Scope-related cumulativity asymmetries and cumulative composition∗

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Balaton Linguistics summer school
August 29, 2018
(material previously presented at SALT 28)

Outline

(i) Empirical problem: Cumulative readings of every DPs, ‘distributive’ conjunctions


(iii) Comparison to existing analyses (time allowing)

1 Singular universals and distributive conjunctions

1.1 Cumulativity asymmetries: English singular universals

Distributivity wrt. lower plural expressions

(1) Every girl in this town fed (the) two dogs.

scenarios: girls Ada and Bea, dogs Carl and Dean

‘distributive’ scenario: Ada fed Carl and Dean. Bea fed Carl and Dean. TRUE
‘cumulative’ scenario: Ada fed Carl. Bea fed Dean. FALSE

∗We would like to thank Magdalena Roszkowski, Dóra Kata Takács and Marcin Wągiel for language consultancy. Further, we thank Kübra Atasoy, Daniel Büiring, Enrico Flor, Izabela Jordanoska, Clemens Mayr, Max Prüller, Eva Rosina, Magdalena Roszkowski, Marcin Wągiel, Susi Wurmbrand, Henk Zeevat, Eytan Zweig as well as the audiences at SALT 28, the University of Göttingen, the University of Tübingen, Masaryk University in Brno and the 19th Szklarska Poręba Workshop on the Roots of Pragmasemantics for relevant comments on earlier versions of this work. We acknowledge the support of the Austrian Science Fund (FWF), project P-29240 ‘Conjunction and disjunction from a typological perspective’.
Cumulativity wrt. higher plural expressions

(2) *(The) two girls fed every dog in this town.*

SCENARIOS:
'distributive' scenario: Ada fed Carl and Dean. Bea fed Carl and Dean. TRUE
'cumulative' scenario: Ada fed Carl. Bea fed Dean. TRUE


1.2 Cumulativity asymmetries: German distributive conjunction

CONTEXT: There are two skiing World Cup races this weekend. Ada and Bea are the only Austrian participants. Ada is competing in the downhill and Bea in the slalom.

Cumulativity wrt. higher plural expressions

(3) Zum Glück haben die zwei Österreicherinnen sowohl die Abfahrt als auch den Slalom gewonnen!

'the fortune have the two Austrians PRT the downhill PRT also the slalom won'

'Fortunately, the two Austrians won both the downhill and the slalom.'

'cumulative' scenario: Ada won the downhill. Bea won the slalom. TRUE

Distributivity wrt. lower plural expressions

(4) Zum Glück haben sowohl die Ada als auch die Bea die zwei Rennen gewonnen!

'PRT the Ada PRT also the Bea the two races won'

'Fortunately, both Ada and Bea won the two races.'

'cumulative' scenario: Ada won the downhill. Bea won the slalom. FALSE

1.3 Distributive conjunction: Analogous data from other languages

- Polish

(5) Generalization from the literature (similar in several other Slavic languages, cf. e.g. Mitrović & Sauerland [2014, 2016])

a. A i B: cumulative/distributive
b. i A i B: distributive only
Data due to Magdalena Roszkowski and Marcin Wągiel (p.c.):

(6) a. SCENARIO: Sabina called ‘Express Catering’. Magda called ‘Star Catering’.
    b. Sabina i Magda dostatecznie wcześnie zadzwonili do tych dwóch restauracji.
    c. I Sabina i Magda dostatecznie wcześnie zadzwonili do tych dwóch restauracji.

i Sabina i Magda enough early called to these two restaurants
‘Sabina and Magda called these two restaurants early enough.’ TRUE in (9-a)

(7) a. SCENARIO: Sabina called Adam. Magda called Piotr.
    b. Na szczęście dwie organizatorki dostatecznie wcześnie poinformowały i Adama i Piotra.
    c. Both Sabina and Magda called these two restaurants early enough.’

FALSE in (9-a)

• Hungarian

(8) Generalization from Szabolcsi (2015): Additive particle within the conjunction removes cumulativity
a. A és B: cumulative/distributive
b. A is (és) B is: distributive only

Data due to Dóra Kata Takács (p.c.):

(9) a. SCENARIO: Sára called ‘Express Catering’. Marcsi called ‘Star Catering’.
    b. Sára és Marcsi időben felhívta a két kiszállító céget.

Sára and Marcsi on-time called the two catering companies.ACC
‘Sára and Marcsi called the two catering companies ahead of time.’ TRUE in (9-a)

(10) a. SCENARIO: Sára called Bálint. Marcsi called Péter.
    b. Szerencsére a két szervező időben felhívta Bálintot és Pétert.

fortunately the two organizers on-time called Bálint.ACC is and Péter.ACC is.

is
‘Fortunately, the two organizers called both Bálint and Péter ahead of time.’
TRUE in (10-a)
1.4 Interim summary: Asymmetrically distributive universals (ADUs)

(i) always have a distributive reading wrt. semantically plural expressions in their scope

(ii) allow for cumulative readings if they occur in the scope of a semantically plural expression

(iii) Assumption here: asymmetry tied to scope (following Champollion (2010), further research needed)

ADUs cross-linguistically

• singular universals: English every DPs, German jed- DPs

• German distributive conjunction: sowohl A als auch B ‘A as well as B’

• possibly other distributive conjunctions: Hungarian A is és B is, Polish i A i B (preliminary data)

Next point: Why ADUs represent a problem for a theory of cumulativity.

1.5 Schein sentences

(11) The two girls wanted to buy the two dogs. adapted from Beck & Sauerland (2000)

• Cumulative truth conditions:
  Each of the two girls wanted to buy at least one of the two dogs &
  for each of the two dogs, at least one of the two girls wanted to buy it

• ⇒ Relation $[\lambda x.\lambda y. y$ wanted to buy $x]$ applies cumulatively to the girls and the dogs

• Cumulative relation may be derived by LF-movement Beck & Sauerland (2000)

Schein sentences
ADUs ‘sandwiched’ between two other plural expressions.


(12) Ada and Bea taught every dog two new tricks. adapted from Schein (1993)

scenario: There are two dogs, Carl and Dean.

Ada taught Carl trick 1 &
Ada taught Carl trick 2 &
Ada taught Dean trick 3 &
Bea taught Dean trick 2

(i) it is not the case that for every dog each of the girls taught it two tricks
⇒ every dog cumulative wrt. Ada and Bea

(ii) every dog was taught two tricks, tricks can be different
⇒ every dog distributive wrt. two tricks

Why a cumulative relation between individuals isn’t enough
• Cumulative relation R1, which takes the arguments Ada+Bea and Carl+Dean:

(13)  \[ R1 = \lambda x.e.\lambda y.e.\lambda y.\text{taught} x \text{two new tricks} \]

No cumulation with two tricks \( \rightsquigarrow \) each girl taught two tricks to some dog. \( \text{predicted FALSE} \)

• Cumulative relation R2, which takes the arguments a+b and two tricks:

(14)  \[ R2 = \lambda x.e.\lambda y.e.\lambda y.\text{taught} x \text{to every dog} \]

No cumulation with every dog \( \rightsquigarrow \) The two tricks must be the same for each dog. \( \text{predicted FALSE} \)

1.6 Our approach: Predicate pluralities

(15)  \textit{Ada and Bea taught every dog two new tricks.} \hspace{1cm} \text{adapted from}\textbf{ Schein (1993)}

Existing approaches

• Cumulative relations between events and individuals \[ \textbf{Schein (1993), Kratzer (2000), Zweig (2008)} \]

• Cumulative relations between individuals plus more complex LF \[ \textbf{Champollion (2010)} \]

Our basic idea

• Cumulation between individuals and predicate pluralities
• Ada+Bea must be in a cumulative relation with one of the elements of this set:

(16)  \{ taught C T1 + taught C T2 + taught D T1 + taught D T2, \\

taught C T1 + taught C T2 + taught D T2 + taught D T3, \\

taught C T3 + taught C T2 + taught D T1 + taught D T2, \ldots \} \}

• We only consider those pluralities of predicates that assign two tricks to each dog.

Interim summary

• Traditional approach to cumulative truth-conditions: Binary relations between individuals apply cumulatively. Relations may be syntactically derived.
• Schein-sentences problematic for this approach
• Our idea: use cumulation with pluralities of predicates.
• Independent motivation: Flattening effects
2 Analysis in the Plural Projection framework

What we will do

• We will give a new meaning for every that captures cumulativity asymmetries:

(17) a. Every girl fed (the) two dogs.
b. (The) two girls fed every dog in this town.

• Rationale based on Schein sentences: We want predicate pluralities that ‘cover’ every dog and assign two tricks to each dog.

(18) Ada and Bea taught every dog two new tricks.
(19) \{taught C T1 + taught C T2 + taught D T1 + taught D T2, 
taught C T1 + taught C T2 + taught D T2 + taught D T3, 
taught C T3 + taught C T2 + taught D T1 + taught D T2, \ldots \}

2.1 every DPs, informally

• Function of type \langle \langle e,a \rangle^*, a^* \rangle – directly manipulates plural sets of predicates.

(20) every girl fed two pets
\[ \text{every girl}(\text{feed}(C) + \text{feed}(D), \text{feed}(C)+\text{feed}(E), \text{feed}(D)+\text{feed}(E)) \]

• For each atomic individual \( x \) in the restrictor, we choose a predicate-plurality \( P \) from the scope, apply each \( P' \leq a P \) to \( x \) and take the sum (\( P \) applies ‘distributively’ to \( x \))

(21) \text{feed}(C)(A) + \text{feed}(D)(A), \text{feed}(C)(B) + \text{feed}(E)(B), \ldots \]

• For each such assignment of predicate-pluralities, we take the sum over all individuals and form the plural set of all such sums

(22) \[ \text{every girl}(\text{feed}(C) + \text{feed}(D), \text{feed}(C)+\text{feed}(E), \text{feed}(D)+\text{feed}(E)) = \text{feed}(C)(A) + \text{feed}(D)(A) + \text{feed}(C)(B) + \text{feed}(E)(B), \]
\[ \text{feed}(C)(A) + \text{feed}(E)(A) + \text{feed}(C)(B) + \text{feed}(D)(B), \]
\[ \text{feed}(C)(A) + \text{feed}(E)(A) + \text{feed}(D)(B) + \text{feed}(E)(B), \ldots \]

• The resulting value is a plural set containing predicates/propositions
2.2 Deriving cumulativity asymmetries

*every* DP in subject position

(23) *Every girl in this town fed the two dogs.*

only distributive

(24)

\[
\begin{array}{c}
\text{[A fed C + A fed D + B fed C + B fed C]} \\
\text{every girl} \\
\text{fed} \\
\text{the two dogs}
\end{array}
\]

\[\text{[fed(C) + fed(D)]}\]

**Prediction:** Singular universals always distributive wrt. material in their scope

*every* DP in object position

(25) *The two girls fed every dog in this town.*

cumulative possible

(26)

\[
\begin{array}{c}
C([A+B])((\text{fed C + fed D})) \\
= [ A \text{ fed } C + B \text{ fed D, B fed C + A fed D, ...]}
\end{array}
\]

\[
\begin{array}{c}
[A+B] \\
\text{the two girls} \\
\text{fed} \\
\text{every dog}
\end{array}
\]

\[\text{[fed C + fed D]}\]

**Prediction:** Cumulation with material outscoping *every* possible, since *every P Q* returns a plurality
Schein sentences

(27) Ada and Bea taught every dog two new tricks.

\[
\begin{array}{c}
A \text{ taught } C \quad T1 + A \text{ taught } C \quad T2 + B \text{ taught } D \quad T2 + A \text{ taught } D \quad T3,
\end{array}
\]
\[
\begin{array}{c}
A \text{ taught } D \quad T1 + B \text{ taught } D \quad T2 + B \text{ taught } C \quad T2 + B \text{ taught } C \quad T3, \ldots
\end{array}
\]

Interim summary

- every DPs take plural sets as their argument. They ‘distributively’ apply elements to atoms in the restrictor.
- The result is a plural set, which can be cumulated with higher pluralities.
- This analysis can be generalized to distributive conjunction (see handout)

3 Comparison with existing theories

3.1 Event-based analyses (Schein 1993, Kratzer 2000, Zweig 2008)

Basic idea: Cumulation targets relations between events and individuals.

(28) The two girls taught every dog two new tricks.

(29) \[ \exists e [ \text{teach}(e) \wedge \text{AGENT}(e)(A+B) \wedge \text{BEN}(e)(+(\text{dog})) \wedge \forall y \leq A \quad +(\text{dog})[\exists Z \in \text{[two tricks]}, \exists e' \leq \text{adapted from Zweig (2008)}]
\]

Differences to our proposal

- We don’t require events, so we can maintain that some predicates that allow for cumulativity don’t have an event/state argument.
- No special story needed for cumulation across predicates where arguments are neither indi-
individuals nor events, such as (some) attitude verbs (30).

(30) The Georgian ambassador called this morning, the Russian one at noon. They think that Trump should take a walk with Putin and build a hotel in Tbilisi, but neither addressed the Caucasus conflict!

3.2 Individual-based analysis (Champollion 2010)

- No appeal to events
- every DPs denote pluralities of individuals

(31) \[ \text{every dog} = \text{the dogs} = C + D \]
- every must directly c-command a distributivity or cumulation operator (*, **, ...) 
- traces of every DPs must range over atoms

(32) The two girls taught every dog two new tricks.

(33) \[ \text{the two girls} \ [\text{every dog} \ast \ast \ 2 \ [\text{two new tricks} \ 3 \ [t_1 \ast \ast \text{taught} \ t_2 \ t_3 ]]] ] \]

(34) \[ \lambda x. x \text{atomic} \cdot \lambda y. \text{there is a plurality Z of two tricks such that y cumulatively taught Z to x} \]

Differences to our proposal:

- Champollion (2010) must assume that traces of ADUs range over atoms: No straightforward account for distribution to non-atomic subpluralities.

(35) Sowohl die Mädchen als auch die Buben haben zwei Hunde gefüttert

PRT the girls PRT also the boys have two dogs fed

‘Both the girls and the boys fed two dogs.’ (German)

SCENARIO: The girls fed two dogs between them and the boys fed two dogs between them.

TRUE

- Our account, as opposed to theories working with syntactically derived predicates, generalizes to flattening effects with conjunction.

4 Conclusion

- We presented a system that derives cumulativity without syntactically derived cumulative relations
- This system derives cumulative truth-conditions step-by-step, along the lines of the hierarchical structure
- This system accounts for the ‘flattening’ effects with conjunction
• We looked at cumulative asymmetries and ‘flattening’ effects with ADUs
• We showed how our system accounts for these data

Questions/problems

• Some technical issues (see appendix below)
• Expansion to collective predicates?
• Expansion to non-upward-monotone DPs (less than five, exactly five . . .)
• Cross-linguistic differences concerning conditions on cumulative reading – scope vs. grammatical function

References


A Technical details

A.1 Ontology

- For every type $a$, there is a set $A_a$ of atomic elements and the domain $D_a$ contains the elements from $A_a$ and pluralities thereof (cf. Schmitt 2013 for independent motivation).

- Pluralities correspond to nonempty subsets of $A_a$ and are formed via a sum operation $\sum$.

- In addition, we introduce separate semantic types for ‘plural sets’ that form the input to cumulation. In particular, a plural set with elements of type $a$ is itself a possible denotation of type $a^\ast$. We assume that $D_{a^\ast}$ and $A_{(a,t)}$ are isomorphic, but disjoint. This allows us to define semantic operations that are sensitive to whether their argument is a plural set.

Definition 1 (Types). The set $T$ of semantic types is the smallest set such that:

(i) $e \in T, t \in T$

(ii) for any $a,b \in T$, $(a,b) \in T$

(iii) for any $a \in T$, $a^\ast \in T$

Definition 2 (Semantic domains). Let $A$ be the (nonempty) set of atomic individuals. For each type $a$, there is an ‘atomic domain’ $A_a$ and a full domain $D_a$ with the following properties:

(i) $A_e = A, A_t = \{0, 1\}$

(ii) For any type $a$:

(a) $D_a$ is a set such that $A_a \subseteq D_a$ and there is an operation $\sum_a : \mathcal{P}(D_a) \setminus \{\emptyset\} \rightarrow D_a$.

(b) There is a bijection $pl_a : \mathcal{P}(A_a) \setminus \{\emptyset\} \rightarrow D_a$ such that $pl_a(\bigcup S) = \sum_a(\{pl_a(S) : S \in S\})$ for any $S \subseteq (\mathcal{P}(A_a) \setminus \{\emptyset\})$ and $pl_a(\{x\}) = x$ for each $x \in A_a$.

(c) $D_a$ and $\mathcal{P}(A_a)$ are disjoint.

(iii) For any types $a,b$: $A_{(a,b)} = D_a^a$, the set of all partial functions from $D_a$ to $D_b$.

(iv) For any type $a$, there is a set $D_{a^\ast}$ disjoint from $\mathcal{P}(D_a)$ on which the operations $\cup, \cap$ and $\setminus$ are defined and a function $pl_a^\ast : \mathcal{P}(D_a) \rightarrow D_{a^\ast}$ that is an isomorphism wrt. $\cup, \cap$ and $\setminus$.

(v) For any type $a$, $A_a^\ast = D_{a^\ast}$.

(vi) For any types $a,b$, $D_a$ and $D_b$ are disjoint. [issue with empty set?]

Notational conventions:
• In the metalanguage, we use natural language expressions in boldface to denote the meanings of lexical elements (e.g. girl, anna, show).
• We use ‘starred’ variables like $x^*$, $P^*$ etc. for types of the form $a^*$.
• We occasionally omit type subscripts on operations that are defined cross-categorially, like $p^a_+$, $pl_a$, $\oplus_a$ etc.
• If we apply $\cup, \cap, \in$, cardinality etc. to plural sets, these operations are defined via the isomorphism between $\mathcal{P}(D_a)$ and $D_a^*$.
• For any type $b$ and $x, y \in D_b$:
  
  $- x +_b y = 
  \begin{cases} 
  \mathbf{1}_b(\{x, y\}) & \text{if } x \neq y \\
  x & \text{if } x = y 
  \end{cases}$

  $- x \leq y \iff x +_b y = y$

  $- x \leq_a y \iff x \leq y \wedge y \in A_b$

• We occasionally write $[x_1, \ldots, x_n]$ for $pl^a(\{x_1, \ldots, x_n\})$.

### A.2 Building cumulativity into the compositional system

• We define a cross-categorial operation $\oplus$ that occurs both in the semantics of conjunction and in cumulativity.

• In most cases $\oplus$ is equivalent to the usual sum operation $+$, but if the elements to be summed up are plural sets, it has a ‘distributive’ effect: Intuitively it produces the set of all pluralities that can be obtained by applying some choice function to the plural sets and summing up all the values.

• In the case of nested plural sets (i.e. type $(a^*)^*$), this is done ‘recursively’. This stipulation is needed for the analysis of conjunction particles.

**Definition 3** (Recursive sum). The operation $\oplus_a : \mathcal{P}(D_a) \setminus \{\emptyset\} \rightarrow D_a$ is defined for any type $a$ in the following way:

(i) For any type $a$ that is not of the form $b^*$, and any nonempty $S \subseteq D_a$, $\oplus S = \mathbf{1} S$.

(ii) For any type $b^*$ and any nonempty $S \subseteq D_{b^*}$, $\oplus_{b^*} S = pl^a(\{X \mid \exists f : f \text{ is a function from } S \text{ to } D_b \wedge \forall X^* \in S : f(X^*) \in pl^{b-1}(X^*) \land X = \oplus_b(\{f(X^*) \mid X^* \in S\})\})$

**Notational convention:** For any type $a$ and any $x, y \in D_a$: $x \oplus_a y = \oplus_a(\{x, y\})$.

The notion of a ‘cover’ formalizes the weak truth conditions of cumulative sentences.

**Definition 4** (Cover). Let $P \in D_a, x \in D_b$. A relation $R \subseteq A_a \times A_b$ is a cover of $(P, x)$ iff $\mathbf{1}((P', x') : (P', x') \in R) = P$ and $\mathbf{1}((x' : \exists P' : (P', x') \in R)) = x$.

We now define a way of combining plural sets. Intuitively, we take any cover of some plurality in the functor set and some plurality in the argument set, and then sum up the values for all function-argument pairs in the cover.

**Definition 5** (Recursive Cumulation). For any $P^* \in D_{(a,b)^*}$ and $x^* \in D_a$:

$C(P^*, x^*) = pl^a(\{\oplus(\{P(x) : (P, x) \in R\}) \mid \exists P \in pl^{b-1}(P^*), x \in pl^{a-1}(x^*) : R \text{ is a cover of } (P, x)\})$
Definition 6 (Composition rules). We assume that for each type \( a \), there is a (possibly empty) set \( L_a \) of lexical expressions of type \( a \), and there is a function \( M \) that assigns any \( \alpha \in L_a \) some denotation in \( D_a \). For any type \( a \), we can then define the meaningful expressions of type \( a \) and their denotations as follows:

(i) For any lexical expression \( \alpha \in L_a \), \( \alpha \) is a meaningful expression of type \( a \) and \( \llbracket \alpha \rrbracket = M(\alpha) \).

(ii) For any meaningful expressions \( \alpha \) of type \( \langle a, b \rangle \) and \( \beta \) of type \( a \), \( \llbracket \alpha \beta \rrbracket \) is a meaningful expression of type \( b \). In this case, 

\[
\llbracket \alpha \beta \rrbracket = \llbracket \alpha \rrbracket(\llbracket \beta \rrbracket)
\]

if \( \llbracket \alpha \rrbracket \) and \( \llbracket \beta \rrbracket \) are both defined, \( \llbracket \alpha \rrbracket \in A_{\langle a, b \rangle} \) (i.e. \( \llbracket \alpha \rrbracket \) is not a proper plurality) and \( \llbracket \beta \rrbracket \in \text{dom}(\llbracket \alpha \rrbracket) \). Otherwise, \( \llbracket \alpha \beta \rrbracket \) is undefined.

(iii) For any meaningful expressions \( \alpha \) of type \( \langle a, b \rangle^* \) and \( \beta \) of type \( a^* \), \( \llbracket \alpha \beta \rrbracket \) is a meaningful expression of type \( b^* \). In this case, 

\[
\llbracket \alpha \beta \rrbracket = C(\llbracket \alpha \rrbracket, \llbracket \beta \rrbracket) \quad \text{(see Definition 5)}
\]

if \( \llbracket \alpha \rrbracket \) and \( \llbracket \beta \rrbracket \) are both defined. Otherwise, \( \llbracket \alpha \beta \rrbracket \) is undefined.

(iv) Nothing else is a meaningful expression of type \( a \).

Note: This definition is vague in that it does not say anything about what \( C(P^*, x^*) \) should look like in cases where functional application fails ‘within’ the cumulation rule (i.e. some atomic part of some \( P \in P^* \) is undefined for some atomic part of some \( x \in x^* \)). We do not attempt a serious treatment of presupposition projection here. The empirical issue of how presuppositions – and other cases of partiality in semantics – actually interact with cumulation is presently not clear to us.

We also did not include variable binding as there are technical problems involved in combining variable binding with a semantics based on alternative sets. These problems arise independently in the semantics of questions and focus.

### A.3 Lexicon

Note: Here, we assume that open-class elements always denote atomic objects of types that do not contain any \(*\), i.e. only the functional lexicon and the composition rules can manipulate plural sets. Lexically collective predicates are not analyzed correctly within the present system.

Type-shifts mediate between ‘singular’ denotations and plural sets. Here, we represent these type-shifts in the syntax for simplicity, but ultimately we do not think this is necessary (or even well motivated).

Definition 7 (Type shifts). For any type \( a \):

(i) \( \llbracket \uparrow_{\langle a, a^* \rangle} \rrbracket = \lambda x_a.p^r(\{x\}) \)

(ii) \( \llbracket \downarrow_{\langle a^*, a \rangle} \rrbracket = \lambda S_{a^*}.p^r(\bigcup\{p^{r^{-1}}(S) \mid S \in S^*\}) \)

We take non-distributive conjunction to be the natural-language counterpart of the cumulation operation \( \oplus \).

Definition 8 (Coordination). For any type \( a \):
\[ \text{AND}_{(a,\langle a, a \rangle)} = \lambda x_a \cdot \lambda y_a \cdot x \oplus_a y \]
\[ \text{OR}_{(a^*, \langle a^*, a^* \rangle)} = \lambda X^*_{a^*} \cdot \lambda Y^*_{a^*} \cdot p^*(p^{r-1}(X^*) \cup p^{r-1}(Y^*)) \]

**Note:** The operation \( \oplus_a \) as defined here is problematic as a potential meaning for conjunction because it is idempotent and commutative, but not associative. So \( (D_{a^*}, \oplus) \) does not form a semilattice even though \( (D_a, +) \) does. However, \( (D_a, \oplus) \) is a semilattice for any type \( a \) that is not of the form \( b^* \), due to the isomorphism with subsets. The cases where associativity fails are all such that two or more conjuncts are identical in meaning. At present, it is unclear to us how the cumulation operation \( \oplus \) could be redefined in such a way that conjunction always comes out as associative.

We assume that definite and indefinite plural DPs are of type \( e^* \). In particular, a plural indefinite denotes a set of pluralities of individuals. Note that this approach will not extend to non-upward-monotonic indefinites such as *exactly ten people* or *at most ten people*.

**Definition 9** (Plural definites and upward-monotonic indefinites).

- \( \mathcal{A}(P^*_{(e,t)^*}) = \lambda x_{e^*}.[\exists P_{(e,t)^*}. P \in p^{r-1}(P^*) \land \exists P'_{(e,t)^*}. P' \leq_a P \land P'(x)] \)
- \( \text{PL}_{(\langle e,t \rangle^*, e^*)} = \lambda P^*_{(e,t)\cdot} \cdot p^*(\lambda x_{e^*} \forall y_{e^*}[y \leq_a x \rightarrow \mathcal{A}(P^*)(y)]) \)
- \( \text{DEF}_{(e,t)^*, e^*} = \lambda x_{e^*} : \exists x \in p^{r-1}(x^*).[\forall y \in p^{r-1}(x^*). y \leq x]. p^*(\{lx \in p^{r-1}(x^*). \forall y \in p^{r-1}(x^*). y \leq x\}) \)
- \( \text{TWO}_{(e,t)^*, e^*} = \lambda x_{e^*} \cdot p^*(\lambda x. p^{r-1}(x^*)(x) \land |x| = 2) \) where \(|x|\) is the number of atomic parts of \( x \).

**Notes:**

- Conjunction and disjunction in the restrictor of a determiner are treated analogously to avoid generating plurality inferences for conjunctions in the restrictor. This is done by the operation \( \mathcal{A} \), which forms the set of all atomic parts of some plurality in the plural set. However, since there are empirical differences between conjunctive and disjunctive NP-coordination, this analysis will have to be modified.

- The pluralization operator \( \text{PL} \) intuitively forms the union of all the atomic parts of all elements in a plural set. Dissociating this operation from the determiner allows us to analyze phrases like two girls and the two girls without positing silent existential quantifiers and without type mismatches.

The basic idea behind the analysis of *every/jeder* is the following: When an *every* DP combines with a plural set of predicates, we take those sums that can be formed by choosing some predicate plurality in the plural set for each individual, combining all individuals with their respective predicate pluralities and summing up all the values.

**Definition 10** (Singular universals).

- for any \( P_{(a,b)} \cdot x_a \cdot D(P, x) = \top(\{Q(x) : Q \leq_a P\}) \)
- \( \text{EVERY}_{\langle (e,t)^*, \langle (e,a)^*, a^* \rangle \rangle} = \lambda P^*_{(e,t)^*} \cdot \lambda R^*_{(e,a)^*} \cdot p^* \{ \top(\{D(f(x), x) : x \in A(P^*)\}) : f \text{ is a function from } A(P^*) \text{ to } p^{r-1}(R^*) \} \)
In the case of conjunction particles, cumulation is built into the lexical entry. This licenses cumulative readings of the plural conjuncts within a distributive conjunction of the type sowohl A als auch B ‘A as well as B’. The distributive effects of these conjunctions are derived from the effects of $\oplus$ and the fact that such conjunctions have to be type-shifted in order to combine with the predicate via cumulation (see examples below).

**Definition 11** (Conjunction particles).
\[
\left[\mu_{e,\langle (e,a)^*,a^*\rangle}\right] = \lambda x^* \cdot \lambda P^* \cdot C(P^*, x^*)
\]

In principle, our approach can also account for distributive readings of predicates that combine with plural arguments.

**Definition 12** (Predicate-level distributivity operators).
\[
\left[\text{dist}_{\langle (a,b)^*,(a^*,b^*)\rangle}\right] = \lambda P^* \cdot \langle a,b \rangle^* \cdot \lambda x_{a,b} \cdot \text{pl}^* (\{Q | \exists f: f \text{ is a function from } \{y | y \leq a x\} \to \text{pl}^* - 1 (P^*) \land Q = \bigcup (\{D(f(y), y) | y \leq a x\})\})
\]

**A problem:** In the present system, distributive predicates with $\text{DIST}$ and cumulative predicates differ in semantic type ($\langle e^*, t^* \rangle$ vs. $\langle e, t \rangle^*$). This means that we can’t get ‘mixed’ readings for predicate conjunctions like (36) where only one conjunct has a distributive interpretation (such readings are discussed by Dowty (1987)). At a technical level this could be addressed by replacing the cumulation rule with a cumulation operator that applies freely in the syntax (of type $\langle (a,b)^*, (a^*,b^*) \rangle$ like $\text{DIST}$), but we do not want to do this since we have not seen an overt counterpart of this operator in any language.

(36) Ada and Bea ate two pizzas and had a glass of wine.

**B Examples**

**Note:** In the following examples we gloss over the problematic behavior of $\oplus$ with conjuncts identical in meaning that was mentioned above.

(37) A toy model
   a. \(M(\text{dog}) = \{\text{carl, dean, eric}\}\)
   b. \(M(\text{girl}) = \{\text{ada, bea}\}\)
   c. \(M(\text{boy}) = \{\text{che, ed}\}\)
   d. \(M(\text{trick}) = \{t_1, t_2, t_3\}\)

**DP meanings**

(38) Ada and Bea
   a. 
      \[
      e(1) \\
      \text{Ada} \\
      \langle e, e \rangle \\
      \langle e, \langle e, e \rangle \rangle \\
      \text{Bea}
      \]
   b. \(\left[(1)\right] = \text{ada} \oplus \text{bea} = \text{ada} + \text{bea}\)

(39) two dogs
(40)  the two girls

(41)  every dog

(42)  sowohl Ada als auch Bea

b. \[\lambda R^{e,a}_{(e,a)} \cdot C(R^e, [ada])\]
c. \[\lambda R^{e,a}_{(e,a)} \cdot C(R^e, [bea])\]
d. \((\lambda R^{e,a}_{(e,a)} \cdot C(R^e, [ada])) + (\lambda R^{e,a}_{(e,a)} \cdot C(R^e, [bea]))\)

(43) sowohl die Buben als auch die Mädchen
PRT the boys PRT also the girls
‘the boys as well as the girls’ (German)

a. \((e, a)^*, a^*\) (3)

\[
\begin{array}{c}
\langle e^*, \langle (e, a)^*, a^* \rangle \rangle \\
\text{AND} \\
\langle (e, a)^*, a^* \rangle (2)
\end{array}
\]

b. \[\lambda R^{e,a}_{(e,a)} \cdot C(R^e, p\{1 \{ x \mid \text{boy}(x) \}\})\] if \(\exists x. \text{boy}(x)\), otherwise undefined
\[= \lambda R^{e,a}_{(e,a)} \cdot C(R^e, [che + ed])\]
c. \[\lambda R^{e,a}_{(e,a)} \cdot C(R^e, p\{1 \{ x \mid \text{girl}(x) \}\})\] if \(\exists x. \text{girl}(x)\), otherwise undefined
\[= \lambda R^{e,a}_{(e,a)} \cdot C(R^e, [ada + bea])\]
d. \[\lambda R^{e,a}_{(e,a)} \cdot C(R^e, [che + ed]) + (\lambda R^{e,a}_{(e,a)} \cdot C(R^e, [ada + bea]))\]

Simple examples with cumulation

(44) Ada and Bea fed two dogs.

a. \(t^*\) (2)

\[
\begin{array}{c}
\langle e, t^* \rangle (1) \\
\text{Ada and Bea} \\
\text{two dogs}
\end{array}
\]

b. \[\lambda R^{e,a}_{(e,a)} \cdot C(R^e, [two \ dogs])\]
\[= \lambda R^{e,a}_{(e,a)} \cdot C([fed], [carl + dean, carl + eric, dean + carl + eric]) = [fed(carl) + fed(dean), fed(carl) + fed(eric), fed(dean) + fed(eric)]\]
c. \[\lambda R^{e,a}_{(e,a)} \cdot C([ada + bea]) = [fed(carl)(ada) + fed(dean)(bea), fed(carl)(bea) + fed(ada)(dean), fe(\.\.\.]]\]

(45) Ada and Bea fed Carl and dean and brushed Eric.
Two girls fed every dog. (cf. Schein 1993, Kratzer 2000)

Distributive items

Note: We are not discussing the examples with made x do P here as they would require a generalized version of the cumulation rule that works with intensional predicates.


b. [[(1)]] = [[brushed(eric)]]

c. [[(2)]] = C([fed], [carl + dean]) = [fed(carl) + fed(dean)]

Distributive items


b. [[(1)]] = (λR∗\text{\textsubscript{e}}\langle e, t \rangle \cdot pℓ\{+\left\{D(f(x), x) : x \in A_e \land \text{dog}(x)\right\}\} : f \text{ is a function from } \{x \mid x \in A_e \land \text{dog}(x)\} \text{ to } pℓ\left\{R∗\right\}([\text{fed}])

= pℓ\{+\left\{D(\text{fed}, x) : x \in A_e \land \text{dog}(x)\right\}\} = pℓ\left\{+\left\{\text{fed}(x) : x \in A_e \land \text{dog}(x)\right\}\right\} = [\text{fed(carl)} + \text{fed(dean)} + \text{fed(eric)}]

c. [[(2)]] = C([fed(carl)] + [fed(dean)] + [fed(eric)], [ada + bea])

= [fed(carl)(ada) + fed(dean)(ada) + fed(eric)(bea) + fed(carl)(bea) + fed(dean)(ada) + fed(eric)(ada) + fed(carl)(ada) + fed(dean)(bea) + fed(eric)(ada),...]

Note: We are not discussing the examples with made x do P here as they would require a generalized version of the cumulation rule that works with intensional predicates.

Distributive items
\[
\text{fed(carl)(ada) + fed(dean)(bea) + fed(eric)(bea),} \ldots
\]

(47) Every girl fed two dogs.

\[
\begin{align*}
\langle e, t \rangle^* (2) \quad & \quad \langle e, t \rangle^* (1) \\
\text{every girl} \quad & \quad \text{two dogs}
\end{align*}
\]

b. \([1] = [\text{fed(carl)} + \text{fed(dean)}, \text{fed(carl)} + \text{fed(eric)}, \text{fed(dean)} + \text{fed(eric)}]
\]

c. \([2] = (\lambda R_{(e,a)}^* \cdot p^\ell \{ + (\{ D(f(x), x) : x \in A_e \land \text{girl}(x) \}) : f \text{ is a function from } \{ x \mid x \in A_e \land \text{girl}(x) \} \text{ to } p^\ell financier(R^*) \}) \}([1])\]

(48) Ada and Bea taught every dog two tricks. (cf. [Schein 1993])

\[
\begin{align*}
\langle e, t \rangle^* (2) \quad & \quad \langle e, t \rangle^* (1) \\
\text{Ada and Bea} \quad & \quad \text{two tricks}
\end{align*}
\]

b. \([1] = C[\text{taught}, [t_1 + t_2, t_1 + t_3, t_2 + t_3]] = [\text{taught}(t_1) + \text{taught}(t_2), \text{taught}(t_1) + \text{taught}(t_3), \text{taught}(t_2) + \text{taught}(t_3)]
\]

c. \([2] = (\lambda R_{(e,a)}^* \cdot p^\ell \{ + (\{ D(f(x), x) : x \in A_e \land \text{dog}(x) \}) : f \text{ is a function from } \{ x \mid x \in A_e \land \text{dog}(x) \} \text{ to } p^\ell financier(R^*) \}) \}([1])\]

\[
\begin{align*}
\text{taught} \quad & \quad \text{taught}
\end{align*}
\]
taught(t_2)(eric) + taught(t_3)(eric),
\text{taught}(t_1)(carl) + \text{taught}(t_2)(carl) + \text{taught}(t_1)(dean) + \text{taught}(t_3)(dean) + \text{taught}(t_1)(eric) + \text{taught}(t_3)(eric), \ldots]

\begin{enumerate}
\item Sowohl Ada als auch Bea haben die zwei Cateringfirmen angerufen.
\item Die Lehrerinnen haben sowohl den Buben als auch den Mädchen zwei Hunde gezeigt.
\end{enumerate}

\textbf{Note:} The additional $\uparrow$ shifts on the subject and the VP are necessary here since we have a plurality of quantifiers which can’t directly combine with the predicate via functional application. After the shift, the subject and the predicate can combine via cumulation. The additional ‘level’ of plural sets created by these shifts accounts for the distributivity of quantifier conjunctions.
a.  \[
\begin{array}{c}
ed
\end{array}
\]

\[
\begin{array}{c}
ed
\end{array}
\]

\[
\begin{array}{c}
ed
\end{array}
\]

die Lehrerinnen

\[
\begin{array}{c}
ed
\end{array}
\]

b.  \[
\begin{array}{c}
ed
\end{array}
\]

(1) [show(carl + dean, carl + eric, dean + eric)]

(2) [show(carl) + show(dean), show(carl) + show(eric), show(dean) + show(eric)]

c.  \[
\begin{array}{c}
ed
\end{array}
\]

(3) [(λR_\{e,(e,t)\}^* C(R^*, \{che + ed\})) + (λR_\{e,(e,t)\}^* C(R^*, \{ada + bea\})]

d.  \[
\begin{array}{c}
ed
\end{array}
\]

(4) [(λR_\{e,(e,t)\}^* C(R^*, \{che + ed\})) + (λR_\{e,(e,t)\}^* C(R^*, \{ada + bea\})]

e.  \[
\begin{array}{c}
ed
\end{array}
\]

(5) [C([(λR_\{e,(e,t)\}^* C(R^*, \{che + ed\})) + (λR_\{e,(e,t)\}^* C(R^*, \{ada + bea\})], [show(carl) + show(dean), show(carl) + show(eric), show(dean) + show(eric)])

(6) [(λR_\{e,(e,t)\}^* C(R^*, \{che + ed\})) + (λR_\{e,(e,t)\}^* C(R^*, \{ada + bea\})]

(7) [C([show(carl) + show(dean), show(carl) + show(eric), show(dean) + show(eric)], [che + ed]) + C([show(carl) + show(dean), show(carl) + show(eric), show(dean) + show(eric)], [ada + bea])]

(8) [show(carl)(che) + show(dean)(ed), show(carl)(ed) + show(dean)(che), show(carl)(che) + show(eric)(ed), show(carl)(ed) + show(eric)(che), show(dean)(che) + show(eric)(ed), show(dean)(ed) + show(eric)(che), ...]

(9) [show(carl)(ada) + show(dean)(bea), show(carl)(bea) + show(dean)(ada), show(carl)(ada) + show(eric)(bea), show(carl)(bea) + show(eric)(ada), ...]

(10) [show(carl)(che) + show(dean)(ed) + show(carl)(ada) + show(dean)(bea), show(carl)(che) + show(dean)(ed) + show(carl)(ada) + show(dean)(bea), show(carl)(che) + show(dean)(ed) + show(carl)(ada) + show(dean)(bea), show(carl)(che) + show(dean)(ed) + show(carl)(ada) + show(dean)(bea), ...]