts a price range within which he will price the IPO. As discussed above, if the underwriter commits not to price outside the range, then the posting of the range is not just cheap talk.
when-issued trading may break down after the opening. Finally, false reporting gives rise to profitable trading opportunities only in the case that a false report results in reported agreement between the polled investors.

Trading profits of investors who manipulate the outcome of bookbuilding: Before we can characterize the optimal direct mechanism, we must first analyze the expected profit of an investor who trades in the when-issued market after manipulating the outcome of bookbuilding by false reporting. A polled investor who sees a negative signal (−), but reports +, makes no insider profits if the other polled investor reports −. This is because the when-issued market does not open (z = 0). If the other polled investor reports + and the liar sells at the open, then she earns expected trading profits of \((B_1(++) - v_0)\eta\), where \(v_0\) is her expected value of the IPO and \(B_1(++)\) is the market makers’ opening bid price, given that two positive signals were reported in bookbuilding. \(\eta\) is the size of a unit trade (consistent with our model of when-issued trading, presented in Section 5), relative to the IPO issue size: 0 < \(\eta\) < < 1. An investor who sees + and reports −, makes no insider profits if the other polled investor reports +. If the other polled investor reports − and the liar buys at the open, then she earns insider trading profits of \((v_0 - A_1(--)\eta\), where \(A_1(--)\) is the market makers’ opening ask price, given that two negative signals were reported in bookbuilding.

It is shown in the proof of Lemma 2 in the Appendix that \((B_1(++) - v_0) = (v_0 - A_1(--)\eta\) and that the profit at the open is bounded:

\[
(B_1(++) - v_0)\eta \leq \left(1 - 2q(1 - q)\alpha(2)\right) w_L \eta \equiv \psi^w_0 \eta \tag{28}
\]

Furthermore, this bound is decreasing in \(\alpha\) and increasing in \(q\).

Lemma 2. The upper bound on the expected informed profit from trading at the open, as given in equation (28), is

i) decreasing in \(\alpha\), and

ii) if there are two polled investors, increasing in \(q\).

The bound, \(\psi^w_0 \eta\), is increasing in \(q\), largely because the impact on expected share value from a lie, \(w_L\), is increasing in \(q\).\footnote{This is true with only 2 polled investors. If there are more than 2 polled investors, then a single disagreeing investor has }
issued market from opening is also increasing in \( q \). Thus, we cannot say that in general the expected profit from lying is increasing in \( q \).

An insider may trade later than at the open, and may trade more than once. However, we expect that the upper bound on the insider’s total expected insider trading profit is directly related to \( \psi_0^w \). As a simplification, we will model the upper bound on the total expected insider trading profits as some factor times the expression in equation (28):

\[
\psi_L^- \leq x^- \psi_0^w \quad \text{and} \quad \psi_L^+ \leq x^+ \psi_0^w
\]

If the investor could trade the entire issue at the opening quote, then the factor \( x^a \) would equal one. Instead, \( x^a \) is clearly less than one. The earlier and more often that a liar expects to be able to trade, the higher is \( x^a \). Thus, \( x^a \) is decreasing in \( \alpha \). As such, both \( x^- \psi_0^w \) and \( x^+ \psi_0^w \) are decreasing in \( \alpha \) and increasing in \( q \).

**The optimal direct mechanism:** The optimal mechanism has characteristics of each of the previous two mechanisms: the optimal mechanism without when-issued trading and the optimal mechanism in the presence of when-issued trading that can open without informative bookbuilding. For example, suppose that the two polled investors report different signals. Then, when-issued trading cannot open and the underwriter optimally follows an allocation policy that is the same as in the case without such trading. The investor who has reported a positive signal receives the maximum possible allocation; the other investor receives no allocation. If the two polled investors report identical signals, then their allocations will depend on the parameters of the model. For many parameter values, investors will receive larger allocations when they report positive signals rather than negative signals, as in the mechanism without when-issued trading. For some parameter values, polled investors will receive no rents when they agree with each other.

An important parameter is \( \alpha \), the fraction of potential investors who hold private information. An increase in \( \alpha \) has two effects on the incentive compatibility constraints. First, the expected trading profits for a polled investor who lies will be lower, both because of greater competition to trade on information, and because the expression in equation (28)
will be lower. Second, the cost of underpricing due to adverse selection risk when no information is learned, $u_{AS}(0)$ will be higher. If $\alpha$ is large enough so that $\psi_L^+ - \psi_L^-$ are small relative to $u_{AS}(0)$, then incentive compatibility can be satisfied by paying rents to a polled investor only in the case that she reports positive information while the other polled investor reports negative information. As in the case with no when-issued trading (equation (27)), underpricing will be determined entirely by residual adverse selection risk, not by the need to induce truth telling. The only difference is that, as long as the when-issued market does not break down, residual adverse selection risk will be lower than without when-issued trading. It is possible that the when-issued market can break down due to wrong information having been observed by both polled investors, and then subsequently contradicted by the when-issued market. Thus, ex post it is possible that the when-issued market will make underpricing greater. However, if $\alpha$ is not low, then the a priori expected value of underpricing due to adverse selection risk, with a when-issued market, is less than the expected value of underpricing due to adverse selection risk without a when-issued market. In addition, if the when-issued market breaks down due to wrong information being learned in bookbuilding, then a potentially costly over- or underpricing mistake has been avoided. We thus have the following result:

**Lemma 3.** If informative bookbuilding is a prerequisite for when-issued trading to open, and if $\alpha$ (the fraction of potential investors who hold private information) is large enough so that $(1 - h_R) \times u_{AS}(0) \geq \psi_L^+$, then

i) as is the case with no when-issued trading, underpricing is determined entirely by residual adverse selection risk, not by the need to induce truth telling.

ii) when-issued trading is beneficial for the issuer.

If instead, the fraction of potential investors who hold private information, $\alpha$, is small, then the presence of when-issued trading can make bookbuilding more expensive. The reason is twofold. First, expected underpricing without when-issued trading is not very high, because of the low residual adverse selection risk. Second, expected informed trading profits are higher with smaller $\alpha$, due to the lower competition. However, as was shown in Section 5 (expression (18)), a smaller value of $\alpha$ also means that bookbuilding is less likely to be needed
in order for the when-issued market to function. If α is small, then informative bookbuilding is a prerequisite for when-issued trading to open only for some very special cases. The following proposition summarizes our results in the case that informative bookbuilding is a prerequisite for when-issued trading to open.

**Proposition 5. Bookbuilding with when-issued trading – Part II.** If the parameter values are such that informative bookbuilding is a prerequisite for the opening of when-issued trading, then

1. As in bookbuilding with no when-issued trading, it is easier to induce truthtelling from investors who have observed negative information than from investors who have observed positive information.

2. If there are many informed investors (the fraction of potential investors who hold private information is not too small), then the when-issued market does not make bookbuilding more expensive.

3. The existence of a when-issued market can make bookbuilding more expensive if one of the following two conditions holds:

   (a) There are few informed investors and the when-issued market is also very illiquid in the sense that liquidity traders have very low incentives to participate.

   (b) There are few informed investors and these investors have very valuable information.

As shown earlier, if bookbuilding is not needed for the when-issued market to function, then this market may increase the cost of gathering information through bookbuilding. But, in this case it is not necessary to gather information through bookbuilding. If bookbuilding is needed for the when-issued market to function, this will be for one of two reasons. Either there is a large adverse selection risk without prior information gathering, or the liquidity traders do not have very strong incentives to trade (ρ is small), or both of these occur. A large adverse selection risk occurs either because there is a large number of informed investors (α large), or because the information held by these investors has a large potential impact on the IPO value (w large), or both. We have shown that if bookbuilding is needed because of a large number of informed investors, then the presence of the when-issued market does not increase the cost of bookbuilding. Thus, it is only when the when-issued market
is very illiquid (in the sense that liquidity traders do not want to trade), or when there is a very small number of informed investors with very good information, that the existence of a when-issued market can increase the cost of bookbuilding!

As already described in this paper, the when-issued market has very clear positive benefits.\textsuperscript{32} For this reason, we cannot say that allowing for a when-issued market is detrimental to the issuer, even if the two conditions described here are satisfied. However, we can say that a necessary condition for the when-issued market to be detrimental is that either the market is very illiquid, or there is only a very small number of very well informed investors. Thus, in the case that information is broadly held and that the when-issued market is able to attract broad participation, it appears that when-issued trading is strictly beneficial to the issuer.

\textsuperscript{32}For example, if there is a significant cost to overpricing, then the ability of the when-issued market to correct a mistake is valuable.
7 Appendix

Notation:
Random variables:
\[ \tilde{V} = \text{secondary market value (post IPO)} \in \{v_0 + w + \tilde{d}, v_0 - w + \tilde{d} \} \]
\[ \tilde{d} \sim U[-\delta, \delta] \]
\[ \tilde{s} \equiv \frac{Y - v_0 - \tilde{d}}{w} \in \{1, -1\} \]
\[ \tilde{\varsigma}_i = \text{informed trader } i\text{'s signal of } \tilde{s} \text{.} \]

Exogenous parameters:  (The exogenous parameters are all common knowledge)
\[ v_0 = \text{prior expected value of } V \]
\[ w = \text{constant (See above for } \tilde{V} \) \]
\[ \delta = \text{bound on the distribution of } \tilde{d} \]
\[ q = \text{probability that } \varsigma_i = s, \forall i; \text{ with probability } 1 - q, \varsigma_i = -s \]
\[ \alpha = \text{fraction of traders who are informed.} \]
\[ c = \text{cost parameter, for cost of overpricing} \]
\[ \rho = \text{valuation parameter for liquidity traders } \geq 0; \text{ buyers have parameter } \rho_t = \rho; \]
\[ \text{sellers have parameter } \rho_t = -\rho \]
\[ \eta = \text{size of a single trade unit, relative to the total IPO issue size} \]

Other variables:
\[ p_I = \text{IPO offer price} \]
\[ z = \text{sum of reported signals by investors polled in bookbuilding} \]
\[ = \text{number of positive reports } - \text{number of negative reports} \]
\[ \pi(z) = \text{probability that } s = 1, \text{ given } z \]
\[ A_t = \text{market makers’ ask price at time } t \]
\[ B_t = \text{market makers’ bid price at time } t \]
\[ S_t = A_t - B_t = \text{market makers’ time } t \text{ quoted spread} \]
\[ y_t = \text{number of buyers } - \text{number sellers, up to and including time } t \text{ trade} \]
\[ \pi_t(z, y_t) = \text{probability that } s = 1, \text{ given } z, t \text{ and } y_t \]

Bookbuilding variables:
\[ h_R = \text{minimum fraction of the offering that must be allocated to retail investors} \]
\[ \gamma = \text{participation cost for each polled investor} \]
\[ u^{ab} = \text{expected underpricing when one polled investor reports } a \text{ and the other reports } b \]
\[ h^{ab} = \text{fraction of offering allocated to polled investor who reports } a \text{ when other reports } b \]
\[ u^{ab}_{\text{AS}} = \text{expected underpricing due to residual adverse selection risk after one polled investor reports } a \text{ and the other } b \]
7.1 Derivations for Section 3

Derivation of $u_M$, given uncertainty about $\tilde{s}$: We derive here the optimal issue price $p_I$, given the objective function (2) and ignoring the constraint (3). $\pi(z)$ is the probability that $\tilde{s} = 1$, given $z$. We assume here that $0 < \pi(z) < 1$, so that uncertainty about $\tilde{s}$ affects the expected cost due to overpricing. If $\delta \geq w$:

$$E[p_I - \hat{V}|p_I > \hat{V}, z] \, \text{prob}\{p_I > \hat{V}|z\} =$$

$$= \begin{cases} 
\frac{\pi(z)}{4\delta} (p_I - v_0 - w + \delta)^2 + \frac{1 - \pi(z)}{4\delta} (p_I - v_0 + w + \delta)^2 & \text{if } v_0 + w - \delta < p_I < v_0 - w + \delta, \\
\frac{\pi(z)}{4\delta} (p_I - v_0 - w + \delta)^2 + (1 - \pi(z))(p_I - v_0 + w) & \text{if } v_0 + w - \delta \leq v_0 - w + \delta \leq p_I, \\
\frac{1 - \pi(z)}{4\delta} (p_I - v_0 + w + \delta)^2 & \text{if } p_I \leq v_0 + w - \delta \leq v_0 - w + \delta.
\end{cases} (30)$$

First case:

$$\frac{\partial \Pi}{\partial p_I} = 1 - \frac{c\pi(z)}{2\delta} (p_I - v_0 - w + \delta) - \frac{c(1 - \pi(z))}{2\delta} (p_I - v_0 + w + \delta)$$

$$p_I = \frac{2\delta}{c} + v_0 - \delta + (2\pi(z) - 1)w$$

The first case holds if $(1 - \pi(z))w < \delta/c \leq \delta - \pi(z)w$. Second case:

$$\frac{\partial \Pi}{\partial p_I} = 1 - \frac{c\pi(z)}{2\delta} (p_I - v_0 - w + \delta) - c(1 - \pi(z))$$

$$p_I = \frac{2\delta}{c} \frac{(1 - c(1 - \pi(z)))}{c\pi(z)} + v_0 - \delta + w$$

The second case holds if $\delta/c \geq \delta - \pi(z)w$. Third case:

$$\frac{\partial \Pi}{\partial p_I} = 1 - \frac{c(1 - \pi(z))}{2\delta} (p_I - v_0 + w + \delta)$$

$$p_I = \frac{2\delta}{c(1 - \pi(z))} + v_0 - \delta - w$$

The third case holds if $\delta/c \leq (1 - \pi(z))w$. Thus, if $\delta \geq w$:

$$p_I = \begin{cases} 
\frac{2\delta}{c} + v_0 - \delta + (2\pi(z) - 1)w & \text{if } (1 - \pi(z))w < \delta/c < \delta - \pi(z)w, \\
\frac{2\delta}{c} \frac{(1 - c(1 - \pi(z)))}{c\pi(z)} + v_0 - \delta + w & \text{if } \delta/c \geq \delta - \pi(z)w, \\
\frac{2\delta}{c(1 - \pi(z))} + v_0 - \delta - w & \text{if } \delta/c \leq (1 - \pi(z))w.
\end{cases}$$

$$E[\hat{V}|z] - \delta \left(1 - \frac{2}{c}\right)$$

$$= \begin{cases} 
E[\hat{V}|z] + 2(1 - \pi(z))w - \delta \left(1 - \frac{2(1 - c(1 - \pi(z)))}{c\pi(z)}\right) & \text{if } \delta/c \geq \delta - \pi(z)w, \\
E[\hat{V}|z] - 2(1 - \pi(z))w - \delta \left(1 - \frac{2}{c(1 - \pi(z))}\right) & \text{if } \delta/c \leq (1 - \pi(z))w.
\end{cases} (31)$$
If $\delta < w$:

$$E[p_I - V|p_I > V] \text{prob}\{p_I > V\} =
\begin{cases}
(1 - \pi(z))(p_I - v_0 + w) & \text{if } v_0 - w + \delta < p_I < v_0 + w - \delta, \\
\frac{\pi(z)}{4\delta} (p_I - v_0 - w + \delta)^2 + (1 - \pi(z))(p_I - v_0 + w) & \text{if } v_0 - w + \delta \leq v_0 + w - \delta \leq p_I, \\
\frac{1 - \pi(z)}{4\delta} (p_I - v_0 + w + \delta)^2 & \text{if } p_I \leq v_0 - w + \delta \leq v_0 + w - \delta.
\end{cases}$$

(32)

$$p_I =
\begin{cases}
v_0 + w - \delta & \text{if } v_0 - w + \delta < p_I < v_0 + w - \delta, \\
\frac{2\delta(1-c(1-\pi(z)))}{c(1-\pi(z))} + v_0 - \delta + w & \text{if } v_0 - w + \delta \leq v_0 + w - \delta \leq p_I, \\
\frac{2\delta}{c(1-\pi(z))} + v_0 - \delta - w & \text{if } p_I \leq v_0 - w + \delta \leq v_0 + w - \delta.
\end{cases}$$

(33)

The first case above is just the lower bound of the second case. Thus, if $\delta < w$:

$$p_I =
\begin{cases}
\frac{2\delta(1-c(1-\pi(z)))}{c(1-\pi(z))} + v_0 - \delta + w & \text{if } 1 \geq c(1-\pi(z)), \\
\frac{2\delta}{c(1-\pi(z))} + v_0 - \delta - w & \text{if } 1 < c(1-\pi(z)).
\end{cases}$$

(34)

Putting everything together, the expected underpricing due to the cost of an overpricing mistake is is given by:

$$u_M =
\begin{cases}
\delta \left(1 - \frac{2}{c}\right) & \text{if } w - \pi(z)w < \delta/c < \delta - \pi(z)w, \\
\delta \left(1 - \frac{2(1-c(1-\pi(z)))}{c(1-\pi(z))}\right) - 2(1 - \pi(z))w & \text{if } \delta \geq w \text{ & } \delta/c \geq \delta - \pi(z)w, \text{ or} \\
\delta \left(1 - \frac{2}{c(1-\pi(z))}\right) + 2(1 - \pi(z))w & \text{if } \delta \leq w \text{ & } 1 \geq c(1-\pi(z))
\end{cases}$$

(35)

### 7.2 Derivations and proofs for Section 4

**Derivation of $\pi(z)$:**

$$\pi(z) = \frac{\text{prob}\{\xi_i|s = 1\} \text{prob}\{s = 1|z_{b\sim i} = z - \xi_i\}}{\text{prob}\{\xi_i|s = 1\} \text{prob}\{s = 1|z_{b\sim i} = z - \xi_i\} + \text{prob}\{\xi_i|s = -1\} \text{prob}\{s = -1|z_{b\sim i} = z - \xi_i\}}$$

$$= \frac{\text{prob}\{\xi_i|s = 1\} \pi(z - \xi_i)}{\text{prob}\{\xi_i|s = 1\} \pi(z - \xi_i) + \text{prob}\{\xi_i|s = -1\}(1 - \pi(z - \xi_i))}$$

(36)

If $\xi_i = 1$:

$$\pi(z) = \frac{q \pi(z - 1)}{q \pi(z - 1) + (1 - q)(1 - \pi(z - 1))}$$

(37)
If \( \varsigma_i = -1 \):

\[
\pi(z) = \frac{(1 - q)\pi(z + 1)}{(1 - q)\pi(z + 1) + q(1 - \pi(z + 1))}
\]

\( \pi(0) = 1/2 \quad \pi(1) = q \quad \pi(-1) = 1 - q = 1 - \pi(1) \)

\( \pi(2) = \frac{q^2}{q^2 + (1 - q)^2} \quad \pi(-2) = 1 - \pi(2) \)

Repeating the above we obtain:

\[
\pi(z) \bigg|_{z \geq 0} = \frac{q^z}{q^z + (1 - q)^z}
\]

\[
\pi(z) \bigg|_{z \leq 0} = \frac{(1 - q)^{|z|}}{q^{|z|} + (1 - q)^{|z|}}
\]

For \( z \geq 0 \), \( \pi(z) \) is increasing in \( z \):

\[
\frac{\partial \pi(z)}{\partial z} = \frac{\ln(q)q^z}{q^z + (1 - q)^z} - \frac{q^z(\ln(q)q^z + \ln(1 - q)(1 - q)^z)}{(q^z + (1 - q)^z)^2} = \frac{q^z(\ln(q) - \ln(1 - q))(1 - q)^z}{(q^z + (1 - q)^z)^2} > 0
\]

The above is positive because \( q > 1 - q \). Similarly, for \( z \leq 0 \), \( \pi(z) \) is decreasing in \( |z| \).

**Proof of Lemma 1.** As a simplification, we write \( \pi_T \) as \( \pi \) in this proof.

\[
\frac{u_{AS}}{w\alpha} = \frac{q - 2\pi(1 - \pi) - (2\pi - 1)^2 \alpha}{1 - ((2\pi - 1)q + 1 - \pi)}
\]

\[
\frac{\partial u_{AS}}{\partial \pi} = \frac{2(2q - 1)(1 - 2\pi)(1 - (1 - q)\alpha - (2q - 1)\pi\alpha) + (2q - 1)\alpha(q - 2\pi(1 - \pi) - (2\pi - 1)^2 q)}{(1 - ((2\pi - 1)q + 1 - \pi)\alpha)^2}
\]

This has the same sign as

\[
2(1 - 2\pi) - 2(1 - 2\pi)(1 - q + (2q - 1)\pi)\alpha + (q - 2\pi(1 - \pi) - (2\pi - 1)^2 q)\alpha = 2(1 - 2\pi) - 2(1 - \pi)^2 \alpha + (1 - 2\pi)^2 q\alpha + q\alpha
\]

If \( \pi = 1/2 \), then (43) is strictly positive. (43) is strictly negative if \( \pi = 1/2 + \alpha(q - 1/2) \).

Also, (43) is strictly decreasing in \( \pi \), so \( \partial u_{AS}/\partial \pi \) is positive \( \forall \pi \leq 1/2 \) and negative \( \forall \pi \geq 1/2 + \alpha(q - 1/2) \).

\[
\frac{u_{AS}}{w\alpha}(\pi = 1/2) = \frac{2q - 1}{2 - \alpha} > \frac{u_{AS}}{w\alpha}(\pi = 1/2 + \alpha(q - 1/2)) = \frac{(2q - 1)(1 - \alpha^2(2q - 1)^2)}{2 - \alpha - \alpha^2(2q - 1)^2}
\]

Also, if \( z = 1 \), then \( \pi(z) = q > 1/2 + \alpha(q - 1/2) \).
Proof of Proposition 1. This follows from the first part of Lemma 1 and:

i) If $\pi_0 = 1/2$, then $|\pi_1 - 1/2| \geq \alpha(q - 1/2)$. (As is shown in Section 5, equality holds if $\delta = 0$ and strict inequality holds if $\delta > 0$.)

ii) $\text{prob}\{z \times y_T < 0\} < \text{prob}\{z \times y_T > 0\}$. If $|\pi - 1/2| > 0$, then $|\pi - 1/2|$ is expected to increase due to when-issued trading.

7.3 Derivations and proofs for Section 5

Proof of Proposition 2. The first part of the proposition has already been proved. For the second part, we need to show that for any $|z| \geq 1$, the opening spread is strictly smaller than with $z = 0$.

$\alpha^+(z) =$ probability of an informed arrival at the open, given $z$ and given that a buyer has arrived.

$\alpha^-(z) =$ probability of an informed arrival at the open, given $z$ and given that a seller has arrived.

$$\frac{\alpha^+(z)}{\alpha} = \frac{q^{z+1} + (1 - q)^{z+1}}{(1 - \alpha)(q^z + (1 - q)^z)/2 + \alpha(q^{z+1} + (1 - q)^{z+1})} > 1$$

$$\frac{\alpha^-(z)}{\alpha} = \frac{q(1 - q)(q^{z-1} + (1 - q)^{z-1})}{(1 - \alpha)(q^z + (1 - q)^z)/2 + \alpha q(1 - q)(q^{z-1} + (1 - q)^{z-1})} < 1$$

(The inequalities above come from the fact that $q > 1/2$. Thus, $q^{z+1} + (1 - q)^{z+1} > q^z/2 + (1 - q)^z/2 > q^z(1 - q) + q(1 - q)^z$.) Applying equation (41), it is seen that for $z \leq -1$, $\alpha^+$ ($\alpha^-$) is the same as $\alpha^- (\alpha^+)$ above, but with $z$ replaced by $|z|$. Thus, for $z \geq 1$:

$$\alpha^+(z) = \alpha^-(z) > \alpha > \alpha^-(z) = \alpha^+(z)$$

We thus present the details only for positive $z$. If $z \geq 1$:

$$E[V|z] = v_0 + (2\pi(z) - 1)w = v_0 + \frac{q^z - (1 - q)^z}{q^z + (1 - q)^z}w.$$  

If both informed and uninformed traders participate in when-issued trading, the opening quotes are given by:

$$A_1|_{z \geq 1} = v_0 + (2\pi(z + 1) - 1)w\alpha^+(z) + (2\pi(z) - 1)w(1 - \alpha^+(z)) + \frac{(1 - \alpha^+(z))\delta}{3}$$

$$= v_0 + \frac{q^{(z+1)} - (1 - q)^{(z+1)}}{q^{(z+1)} + (1 - q)^{(z+1)}}w\alpha^+(z) + \frac{q^z - (1 - q)^z}{q^z + (1 - q)^z}w(1 - \alpha^+(z)) + \frac{(1 - \alpha^+(z))\delta}{3}$$  (44)
\[ B_1 \bigg|_{z \geq 1} = v_0 + (2\pi(z - 1) - 1)w\alpha^-(z) + (2\pi(z - 1))w(1 - \alpha^-(z)) - \frac{(1 - \alpha^-(z))\delta}{3} \]
\[ = v_0 + \frac{q^{(z-1)}}{q^{(z-1)}} - (1 - q)^{(z-1)}w\alpha^-(z) + \frac{q^z - (1 - q)^z}{q^z + (1 - q)^z}w(1 - \alpha^-(z)) - \frac{(1 - \alpha^-(z))\delta}{3} \] (45)

The problem is symmetric in that the opening spread is the same regardless of whether \( z \) is positive; it is merely necessary to replace \( z \) with \( |z| \). Thus, the opening spread is:

\[ S_1 \bigg|_{|z| \geq 1} = (A_1 - B_1) \bigg|_{|z| \geq 1} = \frac{(2 - \alpha^+(z) - \alpha^-(z))\delta}{3} + \max \left[ R(q, z)w, \frac{\alpha^+(z) + \alpha^-(z)\delta}{3} \right] \] (46)

where

\[ R(q, z) \equiv \alpha^+(z)\frac{q^{(|z|+1)}}{q^{(|z|+1)}} - (1 - q)^{(|z|+1)} - \alpha^-(z)\frac{q^{(|z|-1)}}{q^{(|z|-1)}} - (1 - q)^{(|z|-1)} - (\alpha^+(z) - \alpha^-(z))\frac{q^{(|z|)}}{q^{(|z|)}} - (1 - q)^{(|z|)} \]

Let \( \Delta\alpha(z) \equiv 2\alpha - \alpha^+(z) - \alpha^-(z) \geq 0 \) and \( \Delta R(z) \equiv 2\alpha(2q - 1) - R(z) \geq 0 \). Thus

\[ S_1 = \frac{2(1 - \alpha)\delta}{3} + \frac{\Delta\alpha(z)\delta}{3} + \max \left[ 2\alpha(2q - 1)w - \Delta R(z)w, \frac{2\alpha\delta}{3} - \frac{\Delta\alpha(z)\delta}{3} \right] \]

If \( \delta/3 \geq (2q - 1)w \), then the spread is determined entirely by uncertainty about demand and bookbuilding will not tighten the opening spread. Otherwise, a sufficient condition for bookbuilding to tighten the opening spread is: \( \Delta R(z)w - \frac{\Delta\alpha(z)\delta}{3} > 0 \). Or, because \( \delta/3 < (2q - 1)w \), a sufficient condition is:

\[ X(z) \equiv \Delta R(z) - (2q - 1)\Delta\alpha(z) > 0 \] (47)

We can rewrite \( X(z) \) as:

\[ X(z) = \alpha^+(z)\left(2q - 1 - (2\pi(z + 1) - 1) + (2\pi(z - 1)) \right) + \alpha^-(z)\left(2q - 1 - (2\pi(z) - 1) + (2\pi(z - 1)) \right) \]
\[ = \alpha^+(z)\left(2q - 1 - 2(\pi(z + 1) - \pi(z)) \right) + \alpha^-(z)\left(2q - 1 - 2(\pi(z) - \pi(z - 1)) \right) \]

It is easy to show that the above is strictly positive when \( z = 1 \). Using (42) and the fact that \( q > 1/2 \), we see that \( (\pi(z) - \pi(z - 1)) \) is decreasing in \( z \), \( \forall z \geq 1 \). Thus, condition (47) is satisfied \( \forall |z| \geq 1 \).

**Proof of Proposition 3.** We only present results where \( z \geq 0 \). Because of symmetry, the results will carry through to \( z \leq 0 \). We have already shown that \( \partial\pi(z)/\partial z > 0 \) for \( z \geq 0 \). Also, the probability that the next arrival is a buyer is increasing in \( \pi(z) \), because \( q > 1/2 \). Thus, this probability is increasing \( z \).
In addition, \(X(z)\) (defined in the proof of Proposition 2) is increasing in \(z\). Thus, the opening spread is decreasing in \(z\). Thus, there must exist some \(g^* \geq 0\) such that the market will open (or stay open) iff \(|\pi_t(z, y_t) - 1/2| \geq g^*. If \(\rho\) is large enough, then \(g^*\) will be zero and there will be no concern about market breakdown. Suppose instead that \(g^*\) is strictly positive. Then, even if \(|z|\) is large enough for the when-issued market to open, there is a strictly positive probability of market breakdown at some time in the future. This probability is strictly decreasing in \(z\), for two reasons: i) For larger \(z\), more sell orders must arrive for \(|\pi_t(z, y_t) - 1/2|\) to fall below \(g^*\); ii) the probability that a buyer (instead of a seller) arrives at any time in the when-issued market is strictly increasing in \(z\).

### 7.4 Derivations and proofs for Section 6

**Underpricing due to residual adverse selection risk, without when-issued trading.** From equations (11) and (10):

\[
\begin{align*}
\frac{u_{AS}(0)}{w\alpha} &= \frac{2q - 1}{2 - \alpha} \\
\frac{u_{AS}(z)}{w\alpha} &= \frac{q - 2\pi(z)(1 - \pi(z)) - (2\pi(z) - 1)^2q}{1 - (2\pi(z) - 1)q + 1 - \pi(z)\alpha} \\
\pi(2) &= \frac{q^2}{q^2 + (1 - q)^2} = 1 - \pi(-2) \\
\frac{u_{AS}(2)}{w\alpha} &= \frac{q - \frac{2q^2(1-q)^2}{(q^2+(1-q)^2)^2} - \left(\frac{2q-1}{q^2+(1-q)^2}\right)^2q}{1 - \left(\frac{2q-1}{q^2+(1-q)^2}\right)q\alpha - \left(\frac{1-q}{q^2+(1-q)^2}\right)\alpha} \\
&= \frac{(q^2 + (1-q)^2)(q^2 + (1-q)^2 - \alpha(1-3q(1-q)))}{(2q-1)2q^2(1-q)^2} \\
\frac{u_{AS}(-2)}{w\alpha} &= \frac{q - \frac{2q^2(1-q)^2}{(q^2+(1-q)^2)^2} - \left(\frac{q^2-(1-q)^2}{q^2+(1-q)^2}\right)^2q}{1 + \left(\frac{2q-1}{q^2+(1-q)^2}\right)q\alpha - \left(\frac{q^2}{q^2+(1-q)^2}\right)\alpha} \\
&= \frac{(q^2 + (1-q)^2)(q^2 + (1-q)^2 - \alpha q(1-q))}{(2q-1)2q^2(1-q)^2}
\end{align*}
\]

Note: \(q > 1/2 \implies 1 - 3q(1-q) > q(1-q) \implies u_{AS}(2) > u_{AS}(-2)\)

\(u_{AS}(0)\) is strictly increasing in \(q\). When \(q\) is close to 1/2, \(\partial u_{AS}(2)/\partial q\) and \(\partial u_{AS}(-2)/\partial q\) are positive; when \(q\) is close to one, \(\partial u_{AS}(2)/\partial q\) and \(\partial u_{AS}(-2)/\partial q\) are negative.

**Optimal bookbuilding mechanism, without when-issued trading.** Rearranging the incentive compatibility constraints:

\[
\begin{align*}
\left(q^2 + (1-q)^2\right)\left(u^{++}h^{++} - (w_L + u^+)h^{++}\right) + 2q(1-q)\left(u^{-}h^{+-} - (w_L + u^-)h^{+-}\right) &\geq 0 \quad (51) \\
\left(q^2 + (1-q)^2\right)\left(u^{-}h^{--} + (w_L - u^+)h^{+}\right) + 2q(1-q)\left(u^{-}h^{-} + (w_L - u^+)h^{++}\right) &\geq 0 \quad (52)
\end{align*}
\]
First assume that $\gamma = h_R = 0$. The underwriter will not be able to fully eliminate the adverse selection risk, so we need to consider two possible solutions to the constrained optimization problem: Solution 1: Everything is allocated to retail in state $-$. $h^+ = h^- = u^+ = u^- = 0$; $u^- = u_{AS}(-2)$. No underpricing is needed for truth-telling, but is needed for residual adverse selection risk. $u_B = u_{AS}(-2) (q^2 + (1 - q)^2) / 2$.

Solution 2: Nothing is allocated to retail. Underpricing is needed for truth-telling, but not for adverse selection risk. $h^+ = u^- = 0$; $u^+, u^- \leq w_L$; $h^+ = h^- = 1/2$; $h^- = 1$. $(IC^+)$ binding gives:

$$u_B = \frac{(q^2 + (1 - q)^2)}{2} (u^+ + 2q(1 - q) u^-) = 0$$

Thus,

$$u_B = \min \left[ \frac{(q^2 + (1 - q)^2)}{2} u_{AS}(-2), q(1 - q) w_L \right]$$

$$(q^2 + (1 - q)^2) u_{AS}(-2) = \frac{q^2 (1 - q)^2 (2q - 1) w \alpha}{(q^2 + (1 - q)^2 - \alpha q (1 - q))}$$

$$q(1 - q) w_L = \frac{q (1 - q) (2q - 1) w}{q^2 + (1 - q)^2}$$

$$(q^2 + (1 - q)^2) q (1 - q) \alpha < q^2 + (1 - q)^2 - \alpha q (1 - q).$$

Thus, if $\gamma = h_R = 0$:

$$u_B = \frac{(q^2 + (1 - q)^2)}{2} u_{AS}(-2).$$

If $h_R > 0$, then underpricing due to residual adverse selection risk occurs in all states. The IC constraints are satisfied because:

i) $u_{AS}(0) < w_L$. This follows directly from comparing equations (26) and (48). And

ii) $u_{AS}(2) < u_{AS}(0)$. (It has been demonstrated that underpricing due to adverse selection is decreasing in $z$ for $z \geq 0$.)

Thus, $h^+ = h^- = 0$ and the expected underpricing is given by (27).

**Proof of Proposition 4.** We start with the case in which allocations cannot be conditioned on when-issued trading. The constraints $(IC^+_U)$ and $(IC^-_U)$ can be rewritten as:

$$(q^2 + (1 - q)^2) (u^+ h^+ - (w_L + u^-) h^-) + 2q(1 - q) (u^- h^- - (w_L + u^-) h^-) \geq (q^2 + (1 - q)^2) \psi^+_U + 2q(1 - q) \psi^-_U \quad (53)$$

$$(q^2 + (1 - q)^2) (u^- h^- + (w_L - u^+) h^+) + 2q(1 - q) (u^+ h^+ + (w_L - u^+) h^+) \geq (q^2 + (1 - q)^2) \psi^-_U + 2q(1 - q) \psi^+_U \quad (54)$$
If the when-issued market is expected to be fully informative, so that \( w_{LT} \to 0 \), then constraints (53) and (54) become:

\[
(q^2 + (1 - q)^2) \left( u^{++}h^{++} - u^{+-}h^{+-} \right) + 2q(1 - q) \left( u^{+-}h^{-+} - u^{--}h^{--} \right) \\
\geq (q^2 + (1 - q)^2) \psi^+ + 2q(1 - q) \psi^- 
\]  

(55)

\[
(q^2 + (1 - q)^2) \left( u^{--}h^{--} - u^{+-}h^{+-} \right) + 2q(1 - q) \left( u^{+-}h^{++} - u^{++}h^{++} \right) \\
\geq (q^2 + (1 - q)^2) \psi^- + 2q(1 - q) \psi^+ 
\]  

(56)

Because \( q^2 + (1 - q)^2 > 2q(1 - q) \), it is optimal to set \( u^{+-}h^{+-} = u^{--}h^{--} = 0 \) and \( u^{++}h^{++} = u^{--}h^{--} > 0 \). Because the right-hand sides of (55) and (56) are strictly positive, and because residual adverse selection risk goes to zero, (55) and (56) are strictly binding and strictly positive rents must be paid to induce truthtelling.

We next consider the case in which allocations can be conditioned on when-issued trading. We will assume here that the when-issued market is informative, so that misinformation on the part of polled investors is revealed, and so that \( w_{LT} \to 0 \). The constraints (\( IC^+_T \)) and (\( IC^-_T \)) become:

\[
q^2 \left( u^+_ch^+_c - u^+h^+_w \right) + (1 - q)^2 \left( u^+_w h^+_w - u^+h^+_c \right) + \\
q(1 - q) \left( u^+(h^-_c + h^-_w) - u^-h^-_c - u^-h^-_w \right) \\
\geq (q^2 + (1 - q)^2) \psi^+_c + 2q(1 - q) \psi^-_c 
\]  

(57)

\[
q^2 \left( u^-h^-_c - u^-h^-_w \right) + (1 - q)^2 \left( u^-h^-_w - u^-h^-_c \right) + \\
q(1 - q) \left( u^+(h^-_c + h^-_w) - u^+_wh^+_w - u^+_wh^+_c \right) \\
\geq (q^2 + (1 - q)^2) \psi^-_c + 2q(1 - q) \psi^+_c 
\]  

(58)

where the subscript \( c \) (\( w \)) represents the state such that when-issued trading indicates that the investor’s report was correct (wrong). Using the same arguments as above (where allocations cannot be conditioned on when-issued trading), the optimal mechanism calls for zero rents whenever the when-issued market provides information indicating that the investor’s report was wrong: \( u^{ab}h^{ab}_w = 0, \forall \text{ pairs } (a, b) \). Also, for the same reasons as above, constraints (57) and (58) are strictly binding and strictly positive rents must be paid to induce truthtelling.

Furthermore, if we can assume that the right-hand sides of the constraints (57) and (58) are equal (This will be supported by the proof of Lemma 2 below.), then the expected rents are equal for those who have observed negative and positive information. This is true both when allocations can be conditioned on when-issued trading, and when they cannot.

**Proof of Lemma 2.** From the proof of Proposition 2:

\[
A_1(--) = v_0 - (2q - 1)w\alpha^+(-2) - \frac{2q - 1}{q^2 + (1 - q)^2}w(1 - \alpha^+(-2))
\]

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\[
B_1(+) = v_0 + (2q - 1)w\alpha^-(2) + \frac{2q - 1}{q^2 + (1 - q)^2}w(1 - \alpha^-(2)) - \frac{(1 - \alpha^-(-2))\delta}{3}
\]

As shown in the proof of Proposition 2, \(\alpha^-(-2) = \alpha^+(-2)\), thus
\[
(B_1(+) - v_0)\eta = (v_0 - A_1(\cdot\cdot\cdot))\eta
= \left(2q - 1\right)\left(1 - \frac{2q(1 - q)\alpha^-(-2)}{1 - 2q(1 - q)}\right)w - \frac{(1 - \alpha^-(-2))\delta}{3}\eta
\]

The first term of equation (61) represents the expected profit related to the impact of a lie on the public expected value of the IPO, \(w_L\). As indicated by the second term, this expected profit is decreased if there is additional uncertainty about the IPO value that cannot be resolved through bookbuilding (\(\delta > 0\)).

Let \(\psi_0 = B_1(+) - v_0\). For shorthand we'll let \(\alpha' = \alpha^-(2)\).

\[
\frac{\partial \psi_0}{\partial \alpha'} = \frac{\delta}{3} - 2q(1 - q)w_L > 0
\]

\[
\frac{\partial \psi_0}{\partial q} = 2(2q - 1)\alpha'w_L + (1 - 2q(1 - q)\alpha')\frac{\partial w_L}{\partial q} + \left(\frac{\delta}{3} - 2q(1 - q)w_L\right)\frac{\partial \alpha'}{\partial q}
\]

\[
\frac{\partial w_L}{\partial q} = \frac{4q(1 - q)w}{(1 - 2q(1 - q))^2} > 0
\]

\[
\frac{\partial \alpha'}{\partial q} = \frac{-\alpha(1 - \alpha)(2q - 1)/2}{((1 - \alpha)(q^2 + (1 - q)^2)/2 + \alpha q(1 - q))^2} < 0
\]

The upper bound on \(\psi_0\) is \((1 - 2q(1 - q)\alpha^-(2))w_L\). This upper bound is clearly decreasing in \(\alpha\) and increasing in \(q\).

**Proof of Lemma 3.** The participation constraints will change in that investors will want a strictly positive return. We assume, however, that there are more than two investors who can reveal information in bookbuilding. The profit to trading on private information, after two other investors have revealed their information, is not nearly as high as the profit that can be earned after lying. We can thus continue to treat the participation constraints as nonbinding and consider only the incentive compatibility constraints. The constraints \((IC_T^+)\) and \((IC_T^-)\) can be rewritten as:

\[
q^2 + (1 - q)^2 \left(u^{++}h^{++} - (w_L + u^{--})h^{--}\right) + 2q(1 - q)\left(u^{--}h^{--} - (w_L + u^{++})h^{++}\right) \\
\geq 2q(1 - q)\psi_L^+
\]

\[
q^2 + (1 - q)^2 \left(u^{--}h^{--} + (w_L - u^{++})h^{++}\right) + 2q(1 - q)\left(u^{++}h^{++} + (w_L - u^{--})h^{--}\right) \\
\geq 2q(1 - q)\psi_L^-
\]

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If the investors agree due to a lie, then with probability 1/2 the market will break down after it opens. If this happens, then this is equivalent (in terms of information available before pricing) to the two investors having truthfully reported +−. If the market does not break down, then it confirms that the lie was actually correct. Thus, if the market is informative, \( w'_{LT} = 0 \). We rewrite constraints (62) and (63), allowing the allocations to be conditioned on when-issued trading (when the market opens):

\[
q^2 u^+ h^+ + (1 - q)^2 u^+ w^+ - \left( q^2 + (1 - q)^2 \right) (w_L + u^+) h^+ + 2q(1 - q)u^+ h^-
- q(1 - q) \left( u_c h^+ + u_w h^- \right) \geq 2q(1 - q)\psi^+_L
\]

(64)

\[
q^2 u^- h^- + (1 - q)^2 u^- w^- + \left( q^2 + (1 - q)^2 \right) (w_L - u^-) h^- + 2q(1 - q)u^- h^+
- q(1 - q) \left( u_c h^- + u_w h^- \right) \geq 2q(1 - q)\psi^-_L
\]

(65)

where the subscript c (w) indicates that the when-issued market indicated that the investor’s report was correct (wrong). As in the proof of Proposition 2, we will solve for the mechanism both in the case in which allocations can be conditioned on when-issued trading and in the case in which they cannot. In the latter case we will simply require that \( h_c^{aa} = h_w^{aa} \). The participation constraint for retail participation in the IPO requires that: \( u^-, u^+, u^- \geq u_{AS}(0) \). From equations (26) and (48):

\[
u_{AS}(0) = \left( q^2 + (1 - q)^2 \right) \left( \frac{\alpha}{2 - \alpha} \right) w_L
\]

(66)

\( \alpha \leq 2/3 \) is sufficient so that \( u_{AS}(0) < w_L/2 \). The optimal mechanism is similar to that without when-issued trading in that it calls for: \( h^+ = 0 \) and \( h^- = 1 - h_R \). The IC constraints are thus:

\[
q^2 u^+ h^+ + (1 - q)^2 u^+ w^+ - q(1 - q) \left( u_c h^- + u_w h^- \right)
+ 2q(1 - q)u^+ (1 - h_R) \geq 2q(1 - q)\psi^+_L
\]

(67)

\[
q^2 u^- h^- + (1 - q)^2 u^- w^- - q(1 - q) \left( u_c h^+ + u_w h^+ \right)
+ \left( q^2 + (1 - q)^2 \right) (w_L - u^-)(1 - h_R) \geq 2q(1 - q)\psi^-_L
\]

(68)

\( (1 - q)^2 u_{AS}(0) < (1/2)(q^2 + (1 - q)^2)(u_{AS}(2) + u_{AS}(-2)) \). Thus, a mechanism that sets \( u^+ = u^+ = u^- = u_{AS}(0) \) and underpricing otherwise equal to zero is strictly less costly than the mechanism without when-issued trading. In such a mechanism \( h_w^{aa} \) is optimally set to zero because \( q > 1/2 \). If allocations cannot be conditioned on when-issued trading, then no allocations are given to the pooled investors when they agree with each other. This results in the following IC constraints:

\[
2q(1 - q)u_{AS}(0)(1 - h_R) \geq 2q(1 - q)\psi^+_L
\]

(69)

\[
\left( q^2 + (1 - q)^2 \right) (w_L - u_{AS}(0))(1 - h_R) \geq 2q(1 - q)\psi^-_L
\]

(70)

If \( \alpha \) is large enough so that \( \psi^+_L \) and \( \psi^-_L \) are small relative to \( u_{AS}^- \) and \( w_L - u_{AS}^+ \), then the above constraints are satisfied.
Proof of Proposition 5. The first two points of the proposition follow directly from the proof of Lemma 3. In constraints (64) and (65) the term $w_L$ does not go to zero, because the market does not open when the polled investors disagree. Thus, constraint (65) is easier to satisfy than constraint (64). The second point follows from expressions (69) and (70). If $\alpha$ is small, these inequalities will not be satisfied, and the incentive compatibility constraints cannot be satisfied just with the underpricing that is caused by residual adverse selection risk.

Continuing from the analysis of Section 5, the following three conditions must be satisfied in order for informative bookbuilding to be necessary, and sufficient, for when-issued trading:

$$\rho - (1 - \alpha)\delta/3 \leq \alpha(2q - 1)w = (2 - \alpha)u_{AS}(0)$$

$$\rho - (1 - \alpha)\delta/3 > \alpha\delta/3$$

$$\rho - \frac{(1 - \alpha)\delta}{3} > \frac{(2\alpha - \alpha^+(2) - \alpha^-(2))\delta}{6} + \frac{(q^3 - (1 - q)^3) w\alpha^+(2)}{2} - \frac{(2q - 1) w\alpha^-(2)}{2} - \frac{(2q - 1) w(\alpha^+(2) - \alpha^-(2))}{2}$$

(We know that: $0 < (2\alpha - \alpha^+(2) - \alpha^-(2))\delta/6 < \alpha\delta/6$.)

If $\delta$ is large, then the when-issued market will not open, regardless of whether there is informative bookbuilding. To simplify, we will let $\delta = 0$. The above conditions become:

$$\frac{(q^3 - (1 - q)^3) w\alpha^+(2)}{2} - \frac{(2q - 1) w\alpha^-(2)}{2} - \frac{(2q - 1) w(\alpha^+(2) - \alpha^-(2))}{2}$$

As $q \to 1$, the above condition $\to \alpha w \geq \rho > 0$. Thus, even a very illiquid when-issued market ($\rho$ small) can be enabled by bookbuilding if $q$ is large. If $\alpha$ is very small, then bookbuilding is needed for when-issued trading to open only if the market is very illiquid ($\rho$ small) or the information is very valuable ($w$ very large).
References


