Pluralities across categories and plural projection *

Viola Schmitt
University of Vienna
viola.schmitt@univie.ac.at

Abstract
This paper proposes an extension of the class of plural expressions, a generalized analysis of the denotations of such expressions and a novel account of how they semantically combine with other elements in the sentence. The point of departure is the observation that definite plural DPs and and-coordinations with coordinates of several semantic categories share certain features – in particular cumulativity – in the context of other plural expressions. Existing analyses of conjunction fail to derive these parallels and I propose that and-coordinations should be analyzed as denoting pluralities (of whatever kind of semantic object their conjuncts denote). This, in turn, raises the question of how pluralities combine with other material in the sentence. I show that a simple expansion of the standard analysis thereof, which puts the workload onto the predicate, is insufficient and subsequently propose an alternative which is based on the idea that all semantic domains contain pluralities and involves ‘plural projection’: The truth-conditions of sentences containing plurality-denoting expressions are not due to the semantic expansion of the predicate (as in existing analyses), but the result of a step-by-step process – once a plurality enters the derivation, the node immediately dominating it will also denote a plurality, namely of the values obtained by a particular combination of the plurality and the denotation of its sister.

1 Introduction
We usually assume that definite plural DPs such as the cats denote special objects, namely pluralities of individuals, because their behavior differs from that of singular (non-collective) proper names and from the behavior of expressions involving universal quantification over atomic individuals. In the following, one of these symptoms of plurality will be particularly relevant: cumulativity. This refers to the observation that a plurality can be attributed a property if that property is the result of ‘adding up’ the properties of the plurality’s parts (cf. Link 1983, Krifka 1986 a.o.): If the two sentences in (1-b) are true – and Abe and Bert are the only salient boys and Carl and Dido are the only salient cats – then (1-a) is true, albeit neither Abe nor Carl...
individually fed the two cats. Note that the plural DP differs from the universal quantifier every NP, as the truth of (1-c) does not follow from the truth of the two sentences in (1-b).

(1) a. *The two boys fed the two cats.*  
    b. *Abe fed Carl. Bert fed Dido.*  
    c. *Every boy fed the two cats.*

If cumulativity is a distinctive trait of plurality in the individual domain, we can use it as a diagnostic for analogous denotations (pluralities) in other domains. Taking this rationale as my point of departure, I submit that the class of expressions denoting pluralities is much bigger than usually assumed and that in fact any semantic domain contains a proper subdomain of pluralities. I then argue that if we admit these new pluralities to our system, we can formulate a new analysis of cumulativity. Both claims are briefly outlined in the following.

### 1.1 Claim 1: Expanding the class of pluralities

Following Schmitt (2013), I will first show that cumulativity (and to a certain extent also other symptoms of plurality) can be observed for English *and*- coordinations (henceforth ‘conjunctions’) with conjunctions of several semantic categories. I argue that therefore all types of conjunctions – e.g. conjunctions with individual-denoting conjuncts as in (2-a) (cf. Link 1983, Schwarzschild 1996 a.o.), predicate conjunctions, (2-b), and sentential conjunctions, (2-c) – denote pluralities (of individuals, predicates of individuals and propositions, respectively). Hence the relation between [{dance and smoke}] and [{dance}] is analogous to that between [{the two boys}] and [{Abe}] etc.

(2) a. *Abe and Bert*  
    b. *dance and smoke*  
    c. *(that) Abe is in jail and (that) Bert is in rehab*

I implement this claim by proposing that the ontology does not only contain pluralities of individuals (cf. Link 1983) or other primitives such as events (cf. e.g. Landman 2000) or worlds (cf. Schlenker 2004) but that any semantic domain $D_a$ (where $a$ ranges over semantic types) includes a set of pluralities made up from objects of $D_a$. This means that there are pluralities of functions (cf. Gawron and Kehler 2004 for a similar claim) and also pluralities of sentence denotations (cf. Beck and Sharvit 2002 for a related proposal concerning pluralities of questions). I will model this by enriching all semantic domains $D_a$ are enriched by two kinds: a set $\text{PL}_a$ of pluralities, which is isomorphic to $\varphi(D_a) \setminus \{\emptyset\}$ and a set $S_a = \varphi(\text{PL}_a)$. A plural expression of type $a$ will denote a singleton from $S_a$, containing an object from $\text{PL}_a$, which is the ‘sum’ (represented by ‘+$’ below) of the denotations of the individual conjuncts, as illustrated in (3) for the examples in (2).

(3) a. $\{\text{[Abe]} + \text{[Bert]}\}$  
    b. $\{\text{[dance]} + \text{[smoke]}\}$  
    c. $\{\text{[Abe is in jail]} + \text{[Bert is in rehab]}\}$

The reason why we need two sets ($\text{PL}$ and $S$) is connected to the way in which plurality-denoting expressions combine with their sisters, which comprises the second claim of this paper – namely, that adding new pluralities will give us a new perspective on how to derive cumulativity as a cross-categorial ‘symptom’ of plurality.
1.2 Claim 2: Cumulativity and plural projection

Sentences like (1-a) above (repeated in (4)), which contain two or more plural expressions (where ‘plural expression’ stands for any expression denoting a plurality given claim 1) exhibit particular weak truth-conditions (cf. Langendoen 1978[a.o.]). I here refer to them as ‘cumulative truth-conditions’: (4) is true iff each of the two boys fed at least one of the two cats and each of the two cats was fed by at least one of the two boys.

(4) [plural1 The two boys] fed [plural2 the two cats].

Theories that only consider pluralities of individuals or other primitives derive those truth-conditions by what I will call the ‘predicate analysis’, namely, by positing cumulation operations on predicate denotations (cf. Link 1983, Krifka 1986, Sternefeld 1998). For (4), this means that the primitive extension of the predicate feed is enriched by all pairs of individuals that we can form by simultaneously adding up feeders and their respective feedees: (5) then comes out as true iff each of the two boys fed at least one of the two cats and each of the two cats was fed by at least one of the two boys.

However, the predicate analysis faces two problems (cf. also Schmitt (2013)). The first one is rooted in its prediction that since cumulation targets predicates (like feed above), we will only find cumulative truth-conditions if the object language provides an adequate predicate that can act as the input to cumulation. Beck and Sauerland (2000) argue that this predicate must sometimes be syntactically derived, because we find cumulative truth-conditions where the required predicate is not a surface constituent. Yet, I will show that the syntactic operations we would require to form this predicate do not always correspond to those independently attested.

The second problem concerns configurations like (5), where, according to the view taken here, one plural expression (plural 2) contains another (plural 3). We will see that the predicate analysis doesn’t consistently derive the correct truth-conditions for such configurations.

(5) [plural1 The boys] [plural2 fed [plural3 the two cats] and watched TV]

I then propose an alternative way of deriving cumulative truth-conditions, which builds on my claim that we find pluralities of objects from any semantic domain. The basic idea is that once a plural enters the derivation, every node above it will also denote a plurality (hence ‘plural projection’). Broadly speaking, if a plural like the two cats combines with its syntactic sister (e.g. fed), the result is a particular subset of the set of those pluralities we would obtain by applying the parts of the function plurality to the parts of the argument plurality: For any function plurality F with parts \( f_{a,b} \), argument plurality X with parts \( x_a \), it gives us the set of the pluralities created by applying each F-part to some X-part and each X-part being the argument of some F-part. (So in effect, our notion of cumulation from above is now part of a compositional rule and concerns function-argument pairs). Glossing over the details, this means that for (6-a) we obtain the denotation in (6-b) – which is analogous to the denotation of predicate conjunctions like dance and smoke sketched in the previous paragraph.

(6) a. fed the two cats
   b. \( \lambda x.x \text{ fed Carl} + \lambda x.x \text{ fed Dido} \)

If the VP in (6-a) combines with a plural subject, as in (7-a), we get a plurality of propositions, (7-b). The final step in the matrix case will be the application of an abstract singular operator, which yields us true iff at least one element of the set is true. (7-a) thus comes out as true if Abe fed Carl and Bert fed Dido or Abe fed Dido and Bert fed Carl and as false otherwise.
(7)  
  a. *The two boys fed the two cats.*
  b. { Abe fed Carl + Bert fed Dido, Abe fed Dido + Bert fed Carl }

This illustrates, if somewhat sketchily, that the system I propose derives the correct truth-
conditions for simple sentences like (8-a). I will show that it can also capture the truth-
conditions of sentences with more than two plurals as well as, crucially for my line of argu-
ment here, those of sentences involving ‘plural-within-plural’ configurations such as [5] above,
which the predicate analysis struggles with. Since no movement is involved in the derivation,
the system, as presented here, is not constrained by locality at all and thus circumvents the
syntactic problem encountered by the predicate analysis.

What I formulate here is the backbone of a theory of plural composition. A number of
configurations – including those with collective predicates and cases where a plural expression
is embedded by quantificational material as in (8) – will require an expansion of this system.
I believe such an expansion to be possible (see Haslinger and Schmitt (2018)), but since a
proper account would not only warrant a technical discussion, but also a detailed empirical
investigation of the phenomena we want to capture (cf. e.g. Heycock and Zamparelli 2005,
Champollion 2015), I only give a general indication of what such an expansion could look like.

(8)  
  *Abe fed {every [plural dog and cat]} in this town.*

1.3 Structure of the paper

The paper is structured as follows: Section 2 presents the empirical parallels between conjunc-
tions and DP-plurals and shows that existing theories of conjunction fail to capture them. In
section 3, I introduce the predicate analysis and show how we could expand it to explain the em-
pirical facts from section 2. I then argue that such an expansion is on the wrong track because
the predicate analysis itself is faced with serious problems. In section 4, I introduce generalized
plural denotations and plural projection and apply the system to the examples discussed up to
this point. Section 5 concludes the paper addresses questions for future research.

2 The empirical motivation for cross-categorial plurality

Since the truth-conditions of (9-a) and (9-b) are similar, examples like (9-a) might suggest that
the denotations of plural DPs are reducible to universal quantification over atomic individuals
(cf. Winter 2001a for parallel examples).

(9)  
  a. *These ten boys are wearing a dress.*
  b. *Every boy is wearing a dress.*

In analogy, examples like (10-a) could suggest that conjunction is ‘intersective’, (10-a), as it is
true iff Abe has all the properties denoted by the individual conjuncts.

(10)  
  a. *Abe smoked and danced.*
  b. \([\text{smoke and dance}] = \lambda x, \text{smoke}(x) \land \text{dance}(x)\)

But once we consider a wider range of contexts, identifying the semantic impact of plural DPs
with universal quantification becomes untenable. Cumulativity, in particular, rules out such a
treatment. The following will show that the contexts where we witness cumulativity for plural
DPs reveal analogous effects for conjunctions with conjuncts of several semantic categories,
e.g. predicate conjunction as in (10-a). Accordingly, conjunctions cannot be analysed as inter-
sective – rather, their behavior mimics that of plural DPs.

The first part of the claim, i.e. that conjunction is not intersective, is not new. [Link 1983, 1984], [Krifka 1990], [Heycock and Zamparelli 2005] and others argue for non-intersective analyses of conjunction and some of the examples discussed below are modelled on examples from this literature. Nethertheless, the parallelism between conjunctions and plural DPs will actually turn out to be incompatible with their basic assumptions.

### 2.1 Cumulativity

Sentences containing more than one plural DP are one context that reveals ‘symptoms’ of plurality. For the moment, I will focus on examples such as the sentence marked by [S ...] in (11), where two plural DPs occur as co-arguments of a transitive predicate. (I will frequently give larger chunks of discourse where the relevant sentence is indicated by [S ...]. Whenever I write ‘the sentence in (n)’, this refers to the bracketed sentence in (n).)

(11) *I walked the dog. [S The two boys fed the two cats]*.

More precisely, such sentences give rise to peculiar truth-conditions (cf. [Langendoen 1978 a.o.]), which I here refer to as ‘cumulative truth-conditions’. The sentence in (11) is true, for instance, in a scenario where there are two cats – Carl and Dido – and my brother Abe fed Carl and my other brother Bert fed Dido.

Generalizing over the verifying scenarios (but maintaining that Abe and Bert are the only salient boys and Carl and Dido the only salient cats), the truth-conditions are those in (12-a).

(12) \[
\text{[The two boys fed the two cats]} = 1 \iff \\
\text{a. } \forall x \in \{a,b\} (\exists y \in \{c,d\} (x \text{ fed } y)) \land \forall y \in \{c,d\} (\exists x \in \{a,b\} (x \text{ fed } y)) \\
\text{b. } \forall x \in \{a,b\} (\forall y \in \{c,d\} (x \text{ fed } y))
\]

In the following, I will consider expressions that display a parallel behavior, i.e. strings like the one schematised in (13-a) with the truth-conditions in (13-b). (If the string has these truth-
conditions, I will say that *A and B* ‘display cumulativity’.) For the time being, I won’t specify the denotations of *A*, *B*, but appeal to an intuitive relation ‘consist-of’ in the meta-language. This should be sufficient for our present purposes, a proper discussion will follow in section 3.1. Furthermore, I discuss only a limited range of data, omitting collective construals of predicates and restricting the examples to cases where *R* is a binary relation.

(13) a. \( A \ R \ B \)

b. \( 1 \iff \forall x \in S_A (\exists y \in S_B (R(y)(x) = 1)) \land \forall y \in S_B (\exists x \in S_A (R(y)(x) = 1)) \)

where \( S_A, S_B \) are sets of objects that \([A]\) and \([B]\) consist of intuitively

Conjunctions with individual-denoting conjuncts are known to display cumulativity (as well as all other hallmarks of plurality, cf. [Link 1983], [Schwarzschild 1994 a.o.]): If Abe and Bert are the only boys, the truth-conditions of (14) and of (11) above are identical.

(14) *Abe and Bert fed the two cats*.

The subsequent paragraphs will discuss cumulativity of conjunctions whose conjuncts denote more complex objects, namely, predicates of individuals and propositions.
2.1.1 Cumulativity with predicate conjunctions

(15) gives two examples which show that predicate conjunctions of syntactic category VP can display cumulativity.\footnote{There is no cross-linguistic survey, but similar facts can be found in several Indo-European languages as well as Hebrew (Yael Sharvit, pc), Iraqi Arabic (Yusuf Al-Tamimi, pc) and Hungarian (Edgar Onea, pc).}

(15) a. Good Lord! The farm is on fire.\[ S [ A the ten animals] are \[ B [crowing]\_P and \[ B [barking]\_Q!] \] And the farmer is singing Auld Lang Syne! adapted from \cite{Krifka1990}

b. \[ S [ A The children in my class] are \[ B [blond]\_P and \[ B [brunette]\_Q]], but the ones in Sue’s class all have red hair... how strange!

The sentence in (15-a) is true in a scenario with five roosters and five dogs, where the dogs are barking and the roosters are crowing. In fact, it is true in any scenario where all of the farm animals are crowing or barking and some are crowing and some are barking – which means that it has the cumulative truth-conditions in (16). (15-b) is analogous: (15-b) is true in a scenario where half of the children in my class are blond and the other half is brunette.

(16) 1 iff \( \forall x \in S_A(\exists Y \in \{P, Q\}(Y(x) = 1)) \land \forall Y \in \{P, Q\}(\exists x \in S_A(Y(x) = 1)) \)

Contrary to \cite{Winter2001b}, who essentially claims that cumulative truth-conditions for VP-conjunctions are only observable if the denotations of the conjuncts are disjoint as in (15-a,b),\footnote{‘disjoint’, in Winter’s sense, means that it is impossible or highly unlikely given our world-knowledge that an atomic individual has both properties expressed by the conjuncts simultaneously. While his claims are too strong \cite{Poortman2014}’s work might be on the right track: She provides experimental evidence for the claim that in conjunctions \( P \) and \( Q \), the availability of cumulativity of the conjunctions decreases with the level of ‘typicality’ that speakers assign to co-occurrence of \( P \) and \( Q \) in an atomic individual.} (17-a,b) show that non-disjointness of the conjuncts’ denotations does not block cumulativity: The denotations of smoking and dancing in (17-a) are not disjoint, but the truth-conditions of the sentence again fit the pattern in (16). It is true, for instance, if four of the children are smoking, while the other six are dancing. Likewise drink and smoke are not disjoint, but (17-b) exhibits cumulative truth-conditions: If 50 villagers are non-smoking drinkers, 20 are smokers that don’t drink, and the remaining ones both drink and smoke, the sentence is true.

(17) a. What a party! \[ S [ A The ten children I invited] are \[ B [smoking]\_P and \[ B [dancing]\_Q] in the street] and the adults are getting drunk in the living room.

b. Absurd! \[ S [ A The people in this village] \[ B [smoke]\_P and \[ B [drink]\_Q]], but none of them has ever eaten a steak!

2.1.2 Cumulativity with propositional conjunctions

Propositional conjunctions also exhibit cumulativity. Consider first the example in (18).

(18) The agency from Chechnya called and the one from the Philippines. The conversations were useless. \[ S [ A The agencies] claimed \[ B [that Kadyrov had acquired WMD]\_P and \[ B [(that) Duterte had hired a death squad]\_Q]], but neither of them had anything to say about Trump’s alleged interactions with Putin.

In a scenario where the agency from Chechnya made the claim about Kadyrov and the one from the Philippines the claim about Duterte, the sentence is true. Generalizing over such verifying scenarios, we obtain the cumulative truth-conditions in (19).

(19)
Cumulativity of propositional conjunctions is also found when the embedding verb is an attitude predicate like believe. The sentence in (20) is true in a scenario where Abe holds the belief about cats but has no opinions about lizards, whereas Bert is certain that Trump is a lizard but agnostic w.r.t. the role of cats in the world, which means that it has cumulative truth-conditions.

(20) Abe may be the head of the cat cult and Bert a member of the reptile cult, but they are not as bad as you think – okay, \([S_A \text{ they believe} R_B \{\text{that cats rule the world}\}_p \text{ and } \{\text{(that) the US president is a lizard}\}_q]\), but neither of them would maintain that aliens run the NYT – as your ‘sane’ friend Gina does.

These data thus suggest that propositional conjunctions, too, mimick the behavior of plural DPs w.r.t. cumulativity.

But couldn’t we say that Abe and Bert hold the collective belief \([p] \cap [q]\) – and that thus \(p\) and \(q\) simply denotes \([p] \cap [q]\)? In the scenario I gave, Abe is agnostic w.r.t. \([q]\) and Bert \([p]\), but maybe the content of collective belief can be described in terms of what Abe and Bert agree on, in the sense that it simply gives us the intersection of Abe’s and Bert’s belief worlds. This would only require \([q]\) to be compatible with Abe’s beliefs, and \([p]\) to be compatible with Bert’s beliefs. However, we can rule out this possibility by examples like (21), which would involve conflicting beliefs. The sentence in (21) is true if Abe believes the next president to be a cat and Bert believes the next president to be a lizard. Thus in all of Abe’s belief worlds, \([q]\) is false, and vice versa for Bert and \([p]\). (And we don’t get the feeling that (21) attributes inconsistent beliefs to Abe or Bert.) Hence (21) is incompatible with \(p\) and \(q\) denoting the set of worlds that Abe and Bert agree on.

(21) Abe may be the head of the cat cult and Bert a member of the reptile cult, but they are not as bad as you think – okay, \([S_A \text{ they believe} R_B \{\text{that the next president will be a cat}\}_p \text{ and } \{\text{(that) the next president will be a lizard}\}_q]\), but neither of them would maintain that the next president will be an alien- as your ‘sane’ friend Gina does.

2.2 Claims of this paper and the meaning of and

The previous paragraphs have shown that individual conjunctions, predicate conjunctions (of VP) and propositional conjunctions display cumulativity. Their behavior thus mimics that of plural DPs. But since conjunctions have been extensively discussed in the semantic literature, it stands to reason that some existing account should derive these observations. In the following, I argue that this is not the case, despite first appearances.
2.2.1 Intersective and non-intersective and

Concerning the underlying meaning of English and and analogous expressions in other languages, we can distinguish two positions: Those that take the meaning of and to be uniformly intersective (‘distributive’, ‘Boolean’) and those that view it as uniformly non-intersective (‘collective’, ‘non-Boolean’).

Intersective theories (cf. von Stechow 1974, Partee and Rooth 1983, Gazdar 1980, Keenan and Faltz 1985, Winter 2001, Champollion 2015 a.o.) start off with the assumption that and in sentential conjunction is analogous to the operation ‘∧’ on truth-values in classical propositional logic: Accordingly, (22) is said to be true iff both p and q are true and false otherwise (but see paragraph 2.3).

(22) \( [p \text{ Abe went to the office}] \text{ and } [q \text{ Bert went to the gym}] \).

The general idea of such proposals (but cf. Keenan and Faltz 1985 for a slightly different view) is to ‘retrieve’ the semantic impact of and on \( D_t \) in other semantic domains for \( t \)-conjoinable types (types ending in \( t \)) and thus to account for the fact that and does not only conjoin sentences but expressions of various semantic types. The meaning of and is defined as the type-polymorphous operation ‘\( \sqcap \)’ which is recursively expanded from \( D_t \), (23):

(23) \[ X \sqcap Y = \begin{cases} X \land Y & \text{if } X, Y \in D_t \\ \lambda Z, Z(X) \sqcap Y(Z) & \text{if } X, Y \in D_{(a,b)} \text{ and } \langle a, b \rangle \text{ is } t \text{-conjoinable} \end{cases} \]

To see the point, take the predicate conjunction in (24): According to (23), conjoining two elements from \( D_{(e,t)} \) gives us another element of \( D_{(e,t)} \), namely that function which maps any individual \( x \) to 1 iff \( x \) both dances and smokes.

(24) \( [[\text{smoke and dance}]] = \lambda x, \text{smoke}(x) \land \text{dance}(x) \).

It should be clear that without any additional assumptions (such as those in Winter 2001a, Champollion 2015), the intersective theory is incompatible with the data discussed above. (24) gives us a distributive predicate of individuals (distributive because one of its conjuncts, ‘smoke’ is distributive). Thus, its truth-conditions in sentences with a plural subject should be analogous to those of sentences with non-conjoined distributive predicates like (25).

(25) The ten children are smoking.

As (25) is true iff each of the ten children are smoking, the example from (17-a) above, repeated in (26), should be true iff each of the ten children is both smoking and dancing.

(26) The ten children are smoking and dancing.

We saw above that the actual truth-conditions of (26) are much weaker: The sentence is true iff each of the ten children is smoking or dancing and there are both smokers and dancers among the children. In other words, the intersective theory of and, in the ‘bare’ version I reproduced here, cannot account for cumulativity of predicate conjunctions.

In fact it was data like those in (26) which, among other observations, motivated non-intersective theories of and (cf. Link 1983, 1984, Krifka 1990, Heycock and Zamparelli 2005).

\[ ^4 \text{Cf. Winter 2001a, Flor et al. (2017) on reasons why the hypothesis that English and is ambiguous between an intersective and a non-intersective meaning is unattractive.} \]

\[ ^5 \text{Cf. Geach 1970 and van Benthem 1991 for the syntactic prerequisites.} \]
The gist of such theories is, in a sense, the inverse of intersective theories: Rather than considering the semantic impact of *and* in propositional conjunction as basic, they take its role in individual conjunction, such as (27), as the point of departure.

(27) *Abe and Bert*

The assumption is that in contexts such as (27), *and* denotes the operation that forms pluralities of individuals from (pluralities of) individuals (cf. [Link 1983]). I have not defined this operation yet, so I will simply represent it here by ‘+’ – it will be sufficient to say that 

\[
\llbracket \text{Abe} \rrbracket + \llbracket \text{Bert} \rrbracket
\]

is identical to 

\[
\llbracket \text{the two boys} \rrbracket,
\]

if \( \llbracket \text{boy} \rrbracket = \{ \text{Abe, Bert} \} \). The idea is to recursively define the denotation of *and* for all conjuncts of e-conjoinable types – defined in (28)– on the basis of ‘+’, as in (29), where \( \sqcup \) represents the (type-polymorphous) denotation of *and*.

(28) \( e \) is an *e*-conjoinable type and if \( a_1, \ldots, a_n \) are *e*-conjoinable types, then \( \langle (a_1) \ldots (a_n) \rangle t \) is an *e*-conjoinable type.

(29) \[
X \sqcup Y = \begin{cases} 
X \cup Y & \text{if } X, Y \in D_e \\
\lambda Z_a, \exists Z_1, Z_2 [ Z = Z_1 \sqcup Z_2 \land X(Z_1) \land Y(Z_2) ] & \text{if } X, Y \in D_{\langle a, t \rangle} \text{ and } \langle a, t \rangle \text{ is } *e*-conjoinable \\
\lambda Z_{a_1}, \ldots, Z_{a_n}, \exists Z_1^1, Z_1^2, \ldots, Z_n^1, Z_n^2 [ Z_1^1 \sqcup Z_2^1 = Z_1^2 \land \ldots \land Z_1^n \sqcup Z_2^n = Z_n^1 \land \ldots \land X(Z_1^1) \ldots (Z_1^n) \land Y(Z_2^1) \ldots (Z_2^n) ] & \text{if } X, Y \in D_{\langle \langle a, t \rangle \ldots (a, t) \ldots \rangle \rangle} \text{ and } \langle a_1, \ldots, a_n, t \ldots \rangle \text{ is } *e*-conjoinable
\end{cases}
\]

For the predicate conjunction in (30), this yields that function that maps any (plurality of) individuals to 1, just in case it exclusively consists of ‘parts’ that smoke and ‘parts’ that dance – the characteristic function of the set of those (pluralities of ) individuals that are the result of adding up smokers and dancers.

(30) \( \llbracket \text{smoke and dance} \rrbracket = \lambda x \ldots \exists y, z[y + z = x \land \text{smoke}(y) \land \text{dance}(z)] \)

Glossing over the problems resulting from my informal treatment, (29) thus derives the correct truth-conditions for our sentence in (17-a) above, repeated again in (31): (31) is predicted true iff we can split up the group of ten children completely into smokers and dancers (which doesn’t exclude the possibility that some or all children do both).

(31) *The ten children are smoking and dancing.*

In other words, non-intersective theories of conjunction derive cumulativity for predicate conjunction – so why not simply use such an analysis to account for the data above?

### 2.2.2 Why existing non-intersective analyses of *and* are insufficient

Ignoring all other problems for non-intersective theories (cf. [Kritka 1990] and most recently [Champollion 2015]), they include one component that makes them unfit to explain cumulativity of conjunctions: They require what I will henceforth call ‘semantic locality’. This means that these proposals essentially capture the impact of *and* in conjunctions with functional denotations (such as predicate conjunction) in terms of its impact on the arguments of that function (the same holds for intersective theories, of course). As a result, the ‘cumulative relation’ we observed above is predicted to only hold between conjunctions and those elements that either denote an argument of the conjunction or themselves take that conjunction as an argument.

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6\( t \) still represents a special case.
Crucially, this predicts that we should never find cumulative truth-conditions for sentences with an plural DP (or an individual conjunction) $A$ and a predicate conjunction $B$ where $A$ is not (on some level) an argument of $B$ – and this prediction is falsified by sentences like that in (32): It contains predicate conjunction in the embedded clause and a plural DP (the ambassadors) as the subject of the matrix clause. Importantly, this plural DP is not an argument of the predicate conjunction in the embedded clause – the subject of the latter is Trump.

(32)  Diplomacy is useless! The Georgian ambassador called this morning and the Russian one this afternoon. [$S_A [A \text{ The ambassadors]}$ kept insisting that Trump must [$B \text{ take a walk with Putin and build a golf club in Tbilisi}$], but neither of them said anything about the really pressing issue – the Caucasian conflict.]

Nevertheless, the sentence has cumulative truth-conditions and the cumulative relation, so to speak, holds between the matrix subject and the embedded predicate conjunction: The sentence is true in a scenario where the Georgian ambassador insists that the president do $P$ and the Russian one insists that he do $Q$ – or, more generally, if (33) holds.

(33)  $\forall x \in S_A (\exists Y \in \{P,Q\}(x \text{ kept insisting that the president must } Y)) \land \forall Y \in \{P,Q\} (\exists x \in S_A (x \text{ kept insisting that the president must } Y))$

(34-a) and (34-b) are analogous to to (33) in the relevant sense and again we find cumulative truth-conditions. The sentences are true if some of the villagers believe that Abe is a murderer, some believe he is a fraud, some believe he is a gambler, some believe he is a hedonist and all of them believe at least one of these things. Crucially, the cumulative relation again holds between the subject of the matrix clause and the predicate conjunction in the embedded clause, but neither is an argument of the other- rather, the conjunction’s argument is he.$^7$

(34)  The people in this village are not as bad as you think. They each have their idiosyncratic theories about Abe, of course...

a.  [$S_A [A \text{ they}]$ believe that he is [$B \text{ a murderer, a fraud, a gambler and a hedonist}$]– but none of them has ever claimed that he is an actual witch.

b.  [$S_A [A \text{ they}]$ consider him (to be) [$B \text{ a murderer, a fraud, a gambler and a hedonist}$]– but none of them has ever claimed that he is an actual witch.

What these data show is that semantic locality is not a prerequisite for cumulativity of a predicate conjunction. What find, once again, that conjunctions (in this case predicate conjunctions), pattern with DP-plurals which also don’t require semantic locality in order to give rise to cumulative truth-conditions (quite obviously so). Existing non-intersective (and, of course, also intersective) theories cannot account for this observation, as semantic locality is built into these theories. (The same holds for all existing theories of conjunction that work with events, rather than individuals, e.g. [Lasersohn 1995, Landman 2000]) Accordingly, we must look for a new explanation for these facts, which I will do in the following.$^8$

---

7As we find the same facts with predicate-topicalization as in (i), which shows that the conjunction forms a constituent, such examples are not (necessarily) reducible to propositional conjunction (at the level of the embedded clause) plus subsequent ellipsis.

(i)  A murderer, a fraud, a gambler and a hedonist, they believe he is – but none of them has ever claimed that he is an actual witch.

8As opposed to the theories discussed above, my proposal below is in fact not about the lexical meaning of and (even though it is currently phrased like this) but rather about denotations of entire coordinate structures.
2.3 Discussion: Parallels between plural DPs and conjunctions

The preceding paragraphs have shown that conjunctions of individuals, VP-predicates and propositions behave analogously to plural DPs in terms of cumulativity and that no existing analysis of conjunction derives these parallels. Before we turn to other analytical options, let me give some examples showing that the parallels between conjunctions (of individuals, predicates and propositions) and plural DPs are not limited to cumulativity – even though my analysis below will primarily deal with the latter.

**Homogeneity** 
Fodor (1970) notes a ‘grey area’ in terms of truth-value judgements when considering sentences with plural DPs and their negation – a phenomenon known as ‘homogeneity’ (cf. Löbner 1987, 2000, Schwarzschild 1994 a.o.). While (35-a) conveys that Dido bit all of the boys, (35-b) conveys that she bit none of them, so neither sentence adequately describes scenarios where she bit some but not all of the boys. Plural DPs thus differ from universal quantifiers over atoms as in (36), where we find no such grey area.

\[(35)\]  
\[\begin{align*} 
& a. \text{Dido bit the two boys.} \\
& b. \text{Dido didn’t bite the two boys.} 
\end{align*}\]

\[(36)\]  
\[\begin{align*} 
& a. \text{Dido bit every boy.} \\
& b. \text{Dido didn’t bite every boy.} 
\end{align*}\]

Judgements relating to homogeneity can be blurry, (Chemla and Križ 2015), but similar effects seem to be observable for conjunctions. For individual conjunctions this has been noted before (cf. Schwarzschild 1994, Szabolcsi and Haddican 2004 a.o.): Whereas (37-a) conveys that Dido bit both Abe and Bert, (37-b) conveys that she bit neither. (Schwarzschild (1994) and Szabolcsi and Haddican (2004) note that it is crucial that \(\text{and} \) is unstressed. This extends to the examples discussed below.)

\[(37)\]  
\[\begin{align*} 
& a. \text{Dido bit Abe and Bert.} \\
& b. \text{Dido didn’t bite Abe and Bert.} 
\end{align*}\]

However, the effect can also be witnessed with predicate and propositional conjunction. (38-a) expresses that Varg was both P and Q, but (38-b) conveys that he was neither (cf. Geurts (2005) for similar data and judgements).

\[(38)\]  
\[\begin{align*} 
& \text{Well, I went on a blind data last with ‘Varg’ night…..} \\
& a. \text{[S He was [kind]_P and [handsome]_Q. But he’s wanted by the police.} \\
& b. \text{[S He wasn’t [kind]_P and [handsome]_Q. But at least he isn’t wanted by the police.} 
\end{align*}\]

This intuition is corroborated by (39): If predicate conjunction \(P\) and \(Q\) didn’t display cumulativity, we shouldn’t observe the (slight) contrast in (39) between the ‘bare’ conjunction in (39-a) and the conjunction modified by \(\text{both}\) in (39-b). In particular, appealing to ‘in-between-scenarios’ without a qualifying \(\text{I mean... or well}\) seems worse in the first case. This is analogous to what we find when constrasting plural DPs and universal quantifiers as in (40).

\[(39)\]  
\[\begin{align*} 
& \text{The party had already been going on for a couple of hours, when Bert arrived.} \\
& a. \text{He didn’t [dance]_P and [smoke]_Q. He only smoked.} \\
& b. \text{He didn’t both [dance]_P and [smoke]_Q. He only smoked.} 
\end{align*}\]

\[(40)\]  
\[\begin{align*} 
& a. \text{Dido didn’t bite the two boys. She only bit Abe.} 
\end{align*}\]
b. Dido didn’t bite every boy/both the boys. She only bit Abe.

Propositional conjunction is analogous: While (41-a) conveys that Abe claimed both p and q, (41-b) seems to prominently convey that he claimed neither.

(41) I just talked to Abe. We discussed his old enemies, Bert, Gina and Joe...

a. He claimed [p that Bert is in jail] and [q (that) Gina is in rehab. But he didn’t say anything about Joe being in hiding.

b. He didn’t claim [p that Bert is in jail] and [q (that) Gina is in rehab. But he did say something about Joe being in hiding.

Selectional restrictions A number of lexical elements combine with both plural DPs as well as conjunctions of various categories and seem to ‘do the same thing’ in each case. respectively in (42) is one such case, (Gawron and Kehler 2004), illustrated here for plural DPs, (42-a), individual conjunction, (42-b), and predicate conjunction, (42-c).

(42) a. The two first amendments refer to the first two commandments, respectively.

b. Abe and Bert fed Carl and Dido, respectively.

c. Abe and Bert smoked and danced, respectively.

Collective predication Finally, one of the reasons why we assume that plural DPs (and individual conjunctions) have ‘special denotations’ is that collective predicates like meet select for them, but not for singular DPs (with count nouns). (Link 1983). von Stechow (1980) discusses the possibility that similar effects can be witnessed for other domains, with (43-a) as a potential case of collective predication with propositional conjunction, and (43-b) as as a potential case of collectivity with predicate conjunction.

(43) a. That Major was good minded and that Napoleon was evil-minded are two distinct facts.

b. To be drunk and to be sober are incompatible properties. (von Stechow 1980:91(64),(65))

3 Pluralities, the predicate analysis and its expansion

So, why do plural DPs and conjunctions with conjuncts of various semantic categories display the parallels witnessed in the preceding section? My proposal will be straightforward: Conjunctions behave like plural DPs because they, too, denote pluralities, namely, of the kind of object their conjuncts denote: Individuals, predicates of individuals, propositions etc. In other words, I will argue that pluralities – objects that are isomorphic to non-empty subsets of the respective domain – exist in any semantic domain.

Implementing this claim, however, will raise a more complex issue: When considering these ‘new pluralities’, we will also have to consider how they semantically combine with other elements of the sentence. At first sight, there seems to be an easy way out: Take the standard story for the denotation of plural DPs and the way they combine with other elements – the ‘predicate analysis’ which I lay out in paragraph 3.1 – and expand it so that it includes pluralities of functions etc. This is sketched in paragraph 3.2. However, closer scrutiny (applied...
in paragraph 3.3) reveals that the predicate analysis itself faces serious problems. Some of them are observable even in simple cases, i.e. when only dealing with plural DPs, but others become evident once we add our ‘new’ pluralities. Accordingly, a simple expansion of it seems to be on the wrong track.

3.1 Pluralsities and the predicate theory

We will start with the standard view of the denotations of plural DPs and of plural predication. I outline a version of what I call the ‘predicate analysis’: Link’s 1983 proposal, which I here sketch using a set-based ontology.\(^{10}\)

3.1.1 The denotations of plural DPs and ε-conjunctions

The basic idea is that the domain of individuals \(D_e\) does not only contain ‘atomic’ individuals, i.e. objects that have no parts but themselves, but also complex objects that do have individuals as their proper parts. In order to capture this idea, we replace the ‘traditional’ domain of individuals, \(A\), by the set of all non-empty subsets of \(A\), (44).

\[(44) \quad D_e := \varphi(A) \setminus \{\emptyset\}, \text{ where } A \text{ is the non-empty set of individuals.}\]

I introduce a simplifying step here and assume, as in (45), that singulars enter the derivation as singletons (but cf. Schwarzchild 1996, Van der Does 1992 for discussion)\(^{11}\) As a consequence, I will now treat every function that standardly would have \(D_e\) has its domain is a partial function with \(PL\) as its domain, i.e. is a function that takes sets of individuals as its arguments (these sets can, of course, be singletons). Hence, an expression like \(Abe\) will have its denotation in \(\{x : x \in PL \& |x| = 1\}\). Plural DPs like the boys will have their denotations in \(PL\).\(^{10}\) gives some terminology conventions that I will adhere to in the following:

\[(45) \quad \begin{align*}
\text{a. } & \text{I write ‘A is a part of B’ i}\,\,\,\text{f} \, \text{iff } A \subseteq B \\
\text{b. } & \text{I write that ‘A is an atomic part of B’ or write ‘A } \subseteq_{\text{AT}} \text{ B’ iff } A \subseteq B \& |A| = 1. \\
\text{c. } & \text{Finally, I will write that ‘C is the sum of A and B’, iff } A \cup B = C.
\end{align*}\]

Given the parallels between plural DPs and individual conjunctions, Link (1983) assumes that and in individual conjunctions is the operation that forms pluralities of individuals from other individuals (atomic or pluralities), (46): It takes any two individuals (atoms or pluralities) and gives us another individual, as illustrated in (47).

\[(46) \quad \text{[]}_{(\varepsilon(\varepsilon))} = \lambda x.\lambda y.\, x \cup y.\]
\[(47) \quad \text{[]}\text{Abe and Bert[]} = \text{[]}\text{Abe}[] \cup \text{[]}\text{Bert}[] = \{\text{Abe }\} \cup \{\text{Bert }\} = \{\text{Abe, Bert}\}.\]

Simplifying greatly, a plural DP denotes the sum of the elements in the NP-denotation, as exemplified by (48).

\[(48) \quad \text{[]}\text{the boys[]} = \bigcup(\text{[]}\text{boys}[])\]

\(^{10}\) Accounts such as Van der Does 1992 and Winter 2001a differ from the one presented here in a number of ways, but none of these differences matter for my present purposes.

\(^{11}\) I won’t address cases where lexical elements of type \(\varepsilon\) are non-atomic.
3.1.2 Cumulation

But when does a predicate hold of a plurality? Ignoring cases where it can do so primitively, i.e. collective predicates, the task is to ensure that ‘cumulative inferences’ are adequately captured by the theory: If both (49-a) and (49-b) are true, then so is (49-c). blond intuitively expresses a property of atomic individuals (but cf. Schwarzschild 1994 for more discussion) – a trait I here take to be lexically specified (but cf. Link 1983). Accordingly, we require a mechanism where the predicate in (49-a) will hold of the plurality \{ Abe, Bert \} in virtue of holding of its atoms, i.e. a mechanism that will allow for a plurality to inherit the properties of its parts.

\begin{align*}
(49) & \text{a. Abe is blond.} \\
& \text{b. Bert is blond.} \\
& \text{c. Abe and Bert are blond.}
\end{align*}

Link (1983) identifies this mechanism with the cumulation-operation ‘*’ on the extension of intransitive predicates, (50). It expands the basic extension of the predicate by closure under set-union.

\begin{align*}
(50) & \text{For any } P \in D_{(e,t)}, \ast P \text{ is the smallest function } f \text{ s.th. for all } x \in D_e, \text{ if } P(x) = 1, \text{ then } f(x) = 1 \text{ and for any } S \subseteq D_e \text{ s.th. for all } y \in S, f(y) = 1, f(\cup S) = 1. \\

\text{In the following, I assume that } \ast \text{ is introduced in the object language by a (silent) morpheme } \text{cum}. \text{ Accordingly, (51-a) is the LF for (49-c), and (51-b) shows that the sentence is true iff both Abe and Bert are in the primitive extension of blond – i.e. iff both Abe and Bert are blond.}
\end{align*}

\begin{align*}
(51) & \text{a. \{[Abe and Bert] \text{cum}^1 \text{blond}\}} \\
& \text{b. } \ast \{\text{blond}\} \text{ ([Abe and Bert])} = 1 \text{ iff } \exists x, y (x \cup y = \{Abe, Bert\} \& \ast \{\text{blond}\}(x) = 1 \\
& \text{& } \ast \{\text{blond}\}(y) = 1)
\end{align*}

Our main focus in section 2, however, was on transitive structures. Crucially, the latter also license cumulative inferences – if both (52-a) and (52-b) are true, so is (52-c) – and accordingly we require a mechanism that derives such cumulative inferences.

\begin{align*}
(52) & \text{a. Abe fed Carl.} \\
& \text{b. Bert fed Dido.} \\
& \text{c. Abe and Bert fed Carl and Dido.}
\end{align*}

It should be evident that we cannot simply reduce the transitive structure to a structure with two intransitives and iterate application of \text{cum}, as in (53) (cf. Sternefeld 1998): The truth-conditions assigned to (53) are much too strong, requiring that each of the two boys fed each of the two cats.

\begin{align*}
(53) & \text{[Abe and Carl] \text{cum}^1 \{1 \text{[Dido and Carl] \text{cum}^1 \{2 [t1 fed t2] \}]}\}}
\end{align*}

Nevertheless, we can make use of the essential idea of cumulation – if not the operation itself. The input, this time, is a transitive predicate, i.e. a function representing a set of pairs of individuals – e.g. the pair of actual feeders and feedees in the case of feed. The cumulation operation ‘**’ (for which I assume the object language representation \text{cum}^2), defined in (54), now ‘passes on’ this property to pairs of pluralities as follows: It expands the original extension of the predicate by adding together feeders while simultaneously adding together their respective feedees (and \text{vice versa}). Accordingly, the cumulated extension of feed will hold of a pair of individuals \langle a, b \rangle iff a exclusively consists of individuals that feed a part of b and b exclusively
consists of individuals that were fed by a part of a (cf. Krifka 1986, Sternefeld 1998).

(54) For any \( P \in D_{e,e,t} \), \( **P \) is the smallest function \( f \) s.th. for all \( x,y \in D_e \), if \( P(x)(y) = 1 \), then \( f(x)(y) = 1 \) and for all \( S,S' \subseteq D_e \), s.th for every \( x' \in S \) there is an \( y' \in S' \) and \( f(x')(y') = 1 \) and for every \( y' \in S' \) there is an \( x' \in S \) and \( f(x')(y') = 1 \), \( f(\bigcup(S))(\bigcup(S')) = 1 \).

(55-a) gives the LF for (52-c), and (55-b) a sketch of the semantic derivation. Note that the result is what we require: The sentence has the ‘cumulative’ truth-conditions observed above. Analogous operations can be defined for \( n \)-transitive predicates with \( n > 2 \) (cf. Sternefeld 1998).

(55) a. \([ \{Abe and Carl\} \{\text{cum}^2 \text{fed}\} \{Carl and Dido\}]\)

b. \( \llbracket \text{cum}^2 \text{fed} \rrbracket \llbracket \{Carl and Dido\} \rrbracket \llbracket \{Abe and Bert\} \rrbracket \)

= ** \( \llbracket \text{fed} \rrbracket \llbracket \{Carl and Dido\} \rrbracket \llbracket \{Abe and Bert\} \rrbracket \)

= 1 if \( \exists x',x,y,y'(x \cup x' = \{Carl, Dido\} \land y \cup y' = \{Abe, Bert\}) \land ** \( \llbracket \text{fed} \rrbracket (x)(y) \land \llbracket \text{fed} \rrbracket (x')(y') \rrbracket \)

The assumption that cumulation operations are realised by actual morphemes \( \text{cum}^n \) in the syntactic structure – rather than being (exclusively) a lexical property of predicates (as argued by Krifka (1986), cf. also Champollion 2010) – is motivated by Beck and Sauerland’s 2000 observation that the predicate targeted by cumulation does not have to be a lexical element. Consider their example in (56): It has cumulative truth-conditions, namely, the sentence is true iff it is the case that of each of the two women wanted to marry at least one of the two men and it holds for each of the two men that at least one of the two women wanted to marry him.

(56) The two women wanted to marry the two men. Beck and Sauerland 2000:356 (19c)

As cumulative truth-conditions, in the theory laid out here, are the result of operations on predicate extensions, it will be the predicate in (57) that has to undergo cumulation. However, none of the lexical elements in (56) corresponds to (57), nor any of the surface constituents.

(57) \( \lambda x.e.\lambda y.e. \) wanted to marry \( x \)

Beck and Sauerland (2000) argue that (57) is syntactically derived by covert ‘tucking-in’ movement, (Richards 1997), resulting in (58), where \( \text{cum}^2 \) is affixed to the constituent denoting (57).

(58) \( \llbracket \{the two women\} \{the two men\} \{\text{cum}^2 [2 \{I \{t_1 \text{ wanted to marry } t_2 \}] \}] \rrbracket \)

We end up with the following picture: Individual conjunctions have their denotations in an identifiable subset of \( D_e \). Their denotations differ from those of singular DPs only in that they have non-trivial parts, i.e. parts other than themselves. Plurals inherit properties of their parts qua predicate cumulation – operations on the predicate’s extension, which, in the case of transitive predicates, derive us the cumulative truth-conditions witnessed in section 2. The predicates targeted by these operations can be syntactically derived.

3.2 Expanding the predicate analysis

In section 2 above I argued that the behavior of predicate and sentential conjunctions mimics the behavior of plural DPs. I will propose a straightforward explanation of these facts:
Conjunctions with conjuncts of type \( a \) behave analogously to plurals because they, too, denote pluralities – namely, pluralities from objects in \( D_a \).

In what follows, I sketch the most salient implementation of this idea, which essentially generalizes the predicate analysis to all semantic domains (cf. also Schmitt (2013)). I then show (in paragraph 3.3) why this account is insufficient. Because it will be replaced by an alternative system in section 4, I keep my sketch of this preliminary analysis as simple as possible.

### 3.2.1 Ontology

I assume that all well-formed expressions (LFs) are semantically categorized, i.e. are assigned a logical type. Here and also in my final proposal in section 4 I will work with the standard set of extensional types in (59).

\[
\text{(59) The set } T \text{ is smallest set } S \text{ s.t. } e \in S, t \in S \text{ and if } a \in S, b \in S, \langle a, b \rangle \in S.
\]

As a first step, I add pluralities to all semantic domains. Note that my treatment of pluralities here deviates slightly (but not in any deep sense) from the story for individuals I gave in the preceding section, owing to the fact that working with simple power sets might lead to confusion when dealing with pluralities of functions to \( D_t \), but also because I will recycle some of the notions from this current, preliminary version in my final treatment in section 4. That being said, we start off with the ‘atomic’ domains in (60-a), which only contain ‘atomic’ objects, i.e. no pluralities of primitives or any other object. We then add, for each domain \( D_a \) the set of pluralities \( \text{PL}_a \) (so we have two sets per type).

\[
\text{(60) a. atomic domains (containing atoms)}
\]

- \( D_e := \) the domain of all atomic individuals
- \( D_s := \) the set of all atomic possible worlds
- \( D_t := \) the domain of truth-values
- \( D_{(a,b)} := \) the set of all atomic functions from \( D_e \)

\[
\text{b. plural domains (containing pluralities)}
\]

For any domain \( D_a \), there is a set \( \text{PL}_a \) and a bijection \( pl_a : (\mathcal{P}(D_a) \setminus \{\emptyset\}) \to \text{PL}_a \)

For the purposes of readability, I will use the notational conventions in (61):

\[
\text{(61) a. (i) I use } a, b, c \text{ for elements of } \text{PL}_e, P, Q \text{ etc. for elements of } \text{PL}_{e,t}, \text{ and } p, q \text{ etc. for elements of } \text{PL}_a \text{ and}
\]

- (ii) \( a, b, c \) for elements of \( D_e, P, Q \) etc. for elements of \( D_{e,t} \), and \( p, q \) etc. for elements of \( D_t \).

\[
\text{b. ‘sums’: For any } a, b \in \text{PL}_a, a \oplus b = pl_a(p|_{pl_a^{-1}(a)} \cup pl_a^{-1}(b)).
\]

\[
\text{c. ‘part-of’: For any } a, b \in \text{PL}_a, a \leq b \iff pl_a^{-1}(a) \subseteq pl_a^{-1}(b).
\]

\[
\text{d. ‘atomic-part-of’: For any } a, b \in \text{PL}_a, a \leq_{\text{AT}} b \iff pl_a^{-1}(a) \subseteq pl_a^{-1}(b) \text{ and } |pl_a^{-1}(a)| = 1.
\]

Lexical meanings are assigned by the function \( L \), which maps any lexical element of primitive type \( a \) to an element in \( D_a \), and lexical elements of functional types to elements in \( D \cup \text{PL} \cup \{f : D \to \text{PL}\} \cup \{f : \text{PL} \to \text{PL}\} \cup \{f : \text{PL} \to D\} \), where \( D \) and \( \text{PL} \) stand for \( \bigcup D_{a \in T} \) and \( \bigcup \text{PL}_{a \in T} \).

\[\text{12This means I don’t introduce special types for plural expressions. Doing so would be more elegant, but would add to notational complexity.}\]
respectively (this simply means that there will be expressions which denote functions from atomic domains to plural domains, from plural domains to plural domains etc.).

I introduce two operations that ‘shift’ the denotations of expressions from one set to another. ‘↓’ in (62-a) takes elements from \( \text{PL}_a \) to \( D_a \). It is only defined if the set-correlate of its argument is a singleton. (62-b) on the other hand gives a \textit{quasi}-trivial ‘shift’-morpheme: For any atomic domain \( D_a \), \( \text{PL}_a \) denotes a function from \( D_a \) to \( \text{PL}_a \).

\[
\begin{align*}
(62) \quad & \text{a. for any type } a, \downarrow a \text{ is a function from } \text{PL}_a \text{ to } D_a. \text{ For any } X \in \text{PL}_a, \downarrow (X) \text{ is defined if } |pl^{-1}(X)| = 1. \text{ If defined } \downarrow (X) = \{ Y \in D_a : pl(Y) = 1 \}.
\end{align*}
\]

I will simplify the derivations for the sake of readability: Unless it could lead to confusion, I drop the type-subscript on functions defined for each semantic domain, omit the ‘trivial’ morpheme in (62-b) from the syntax and drop the correlating operators from the meta-language. This means e.g. that I write \( P(a) \) instead of \( P(\downarrow(a)) \) for some \( P \in D_{(a,t)} \) if \( a \in \text{PL}_a \) if it follows from the context that \( |pl^{-1}(a)| = 1 \). Furthermore, I replace characteristic functions of sets of individuals by upper-case words, e.g. ‘CAT’ stands for ‘\( \lambda x. x \) is a cat’ and accordingly ‘CAT’ for ‘\( pl(\lambda x. x \) is a cat)’.

For the denotations of object-language declaratives, the meta-linguistic sentence will stand for the proposition the object-language sentence expresses.

### 3.2.2 Plural denotations

Definite plurals and conjunctions with conjuncts of any category will denote pluralities. For definite plurals, I assume the definite determiner denotation in (63-a): It takes a predicate extension and gives us the sum of all the individuals in that extension – if that extension is non-empty.

\[
\begin{align*}
(63) \quad & \text{a. } \llbracket \text{the} \rrbracket = \lambda P(\epsilon t) : \{ y : P(y) = 1 \} \neq \emptyset. \bigoplus (\{ pl(y) : P(\downarrow (y)) = 1 \}).
\end{align*}
\]

Conjunctions with conjuncts of type \( a \) will denote the sum of all of the conjuncts’ denotations. Accordingly, I must assume that all conjuncts that don’t have their denotations in \( \text{PL} \) are affixed with \( \text{pl} \). I.e. the LF of \( \text{Abe and Bert} \) is the one given in (64).

\[
\begin{align*}
(64) \quad & \llbracket \text{pl Abe [ and [ pl Bert ]]} \rrbracket
\end{align*}
\]

I assume the meaning in (65-a) for \textit{and} in (65), some examples of its effect are given in (66): A conjunction of with conjuncts of type \( e \) gives us a plurality of individuals, (66-a), a conjunction with conjuncts with functional types, such as \( \langle \epsilon t \rangle \), denotes a plurality of the corresponding functions, (66-b), and a conjunction of declaratives gives us a plurality of propositions, (66-c).

\[
\begin{align*}
(65) \quad & \llbracket \text{and}_{(a,(a,e))} \rrbracket = \lambda X \in \text{PL}_a. \forall Y \in \text{PL}_a. X \oplus Y.
\end{align*}
\]

\[
\begin{align*}
(66) \quad & \text{a. } \llbracket \{ \text{pl Abe} \} \text{ and [ pl Bert ]} \rrbracket = \llbracket \text{and} \rrbracket (\text{Abe})(\text{Bert}) = \text{Abe} \oplus \text{Bert}.
\text{b. } \llbracket \{ \text{pl smoke} \} \text{ and [ pl dance ]} \rrbracket = \llbracket \text{and} \rrbracket (\text{SMOKE})(\text{DANCE}) = \text{SMOKE} \oplus \text{DANCE}
\text{c. } \llbracket \{ \text{pl [ that Kadyrov acquired WMD] } \} \text{ and [ pl [ that] Duterte has hired a death squad] } \rrbracket = \llbracket \text{and} \rrbracket (\text{Kadyrov acquired WMD})(\text{Duterte has hired a death squad}) = \text{Kadyrov acquired WMD} \oplus \text{Duterte has hired a death squad}
\end{align*}
\]

\footnote{For example, an expression of type \( \langle a,b \rangle \) will have its denotation in \( \{ f : D_a \rightarrow D_b \} \cup \{ f : \text{PL}_a \rightarrow \text{PL}_b \} \cup \{ f : D_a \rightarrow \text{PL}_b \} \cup \{ f : \text{PL}_a \rightarrow D_b \}. \]
3.2.3 Generalized Cumulation

The next step is to let these pluralities (of individuals, functions etc.) combine with other material in the clause. First, we need a default rule for matrix pluralities, i.e. conjunction of sentences as in (67). According to (68), (67), which denotes a plurality of propositions, is true in w iff both conjuncts are true in w.

(67) Abe smoked and Bert danced.

(68) $p \in \text{PL}_{(s,t)}$ is true in w iff $\forall q \le_{AT} p, (\downarrow (q))(w) = 1$.

For all other cases, we simply generalise the cumulation rules taken from above, for all t-conjoinable types $a$. In order to avoid any confusion with the operators discussion in section 3.1 I use a new set of symbols, ‘+’, ‘+++’ etc, which expand ‘*’, ‘**’, . . . ’ above.

\[
\begin{align*}
\text{a. } & [ \text{Abe and Bert } ] \text{ cum}^1 [1 \text{ [t slept ]}] \\
\text{b. } & [ a \in \text{PL}_e, \forall y \le_{AT} x \text{ (slept (y))}(\text{Abe } \oplus \text{Bert}) = 1 \text{ iff Abe slept } \land \text{Bert slept} \\
\text{c. } & [\lambda p \in \text{PL}_{(e,t)}, \forall q \le_{AT} p \text{ (Q(Abe))} \text{(SMOKED } \oplus \text{DRANK}) = 1 \text{ iff Abe smoked } \\
\& \text{Abe drank} \\
\end{align*}
\]

Finally, we need syntactic rules that generate the right LFs. For this purpose, I adopt [Beck and Sauerland’s 2000] syntax to all plural expressions, using covert movement of all expressions with denotations in PL, which will be tucking in all those cases where the sentence contains more than one plural expression. The resulting predicates are affixed with +, ++ etc.\textsuperscript{14}

This correctly yields ‘distributive’ truth-conditions for sentences with one plural expression, e.g. an individual conjunction, (70-a), or a predicate conjunction, (71-a). The LFs are given in (70-b) and (71-b), respectively, and the (simplified) semantic derivations in (70-c) and (71-c).

(70) a. Abe and Bert slept.
   b. [ [ Abe and Bert ] cum$^1$[ 1 [ t$_{le}$ slept ]]]
   c. $[\lambda x \in \text{PL}_e, \forall y \le_{AT} x \text{ (slept (y))}](\text{Abe } \oplus \text{Bert}) = 1 \text{ iff Abe slept } \land \text{Bert slept}$

(71) a. Abe smoked and drank.
   b. [ [ smoked and drank cum$^1$[ 1 [Abe [ t$_{le}(e)$.]]]]
   c. $[\lambda p \in \text{PL}_{(e,t)}, \forall q \le_{AT} p \text{ (Q(Abe))} \text{(SMOKED } \oplus \text{DRANK}) = 1 \text{ iff Abe smoked } \\
\& \text{Abe drank}$

For structures with two (or more) plural expressions, e.g. two individual conjunctions, (72-a), or an individual and a predicate conjunction, (73-a), we obtain the LFs in (72-b) and (73-b), respectively. As shown in (72-c) and (73-c), which give the the (simplified) semantic derivations, we obtain the correct cumulative truth-conditions.

(72) a. Abe and Bert fed Carl and Dido.
   b. [ [ Abe and Bert ] [Carl and Dido] cum$^2$[ 2 [1 [t$_{f} \text{ fed t$_{2}$]}}}]
   c. $[\lambda x \in \text{PL}_e, \lambda y \in \text{PL}_e, \exists x', x'', y', y'' (x' \oplus x'' = x \land y' \oplus y'' = y \land \text{fed}(x')(y') \land \text{fed}(x'')(y'')) \text{(C } \oplus \text{D)}(A \oplus B) \\
\text{= 1 iff } \exists x', x'', y', y'' (x' \oplus x'' = C \oplus D \land y' \oplus y'' = A \oplus B \land \text{fed}(x')(y') \land \text{fed}(x'')(y''))$

(73) a. Abe and Bert smoked and drank.
   b. [ [ Abe and Bert ] [smoke and drank] cum$^2$[ 2 [1 [t$_{1} \text{ t$_{2}$]}}}]

\textsuperscript{14}Cf. Schmitt (2013) for the explicit syntactic rules.
c. \( \lambda P \in PL_{(e)} \cdot \lambda x \in PL_e. \exists P', P'', x', x''(P' \oplus P'' = P \land x' \oplus x'' = x \land P'(x') \land (P''(x''))(\text{SMOKED } \oplus \text{ DRANK})(A \oplus B) \land x' \oplus x'' = (A \oplus B) \land P'(x') \land P''(x'')) \)

Since the present system is simply an expansion of the predicate analysis, we don’t require semantic locality w.r.t. the predicate conjunction and the individual plural (as was the case in the non-intersective treatments of conjunction discussed in section 2.2 above). For instance, in (74-a), adapted from (32) above, both expressions are simply treated as plural expressions and we use syntactic movement as in (74-b) to create a relation that can subsequently be cumulated and take the two plurals as its arguments.

(74) a. Abe and Bert insist that the president must take a walk with Putin and build a golf club in Tbilisi.

b. [ [ Abe and Bert ] [take a walk with Putin and build a golf club in Tbilisi] cum² [2 [1 [t₁ insist that the president must t₂ ]]]]

3.3 Why the predicate analysis is insufficient

While this expanded system derives the correct truth-conditions for the sentences considered so far (modulo the simplifications), it cannot be the solution to our problem. The main reason for this doesn’t lie in the expansion – rather, it is the predicate analysis itself that is flawed. Since these flaws are connected to its core assumption, namely, that cumulative truth-conditions are the result of operations targeting the predicate that the plurals occur as (LF)-arguments of, any expansion of this theory will face the same problems.

3.3.1 The syntactic problem for the predicate theory

The ‘syntactic problem’, concerns the question of how the relation targeted by ++ is derived when its object language correlate is not a surface constituent. The data presented in the following show that it cannot be formed by the standard syntactic operations of LF-displacement, as its formation is not constrained by any of the independently attested restrictions on these operations.

Recall that (75), repeated from (56) above, was supposed to show that cumulativity is not (exclusively) a property of lexical predicates but also of syntactically derived predicates – including those that do not correspond to surface constituents. The sentence has cumulative truth-conditions (see above) and the relation \( R \) that must be cumulated, namely (76), is not expressed by any surface constituent.

(75) The two women wanted to marry the two men (Beck and Sauerland 2000:356 (19c))

(76) \( R = \lambda x_1 \lambda y_1. \text{ wanted to marry } x \)

As described above, Beck and Sauerland (2000) (henceforth B&S) argue that the input to ++ is derived by covert tucking-in movement, so we obtain (77) for as the structure for (75).

(77) [ [the two women] [ [the two men] [cum² [2 [1 [t₁ wanted to marry t₂ ]]]]]]

If the relation that forms the input to cumulation is derived by covert movement, it should be subject to the constraints independently attested for this operation – and this exactly is B&S’s
point. They argue that cumulative truth-conditions are only available if the derivation of the syntactic correlate of the relation obeys the constraints on covert movement.

Accordingly, B&S consider configurations where covert movement of quantifiers is blocked\(^{15}\): For instance, quantifiers occurring in finite embedded clauses cannot scope over a quantifier in the matrix (but cf. \cite{Reinhart1997}): (78-a) lacks the reading paraphrased in (78-b)\(^{16}\).

\begin{align*}
(78) & \quad a. \text{ At least one lawyer pronounced that every proposal is against the law.} \\
& \quad b. \text{ For every proposal } x, \text{ there is at least one lawyer } y, \text{ s.th. } y \text{ pronounced } x \text{ to be against the law.}
\end{align*}

B&S predict – correctly, as they argue – that (79-a) should lack the reading in (79-b): It could only be obtained by cumulating the predicate in (79-c), but this predicate cannot be derived by covert movement of the object, as this movement would have to cross a clause-boundary, which (78-a) shows to be impossible.

\begin{align*}
(79) & \quad a. \quad \begin{array}{l}
\{A \text{ The two lawyers}\} \text{ have pronounced that } \{B \text{ the two proposals}\} \text{ are against the law.}
\end{array} \\
& \quad \text{(Beck and Sauerland2000)(43b)} \\
& \quad b. \quad \forall x \leq_{AT} \square [A] (\exists y \leq_{AT} \square [B] (x \text{ pronounced that } y \text{ is against the law})) \land \forall y \leq_{AT} \square [A] (\exists y \leq_{AT} \square [B] (\text{ } x \text{ pronounced that } y \text{ is against the law }))
\end{align*}

The empirical claim that cumulative construals are subject to the same constraints as QR is not correct, however. First, clause-boundedness does not generally block cumulative construals in configurations where it blocks inverse scope. The sentence in (79-a) allegedly lacks the construal in (79-b), but set within the context in (80), this construal is clearly available – the sentence is true in the scenario given.

\begin{align*}
(80) & \quad \text{scenario} \quad \text{The chair of the linguistics department, Dr. Abe, and the chair of the musicology department, Dr. Bert keep coming up with crazy proposals. Last week, Dr. Abe proposed to expel all teachers that didn’t speak Esperanto and Dr. Bert brought forth a motion excluding any student that didn’t play the piano. This morning, there was a meeting with the university lawyers, Dr. Kern, who specialises in the rights of faculty members, and Dr. Marten, the legal representative of the student body. Dr. Kern immediately dismissed Dr. Abe’s proposition, and Dr. Marten declared Dr. Bert’s proposal to be untenable, but both said that the chairs could not be fired on the basis of the behavior.}
\end{align*}

\begin{align*}
& \quad \text{Well, the two lawyers have pronounced that the two proposals are against the law (as was kind of expected) but neither of them supported the dean’s motion to fire Dr. Abe and Dr. Bert immediately.}
\end{align*}

(81-a) makes the same point: The sentence is true in a scenario where the Grosny agency insists that Trump should call Kadyrov and the Manila agency insist that he should call Duterte, which means it has a cumulative construal which would require the predicate in (81-b) to be in input for cumulation – and the syntactic derivation of this predicate would involve the crossing of a clause-boundary.

\(^{15}\)Cf. \citet{Schmitt2013} for a discussion of the argument from English double-object constructions given by B&S.

\(^{16}\)It does not matter here whether covert movement of non-symmetric quantifiers across clause-boundaries is generally impossible (cf. e.g. \cite{Wurmbrand2017} against such a view), since I only consider configurations where inverse scope is impossible and contrast them with analogous sentences that exhibit a cumulative construal.
This morning, our two intelligence agencies called in. First, I spoke to Grosny agency about Kadyrov, then I talked to the Manila agents about Duterte. Both agencies had quite an agenda, and I have no clue how to proceed now. Namely, the two agencies insisted that Trump should call the [two madmen / Kadyrov and Duterte], but I know that he won’t talk to anyone but May.

What is more, we even find cases where the derivation of the required predicate would have to involve movement of the lower plural out of an island for (seemingly much more liberal) overt movement. Assuming a similar context as the one in (81-a), the sentence in (82) is true in the scenario described right above (81). Accordingly it exhibits cumulative truth-conditions, which means the predicate in (81-b) must be cumulated. However, deriving this predicate syntactically would involve movement of the lower plural out of a topicalized clause (cf. Ross 1967).

Well, the two agencies’ plans for Trump were not as horrible as you make it sound. That he should call the two dictators, the agencies insisted, but neither of them said anything about a ‘dictators’ summit’ in the near future.

Analogously, deriving the correct truth-conditions for the sentence in (83-a) would mean that the lower plural would have to move out of an adjunct, namely, the antecedent of the conditional: The sentence is true in the scenario given, accordingly, the predicate in (83-b) would have to undergo cumulation.

If { the two cats / Carl and Dido } move, the two boys have to press a button.

adapted from an example by Manuel Križ (pc)

While all of this runs contrary to the B&S’s point that predicates targeted by cumulation are derived by ‘standard’ covert movement, it does not, as such, falsify the idea that the required predicate is derived by some syntactic mechanism. In particular, we know that indefinites can take scope in positions they cannot ‘reach’ if the standard restrictions on movement applied (cf. e.g. Ruys 1992, Reinhart 1997, Winter 2001a a.o.). This is witnessed by the fact that (84-a) has (84-b) as one of its readings.

If some building in Washington is attacked by terrorists then US security will be threatened.

Winter (2001a) 85 (28)

There is a building in Washington, such that if this building is attacked by terrorists, US security will be threatened.

One could thus argue that definite plurals are as unconstrained as singular indefinites (cf. Winter (2001a) for a parallel treatment of definite and indefinite plurals): Simplifying greatly, they can be interpreted in any position. However, Yoad Winter (pc) points out that this would predict that plural expressions which resist exceptional scope taking should not partake in cumulative construals of sentences where the required relation cannot be derived by standard movement. In particular Winter (2001a) (a.o.) argues on the basis of contrasts like (85) that while indefinites with bare numerals can take exceptional scope (just as some building in (84-a)), those with
modified numerals can’t: According to Winter, (85) does not have the reading paraphrased below.

(85) \[ \text{If exactly two people I know are John’s parents then he is lucky.} \]  

(Winter 2001a:108 (105b))

# There are exactly two people I know, such that if they are John’s parents, he is lucky.

If exceptional scope taking is unavailable for modified numerals, but is the mechanism behind the cumulative construals of examples where a syntactic derivation of the relation would violate a syntactic island (e.g. [80]–[83-a]), then cumulative construals should be unavailable in those configurations if we replace the definite plural by a modified numeral. But (86-a), where the lower plural is a modified numeral, is fine in the context given. Hence, a cumulative construal is available, and in order to derive this construal, we would need to cumulate the predicate in (86-b) – the derivation of which would involve movement out of a tensed clause.

(86) a. \[ \text{My friends Abe and Bert are members of different health cults. So I was hopeful when I consulted each of them on my health issues - I expected at least ten recommendations from each of them. The outcome was disappointing. (Between them) They told me that I should contact exactly two gurus....and that was it! Abe said I should talk to Yasmuheen and Bert insisted I should contact Mr. Leadbeater.} \]

b. \[ \lambda x.\lambda y. y \text{ said I that should contact } x \]

In summary, we find cumulative construals for sentences where the relation that should form the input to cumulation is not a surface constituent and thus has to be derived syntactically. This derivation, however, cannot generally be the result of covert movement, since in a number of cases movement would have to be out of syntactic islands. Furthermore, since at least some of the plural expressions that partake in these cumulative construals are such that they do not license ‘exceptional scope-taking’ in other contexts, we cannot simply assume that the mechanism responsible for this exceptional scope taking in other contexts is the one that will derive us the required relation in the case of cumulative construals. Accordingly, we don’t have a mechanism that derives us the predicates that form the input to cumulation by ++.

3.3.2 The projection problem for the predicate analysis

The second problem for the predicate analysis – the ‘projection problem’ – arises in configurations where one plural expression is contained within another one, as in the examples in (87-b) and (88-b). (This characterisation presupposes the extended view of ‘plural expression’ pursued in this paper.) Crucially for my purposes below, (87-b) is true in the scenario in (87-a), and (88-b) is true in the scenario in (88-a)\[17\]. Generalizing over verifying scenarios, the truth-conditions of (87-b) and (88-b) are those informally paraphrased in (87-c) and (88-c).

(87) a. \[ \text{scenario: Abe fed Carl. Bert fed Dido. Bert brushed golden retriever Eric. Neither Abe nor Bert fed hamster Harry. Harry is dead.} \]

b. \[ \text{Abe and Bert fed Dido and Carl and brushed Eric, but none of them took care of the hamster! (It’s dead!)} \]

\[17\] Examples like (i) with DP-plurals instead of individual conjunctions are completely parallel. Accordingly, we cannot appeal to ellipsis in order to explain the effects witnessed below.

(i) \[ \text{The two boys fed the two cats and brushed the dog, but none of them took care of the hamster! (It’s dead!)} \]
c. Abe and Bert each did one of the following: feed Carl, feed Dido, brush Eric & Carl and Dido were each fed by Abe or Bert & Eric was brushed by Abe or Bert.

(88) a. scenario: Abe made Gina feed his cat Carl, Bert made Gina feed his cat Dido and brush his dog Eric.

b. Abe and Bert made Gina feed Carl and Dido and brush Eric, when all she wanted to do was take care of poor hamster Harry.

c. Abe and Bert each did one of the following: make Gina feed Carl make Gina feed Dido, make Gina brush Eric & Abe or Bert made Gina feed Carl & Abe or Bert made Gina feed Dido & Abe or Bert made Gina brush Eric.

Informally, the embedded plural expression seems to ‘project’ to the embedding plural expression: The VP-conjunctions in (87-b) and (88-b), in which the first conjunct itself embeds an individual conjunction, i.e. (89-a), behaves like the VP-conjunction in (89-b) w.r.t. the cumulative truth-conditions observed above – i.e. each member of the subject plurality has to be in the relevant relation with at least one of the conjuncts in (89-b) and vice versa.

(89) a. \([\text{fed } \{\text{Carl and Dido}\} \text{ and brush Eric}]\)

b. \([[[\text{fed Carl}] \text{ and } [\text{fed Dido}] \text{ and } [\text{brush Eric}]]\]

None of the proposals considered so far – including the expanded predicate analysis – consistently derives us this equivalence. The problem, broadly speaking, is that the embedded plural expression will be inaccessible for any form of cumulated relation with the subject plurality in configurations like (88-b).

Let us first consider why we cannot consistently derive the correct truth-conditions without assuming the expanded predicate theory, namely, by appealing to the non-intersective analysis of conjunction discussed in section 2.2. Recall that under this analysis, a conjunction of predicates \(P, Q\) has the denotation in (90-a) – it holds of all those individuals that have a P-part and a Q-part. If we furthermore assume that the relation expressed by \text{fed} in the first conjunct is cumulated via \(++\), as schematized in (90-b), the conjunction in (87-b) will have the denotation in (90-c). If we apply this function to the subject’s denotation, the sentence is correctly predicted to be true in our scenario in (87-a) (just replace \(x'\) by A\(\oplus\) B and \(x''\) by B).

(90) a. \([P \text{ and } Q] = \lambda x_x''(x' \oplus x'' = x \land P(x') \land Q(x''))\)

b. \([\text{Abe and Bert } [\text{ cum } \text{ fed }] C \text{ and } D \text{ and } \text{ brushed E}]]\)

c. \(\lambda x_x''(x' \oplus x'' = x \land + + [[\text{fed}]](\lambda C D))(C \text{ and } D) \land x'' \text{ brushed E})\)

d. Abe and Bert each did one of the following: feed Carl, feed Dido, brush Eric & Carl and Dido were each fed by Abe or Bert & Eric was brushed by Abe or Bert.

The problem with non-intersective theories of conjunction, however, was semantic locality—they only predict cumulative truth-conditions w.r.t. a predicate conjunction \(P \text{ and } Q\) and some other element \(X\) if \(X\) is an argument of the conjunction or vice versa. In section 2.2 above, I showed this prediction to be false. In analogy, semantic locality prevents a general story for the examples under discussion: The sentence in (88-b) is parallel to (87-b) except that Abe and Bert does not denote an argument of the predicate conjunction (not vice versa). The non-intersective analysis of predicate conjunction, therefore, gives us the (simplified) semantic derivation in (91)—the resulting truth-conditions are clearly too strong.

(91) a. \(\llbracket(88-b)\rrbracket = +[[\text{made}]]([[\text{feed } C \text{ and } D \text{ and } \text{ brush } E}]](G)) ([A,B]) =\)

\(^{18}\)I still use the notational devices introduced for pluralities in the previous paragraphs.
= +[[made]](\lambda_{x_e}.\exists x',x''(x'\oplus x'' = x\land ++[[feed]](C\oplus D)(x')\land x''\ \text{brush} E)(G))(A\oplus B)

b. Abe and Bert each did all of the following: make Gina feed Carl, make Gina feed Dido, make Gina brush Eric.

Accordingly, previous proposals do not derive the correct truth-conditions for all the sentences under consideration. But how does the expanded predicate analysis fare? Let us first consider the simpler sentence in (87-b). The most plausible analysis on the basis of the expanded predicate analysis starts off with the LF in (92-a). This yields us the semantic derivation in (92-b), which, in fact, delivers the correct truth-conditions (A \oplus B cumulatively have the property expressed by the first conjunct, B the property expressed by the second conjunct).

(92) a. \[A and B]\ (\text{cum}^2 \text{fed} C \text{and} D \text{and} \text{brushed} E) \text{cum}^2 (1\iff t_1 t_2)

b. A_\text{P}(e,t),\lambda x_e.\exists P', P'', x', x''(P' \oplus P'' = P \land x' \oplus x'' = x \land P'(x') \land P''(x''))(pl(\lambda_{y_e} + +[[fed]])(C \oplus D)(y)) \oplus pl(\lambda_{y_e} y \text{brushed} E))(A \oplus B)

c. Abe and Bert each did one of the following: feed Carl, feed Dido, brush Eric & Carl and Dido were each fed by Abe or Bert & Eric was brushed by Abe or Bert.

But this proposal, too, runs into problems with (88-b). At LF, the embedding plural expression is moved according to the rules specified above, (93-a). The truth-conditions resulting from the (again simplified) derivation in (93-b) are those paraphrased in (93-c) and they turn out to be too strong – (88-b) is incorrectly predicted false in the scenario in (88-a).

(93) a. \[A and B]\ (\text{cum}^2 \text{fed} C \text{and} D \text{and} \text{brushed} E) \text{cum}^2 (1\iff 2 [t_1 \text{made} G t_2])

b. A_\text{P}(e,t),\lambda x_e.\exists P', P'', x', x''(P' \oplus P'' = P \land x' \oplus x'' = x \land P'(x') \land P''(x''))(pl(\lambda_{y_e} + +[[fed]])(C \oplus D)(y)) \oplus pl(\lambda_{y_e} y \text{brushed} E))(A \oplus B)

c. Abe and Bert each did one of the following: make Gina feed both Carl and Dido, brush Eric & Abe or Bert made Gina feed Carl & Abe or Bert made Gina feed Dido & Abe or Bert made Gina brush Eric.

This means that even though the expanded predicate analysis was tailored to overcome semantic locality, the system breaks down in those cases where one plural expression is embedded in another one. In other words, since the expanded predicate analysis, just as the standard one, derives cumulative truth-conditions by cumulating relations between pluralities, it fails in all those cases, where we cannot form a such a relation directly, because of syntactic embedding.

### 3.4 Summary: The predicate analysis and its expansion

In the previous paragraphs, I outlined the predicate analysis, which derives cumulative truth-conditions by enriching the predicates: For any structure as in (94-a), where A and B denote pluralities from D_\text{a}, and R has its denotation in D_{(a,(a,t))}, cumulative truth-conditions result from a cumulation operation on R.

(94) R(A)(B)

Using this system as a background, I tried to come to terms with our data from section 2 by assuming that all conjunctions denote pluralities of the kind of objects their conjuncts denote and by expanding the predicate analysis so as to allow for cumulation operations that will expand relations between any two (or more) objects from any semantic domain (excluding D_\text{t}).
This expanded version of the predicate analysis – and the predicate analysis in general – ran into two problems. First, when forming the relation $R$ that represents the input to the cumulation operation, we will have to violate various constraints on syntactic movement that are observable otherwise (‘syntactic problem’). Second, the system won’t derive the correct truth-conditions if one plural expression is embedded in another one (‘projection problem’).

4 An alternative proposal: Pluralities and plural projection

In the following, I spell out an alternative analysis for conjunctions and plural composition which will derive cumulative truth-conditions for the examples considered in sections 2 and 3. (It will also derive homogeneity, but not in any ‘deep’ way.)

As in section 3.3 above, I will take the parallels between plural DPs and conjunctions at face value, assuming that all semantic domains contain a subdomain of pluralities (of the relevant kind of semantic object) and that conjunctions with conjuncts of type $a$ denote pluralities of the conjuncts’ denotations. The modelling of these pluralities will be slightly more complex than in section 3.3, for reasons having to do with how these pluralities combine with their sisters. In particular, I argued in section 3 that expanding the predicate analysis to other kinds of pluralities is inadequate. The syntactic problem and the projection problem suggest that rather than forming relations between the pluralities and affixing these relations with cumulation operators, we should look for a system that lets us ‘project’ pluralities bottom up. Again, I will take these facts at face value, proposing a system where once a plurality $a$ enters the semantic derivation, the node immediately dominating it will also denote a plurality, namely, a plurality of values obtained by combining $a$ with its sister. I.e. argument pluralities will ‘project’ in the sense that once they combine with a function the result will be pluralities of those values that the function yields for the different ‘parts’ of the argument, function pluralities ‘project’ in the sense that once they combine with an argument we will obtain a plurality of the values the function ‘parts’ yield for that argument. I schematize this in (95), where $f, g$ stand for functions that have $a, b$ in their domains, and ‘+’ represents plurality-formation. (The actual implementation will again be slightly more complex in order to deal with structures containing more than two pluralities.)

\[
\begin{align*}
\text{f(a)} + \text{f(b)} & \quad \text{f(a)} + \text{g(a)} \\
\text{f} & \quad \text{a+b} \\
\text{f} & \quad \text{g} \quad \text{a}
\end{align*}
\]

The general idea of projection is, of course, strongly reminiscent of focus projection (cf. Rooth (1985) a.o.) or other systems working with a Hamblin-style alternative semantics (cf. e.g. Simmons (2005) for disjunctions). The difference to such systems will lie in how exactly projection is defined, i.e. how exactly we go from an argument/function plurality to a plurality of values. The idea is that projection essentially encodes cumulation: Rather than assuming an operator that modifies predicate extensions, cumulation is part of the compositional system itself.

4.1 Plural denotations

As a first step, I enrich the ontology by pluralities and sets thereof and then introduce the denotations of conjunctions. For the moment, I keep the system completely extensional, which means that I don’t introduce any parametrization w.r.t. worlds or times, and also distributive, meaning that I don’t include collective predicates (see section 5).
4.1.1 Ontology

The ontology will look similar to that in section 3.2.1 above, except that the set of possible denotations for every type \( a \) is expanded not only by the set \( \text{PL}_a \) of pluralities, but also by the ‘plural set’ \( S_a \), which contains all subsets of \( \text{PL}_a \). We thus end up with three ‘levels’ of complexity for any domain. (I will write \( D \) for the union of all atomic domains, \( \text{PL} \) for the union of all plural domains, and \( S \) for that of all plural sets.)

\[
\begin{align*}
\text{(96) a. atomic domains (containing atoms)} \\
&\quad (i) \quad D_e := \text{the domain of all atomic individuals} \\
&\quad (ii) \quad D_t := \text{the domain of truth-values} \\
&\quad (iii) \quad D_s := \text{the set of all atomic possible worlds} \\
&\quad (iv) \quad D_{(a,b)} := \text{the set of all atomic functions from } D_e \\

\text{b. plural domains (containing pluralities)} \\
&\quad \text{For any domain } D_a, \text{ there is a set } \text{PL}_a \text{ and a bijection } p_l_a : (P(D_a) \setminus \{\emptyset\}) \rightarrow \text{PL}_a \\

\text{c. plural sets (containing sets of pluralities)} \\
&\quad \text{For any domain } D_a, \text{ there is a set } S_a = \varnothing(\text{PL}_a)
\end{align*}
\]

(97) repeats the notational conventions from section 3.2.1 and adds the shorthand in (97-a-i) relating to our ‘new’ level of plural sets.

\[
\begin{align*}
\text{(97) a. } &\quad (i) \quad \text{I use } a, b, c \text{ for elements of } S_e, P, Q \text{ etc. for elements of } S_{e,t}, \text{ and } p, q \text{ etc. for elements of } S_t \\
&\quad (ii) \quad a, b, c \text{ for elements of } \text{PL}_e, P, Q \text{ etc. for elements of } \text{PL}_{e,t}, \text{ and } p, q \text{ etc. for elements of } \text{PL}_t \\
&\quad (iii) \quad a, b, c \text{ for elements of } D_e, P, Q \text{ etc. for elements of } D_{e,t}, \text{ and } p, q \text{ etc. for elements of } D_t \\

\text{b. ‘sums’: For any } a, b \in \text{PL}_a, \quad a \oplus b = pl_a(pl_{a}^{-1}(a) \cup pl_{a}^{-1}(b)). \\

\text{c. ‘part-of’: For any } a, b \in \text{PL}_a, \quad a \leq b \text{ iff } pl_{a}^{-1}(a) \subseteq pl_{a}^{-1}(b). \\

\text{d. ‘atomic-part-of’: For any } a, b \in \text{PL}_a, \quad a \leq AT b \text{ iff } pl_{a}^{-1}(a) \subseteq pl_{a}^{-1}(b) \text{ and } |pl_{a}^{-1}(a)| = 1.
\end{align*}
\]

I also keep the ‘shifts’ introduced in section 3.2.1 repeated, in (98-a,b) and add the operation \( \llbracket \_ \rrbracket_a \) in (98-c), which takes elements from \( \text{PL}_a \) to \( S_a \).

\[
\begin{align*}
\text{(98) a. } &\quad \text{for any type } a, \downarrow_a \text{ is a function from } \text{PL}_a \text{ to } D_a. \text{ For any } X \in \text{PL}_a, \downarrow (X) \text{ is defined} \\
&\quad \text{iff } |pl_{a}^{-1}(X)| = 1. \text{ If defined } \downarrow (X) \text{ is the unique } Y \in pl_{a}^{-1}(X). \\

\text{b. } &\quad \llbracket pl_a \rrbracket = \uparrow_a = AX \in D_a, pl(l\{X\}) \\

\text{c. } &\quad \llbracket pl_a \rrbracket = \cap_{a} = AX \in \text{PL}_a \cdot \{X\}
\end{align*}
\]

Again, just as in section 3.2.1 I will often simplify the derivations to keep them readable: Unless it would lead to confusion, I will drop the type-subscript on functions defined for each semantic domain, leave out the ‘trivial’ morphemes in (98-b,c) in the syntactic derivation and furthermore drop the correlating operators from the meta-language.

Lexical meanings are assigned by the function \( L \), which maps any lexical element of primitive type \( a \) to an element in \( D_a \), and any lexical element of type \( \langle a, b \rangle \) to elements in \( D \cup \{ f : D \rightarrow \text{PL} \} \cup \{ f : \text{PL} \rightarrow S \} \cup \{ f : S \rightarrow S \} \). In this new proposal, I add a basic assumption about the correlation of syntactic categories and denotations, namely, that all open class-elements are assigned a denotation in \( D \), (99).\(^{19}\)

\(^{19}\)Note that collective predicates are incompatible with this assumption. See section 5.
a. For any lexical element of syntactic category $V, A$ or $N$, $\mathcal{L}(\alpha) \in D$.  

b. If $\alpha$ is a terminal node, then $\llbracket \alpha \rrbracket = \mathcal{L}(\alpha)$

I furthermore make two syntactic assumptions, namely, that all expressions with a denotation in $D$ must be affixed with $\text{pl}$, and all expressions with a denotation in $\text{PL}$ with $\text{pl}$ – which essentially means that open-class elements will always be ‘carried’ to a denotation in $\mathcal{S}$. This is stated in the (simplified) syntactic rules in (100) and illustrated in (101), which also gives the denotations of each node. Closed-class (functional) elements will be addressed later on.

(100) a. If $\alpha$ is a terminal node with a denotation in $D$, and $\beta$ immediately dominates $\alpha$, then $\beta$ is a well-formed expression iff $\beta$ also immediately dominates $\text{pl}$.

b. If $\alpha$ is a node with a denotation in $\text{PL}$, and $\beta$ immediately dominates $\alpha$, then $\beta$ is a well-formed expression iff $\beta$ also immediately dominates $\text{pl}$.

I will again replace characteristic functions of individuals by upper-case words (‘CAT’ stands for ‘$\lambda x. x$ is a cat’, ‘CAT’ for ‘$pl(\lambda x. x$ is a cat)’). For the denotations of object-language declaratives, the meta-linguistic sentence will (for the moment) stand for the truth-value of the object-language sentence, i.e. if, for the object language sentence $\text{Abe fed Carl}$ I write ‘Abe fed Carl’ in the meta-language, this will stand for ‘1’ iff Abe fed Carl and for ‘0’ otherwise.

### 4.1.2 Denotations for conjunctions

We can now introduce plural denotations for conjunctions (see section 5 for the treatment of definite plurals). For the sake of simplicity, but without any syntactic commitment, I assume a binary-branching structure for coordination. For any type $a$, the meaning of $\text{and}$ is given in (102): It takes the conjuncts’ denotations (plural sets) and yields a plural set containing all those pluralities that we get by adding elements (i.e. pluralities) from one the conjuncts’ denotations to elements (i.e. pluralities) of the other. (103) gives some examples.

(102) $\llbracket \text{and}_{(a(a))} \rrbracket = \lambda X_0. \lambda Y_0. \{ X_0 \cup Y_0 : X_0 \in X, Y_0 \in Y \}$

(103) a. $\llbracket \text{Abe and Bert} \rrbracket = \llbracket \text{and}_{(e(e))} \rrbracket (\text{Abe}) (\text{Bert}) = \{ \text{Abe} \cup \text{Bert} \}$

b. $\llbracket \text{smoke and dance} \rrbracket = \{ \text{SMOKE} \cup \text{DANCE} \}$

c. $\llbracket \text{Abe fed Carl and Bert fed Dido} \rrbracket = \{ \text{Abe fed Carl} \cup \text{Bert fed Dido} \}$

For the sake of completeness – and to show that we don’t inadvertently collapse the meanings of conjunction and disjunction – (104) gives the meaning of $\text{or}$, for any type $a$. It takes the disjuncts’ denotations (plural sets) and yields us their union. (105) gives some examples.

(104) $\llbracket \text{or}_{(a(a))} \rrbracket = \lambda X_0. \lambda Y_0. X_0 \cup Y_0$

(105) a. $\llbracket \text{Abe or Bert} \rrbracket = \llbracket \text{or}_{(e(e))} \rrbracket (\text{Abe}) (\text{Bert}) = \{ \text{Abe}, \text{Bert} \}$

b. $\llbracket \text{smoke or dance} \rrbracket = \{ \text{SMOKE}, \text{DANCE} \}$

---

20This treatment of conjunction and disjunction (in combination with the composition rules below) raises issues concerning scalar implicatures. Just as any non-intersective analysis of conjunction, the meaning given for $\text{and}$ here is too weak to derive those effects that are usually assumed to be the result of lexical contrast between $\text{and}$ and $\text{or}$ (cf. Haslinger and Schmitt (2017) for discussion).
c. \[[\text{Abe fed Carl or Bert fed Dido}] = \{\text{Abe fed Carl, Bert fed Dido}\]\]

### 4.2 Plural composition

The final step is to let plural sets combine with the other elements in the clause.

Let us first consider what happens with matrix-clause conjunction, i.e. cases like (103-c) where we end up with a plural set. We still have to determine how such sets map to 1 and 0, respectively. I assume that the singular morpheme \(sg\), (106), attaches to any root node. (It may optionally attach to other nodes of type \(t\)). Its denotation is supposed to encode the homogeneity effects observed above, adapting a proposal by Löbner (1987), which views them as an ‘all-or-nothing’ presupposition. The function takes a set of truth-value pluralities as its argument. It is defined if at least one of these pluralities reduces to 1 or all of them reduce to 0. A plurality of truth-values, given our definitions of the domains above, will be reducible to 1 just in case of all its atomic parts are 1, and to 0, if all of its atomic parts are 0. If defined, it will yield 1 if at least one of the pluralities reduces to 1, and 0 otherwise.

\[(106) \quad [sg] = T = \lambda p . \exists q \in p (\downarrow (q) = 1) \lor \forall q \in p (\downarrow (q) = 0).\exists q \in p (\downarrow (q) = 1)\]

The full derivation for a matrix-clause conjunction thus looks like (107): (107-a) gives the LF with the singular morpheme attached to the highest node. We get ‘true’ if all of the conjuncts are true, ‘false’ if none of them are and ‘undefined’ otherwise. This means that we obtain the correct truth-conditions, in addition to the homogeneity effects observed in section 2.

\[(107) \quad a. \quad [sg \text{Abe fed Carl and Bert fed Dido}]\\
\quad b. \quad [[(107-a)]] = T([\text{Abe fed Carl } \oplus \text{ Bert fed Dido}])\\
\quad \text{defined iff } \downarrow (\text{Abe fed Carl}) = \downarrow (\text{Bert fed Dido})\\
\quad \text{if defined, } 1 \text{ if } \downarrow (\text{Abe fed Carl}) = \downarrow (\text{Bert fed Dido}) = 1, \text{ and } 0 \text{ otherwise.}\]

Just to make the contrast to disjunction clear, (108) gives a parallel structure with or. It will come out as ‘true’ if at least one of the disjuncts is true, and ‘false’ otherwise.

\[(108) \quad a. \quad [sg \text{Abe fed Carl or Bert fed Dido}]\\
\quad b. \quad [[(108-a)]] = T([\text{Abe fed Carl, Bert fed Dido}])\\
\quad 1 \text{ if } \downarrow (\text{Abe fed Car}) = 1 \text{ or } \downarrow (\text{Bert fed Dido}) = 1, \text{ 0 otherwise.}\]

The more interesting cases, of course, are those where the plurality is not formed at the matrix level, but a proper part of the sentence, as in any of the sentences in (109) (all of which are simplified versions of constructions discussed in sections 2 and 3). We want to derive cumulative truth-conditions for all the sentences involving more than one plurality, which, according to the treatment here, are all the sentences in (109-c)–(109-i). We must furthermore be able to deal with the lack of semantic locality, i.e. examples like (109-g,h,i), the lack of syntactic locality, exemplified by (109-i), and the projection problem, illustrated by (109-g,h).

\[(109) \quad a. \quad \text{Abe and Bert smoked.}\\
\quad b. \quad \text{Abe smoke and drank.}\\
\quad c. \quad \text{Abe and Bert smoked and drank.}\\

---

\(^{21}\)Negation could then be taken to applies to elements of \(D\), as in (i)– hence \(sg\) must apply before negation can do so. Since negation ‘feeds off’ the value of \(sg\), it will not be able to yield a value itself if \(sg\) is undefined.

\[(i) \quad [\text{not}] = \lambda p . \{\text{pl}(\neg p)\}\]
d. Abe and Bert fed Carl and Dido.
e. Abe and Bert introduced Carl and Dido to Eric and Ferdl.
f. Abe and Bert made Gina smoke and drink.
g. Abe and Bert fed Carl and Dido and brushed Eric.
h. Abe and Bert made Gina feed Carl and Dido and brush Eric.
i. Abe and Bert claimed Eric smoked and danced.

I propose that there are two rules of composition: Functional application (FA), and a new rule of composition, cumulative combination (CC), (110). CC will apply whenever the nodes that are to combine semantically denote plural sets – more specifically, a set $F$ of function pluralities $f$ and a set $X$ of argument pluralities $x$. The output of CC will again be a plural set – namely, a set $V$ of value pluralities $v$. This set is derived via the relation $C$, which is defined below and essentially encodes cumulation. $V$ will contain all the smallest pluralities $v$ s.th. there is $f \in F$ and an $x \in x$ and $v$ is the sum of ‘cumulatively’ applying atomic parts of $f$ to atomic parts of $x$: For every atomic part of $f$, there must be an atomic part of $x$ that it applies to, and for every atomic part of $x$ there must be an atomic part of $f$ that it is an argument of. (There is no deeper reason for this set containing only the smallest value-pluralities—this will simply keep the derivations in the following from getting unnecessarily complex.)

\[ (110) \text{ Cumulative combination (CC)} \]

If $\alpha$ is a branching node with daughters $\beta$, $\gamma$, where $[[\beta]] \in S_{(a,b)}$ and $[[\gamma]] \in S_{a}$, $$[[\alpha]] = [[\beta]] \cdot [[\gamma]] = \{ C \in C([[\beta]], [[\gamma]]) : \neg \exists C^c \in C([[\beta]], [[\gamma]]) (C \subseteq C^c \land C^c \neq C) \}$$

where, for any $X \in S_{(a,b)}$, $Y \in S_{a}$, $C(X)(Y) :=$

\[ \{ C \in PL_{b} : \exists X \in X, Y \in Y : \forall C' \leq_{AT} C (\exists X' \leq_{AT} X, Y' \leq_{AT} Y : C' = X'(Y')) \wedge \forall X' \leq_{AT} X (\exists Y' \leq_{AT} Y, C' \leq_{AT} C (C' = X'(Y'))) \wedge \forall Y' \leq_{AT} Y (\exists X' \leq_{AT} X, C' \leq_{AT} C (C' = X'(Y'))) \} \]

The effect of this compositional rule will become clearer if we look at some of the simple examples from (109) above. Consider first (109-a) and (109-b), which each contain one plural expression. Their derivations are given in (111-a) and (111-b). In both cases – or more generally, whenever the sentence contains no plural expression, or only one such expression – we end up with a singleton at the root node, which contains a propositional plurality. Application of the singular-operator will yield us true in (111-a) if both of Abe and Bert smoked, false if neither smoked and undefined otherwise. (111-b) is analogous: It is true if Abe both smoked and danced, false if he did neither, and undefined otherwise.

\[ (111) \]
a. $[[sg[A and B smoked]]] = [[sg]] ( [[smoked]] \bullet [[Abe and Bert]]) =$

\[ = T(\{\text{SMOKED}\} \bullet \{\text{Abe} \oplus \text{Bert}\}) = T(\{\text{Abe smoked} \oplus \text{Bert smoked}\}) \]
b. $[[sg[Abe smoked and danced]]] = [[sg]] ( [[smoke and dance]] \bullet [[Abe]]) =$

\[ = T(\{\text{SMOKED} \oplus \text{DANCED} \} \bullet \{\text{Abe}\}) = T(\{\text{Abe smoked} \oplus \text{Abe danced}\}) \]

For the sentences with more than one plural expression, e.g. (109-c) – (109-e), the sentence-level plural set will contain more than one plurality. The derivation for (109-d) is given in (112-a) and that for (109-c) in (112-b). Note that in (112-b), the DP-plurality ‘projects’ to what is, in fact, a plurality of intransitive predicates. Accordingly, both in (112-a) and in (112-b), we eventually combine the subject plural set with a predicate plural set. In both cases, the singular will map the set to ‘true’ just in case one of the two propositional pluralities reduces to ‘true’. We therefore derive the correct, ‘cumulative’ truth-conditions for (109-c) and (109-d).

\[ (112) \]
a. $[[sg[A and B smoked and drank]]] = [[sg]]([[smoked and drank]] \bullet [[A and B]])$
\[ T((\text{SMOKED} \oplus \text{DRANK}) \bullet (A \oplus B)) = T((A \text{ smoked} \oplus B \text{ drank}, B \text{ smoked} \oplus A \text{ drank})) \]

b. \[ [[\text{sg}[Abe \text{ and Bert fed Carl and Dido]]] = [[\text{sg}][[[\text{fed Carl and Dido}]] \bullet [[Abe \text{ and Bert]]]]
\]
\[ = [[\text{sg}][[[p(\lambda x_1, \lambda y_2, y \text{ fed } x)] \bullet (C \oplus D)] \bullet (A \oplus B)]
\]
\[ = [[\text{sg}][[[\text{FED C} \oplus \text{FED D}]] \bullet (A \oplus B)]
\]
\[ = T((A \text{ fed C} \oplus B \text{ fed D}, B \text{ fed C} \oplus A \text{ fed D})) \]

(109-e) contains three plural expression. It also gets the correct truth-conditions: The VP has the denotation in (113-a) and the entire sentence that in (113-b). Again, it will be true of one of elements of the plural set reduces to ‘true’.

(113) a. \[ [[Dido \text{ and Carl [introduced to Eric and Ferdl]]]] = \{\text{INTRODUCE C TO E} \oplus \text{INTRODUCE D TO F}, \text{INTRODUCE D TO E} \oplus \text{INTRODUCE C TO F}\}
\]
b. \[ [[\text{sg}[Abe \text{ and Bert [Dido and Carl [introduced to Eric and Ferdl]]]]]
\]
\[ = \{A \text{ introduced C to E} \oplus B \text{ introduced D to F}, B \text{ introduced C to E} \oplus A \text{ introduced D to F}, A \text{ introduced D to E} \oplus B \text{ introduced C to F}, B \text{ introduced D to E} \oplus B \text{ introduced D to F}\} \]

4.3 Application

The more complex examples in (109-i)–(109-i) each played a part in rejecting the various analyses discussed in sections 2 and 3. I first discuss (109-g) which is the simple case of the projection problem. (114) gives all the relevant steps of the derivation (I drop the sg-operator, as it should be clear by now when it applies and what it does). For the first conjunct, we derive a set containing a predicate plurality, (114-b). Conjoining this set with the second conjunct gives us again a set containing a predicate plurality, (114-c). Note that we capture the intuition from paragraph 3.3: fed Carl and Dido and brushed Eric will end up being denotationally equivalent to fed Carl and fed Dido and brushed Eric. Combining this set with the subject yields (114-c) and thus the correct truth-conditions.

(114) a. Abe and Bert fed Carl and Dido and brushed Eric.

b. \[ [[\text{fed C and D}]] = \{\text{FED C} \oplus \text{FED D}\}
\]
c. \[ [[\text{fed C and D and brushed Eric}]] = \{\text{FED C} \oplus \text{FED D} \oplus \text{BRUSHED E}\}
\]
d. \[ \{A \text{ fed C} \oplus B \text{ fed D} \oplus B \text{ brushed E}, A \text{ fed D} \oplus B \text{ fed C} \oplus B \text{ brushed E}, B \text{ fed C} \oplus B \text{ fed D} \oplus A \text{ brushed E}, B \text{ fed C} \oplus A \text{ fed D} \oplus A \text{ brushed E}, B \text{ fed D} \oplus A \text{ fed C} \oplus A \text{ fed D} \oplus B \text{ brushed E}\} \]

For the other examples, our extensional system won’t suffice. I here chose the most simple variant of world-parametrization: Type \(s\) is added to our set of types and the set \(W\) of all possible worlds to our semantic domains. All lexical elements are assigned functions from worlds to extensions in that respective world and add two rules on the combination of meanings from \(D\), which essentially encode extensional and intensional functional application. This means that I don’t have to worry with how world-parametrization affects our levels \(PL\) and \(S\) – we will simply form pluralities of intensions and sets thereof, as sketched in (116). (This will also mean that the function \(s\) will have to be relativized to worlds, which I omit here.) For the remainder of this section, I modify my notation as follows: I write ‘SMOKE’, for ‘\(\lambda w. \lambda x_1, x_2, x_3 \text{ smokes in } w\)’, and ‘Abe smokes’, will stand for ‘\(\lambda w. \text{ Abe smokes in } w\)’.

\[ ^{22}\text{This treatment is insufficient once we broaden our empirical scope. In particular, examples like (i-b) might require us to rethink matters: (i-b) is true in the scenario in (i-a) and therefore displays a form of cumulativity}\]
(115) a. For any $A \in D_{(x(a,b))}, B \in D_{(x(a))}, A(B) = \lambda w. A(w)(B(w))$

b. For any $A \in D_{(x(a,b))}, B \in D_{(x(a))}, A(B) = \lambda w. A(B)(w))$

(116) $\llbracket \text{smoke} \rrbracket \bullet \llbracket \text{Abe and Bert} \rrbracket = \{\lambda w. \lambda x.s. \text{x smokes in w} \} \bullet \{\lambda w. A \oplus \lambda w. B \} = \{p\{\lambda w. A \text{smokes in w}\} \oplus p\{\lambda w. B \text{smokes in w}\}\}$

We can now turn to the more complex example of the projection problem, \(\llbracket 109-h \rrbracket\), which none of the previous analyses could derive. Again, I only give the relevant steps of the derivation in (117), and as it is of no consequence for my purposes, I make the simplifying assumptions that \textit{make} denotes a function which takes a propositional argument, e.g. $\lambda w. \lambda p(x). \lambda x.s. x$ does everything to make $p$ true in $w$. (117-b) shows the denotation of the VP-conjunction (it is identical to what we derived in [114]). (117-c) gives the denotation of the embedded clause, a set containing a plurality of propositions. This projects to a set containing a plurality of predicates, (117-d), and combining the latter with the denotation of the subject in (117-e) yields us a set of propositions analogous to those in \(\llbracket 114-d \rrbracket\). Hence, we derive the correct truth-conditions and solve the projection problem.

(117) a. 

\textit{Abe and Bert made Gina feed Carl and Dido and brush Eric.}

b. 

$\llbracket \text{feed C and D and brush Eric} \rrbracket = \{\text{FEED C} \oplus \text{FEED D} \oplus \text{BRUSH E}\}$

c. 

$\llbracket \text{G feed C and D and brush Eric} \rrbracket = \{\text{G feed C} \oplus \text{G feed D} \oplus \text{G brush E}\}$

d. 

$\llbracket \text{made G feed C and D and brush Eric} \rrbracket = \{\text{MADE G FEED C} \oplus \text{MADE G FEED D} \oplus \text{MADE G BRUSH E}\}$

e. 

$\{\text{A made G feed C} \oplus \text{B made G feed D} \oplus \text{B made G brush E}, \text{A made G feed D} \oplus \text{B made G feed C} \oplus \text{B made G brush E}, \text{A made G feed D} \oplus \text{A made G brush E}, \text{B made G feed D} \oplus \text{A made G feed C} \oplus \text{A made G brush E}, \text{A made G feed C} \oplus \text{A made G feed D} \oplus \text{B made G brush E}\}$

This example also highlights two other features of the proposal: First, the system does not require semantic locality in order to derive cumulativity for sentences with predicate conjunctions. Once a plurality enters the system, it will project upwards so that any other plurality higher up in the tree will be able to ‘enter in a cumulative relation’ with it. I.e. sentences like \(\llbracket 109-h \rrbracket\) but also \(\llbracket 109-t \rrbracket\) which the non-intersective analysis of conjunction couldn’t derive, don’t represent a problem for the current system. Furthermore, as pluralities project upwards by a compositional rule that encodes cumulation, we don’t require any cumulation operators (i.e. ‘*, **,...’) and therefore also no predicates that these operators can attach to. This, in turn, means that we don’t need to derive predicates by covert movement in examples like \(\llbracket 109-i \rrbracket\) and therefore – correctly – don’t predict any constraints by syntactic locality in the first place.

– but crucially, both Berta’s and Carl’s belief is \textit{de dicto}. These could be used as yet another argument against the predicate analysis (which would only yield a \textit{de re} reading, unless we radically alter our view on expressions like \textit{two monsters} (cf. e.g. Condoravdi et al. [2001])). Nevertheless, the current system doesn’t derive these facts either. Intuitively, \textit{two monsters} seems to project up to a plurality of propositions cumulatively believed by Berta and Carl, but this projection is not yet captured by the system.

(i) a. 

\text{\textsc{scenario}} Berta and Carl spent the night at Joe’s castle. Berta believes in griffins, Carl in zombies. Around midnight, Berta heard a sound in her bedroom and was certain that it was caused by a griffin. A little later, Carl heard a sound in his room, and took it to be caused by a zombie. In the morning, they each took Joe aside and told him what they believed was going on at his castle. Joe tells me: \textit{Well, I had invited Berta and Carl to spend the night at the castle. Bad idea! I know that people find it a little spooky here, but guess what...}

b. 

these idiots believed that two monsters (altogether) were roaming the castle!
4.4 Interim summary

In the preceding paragraphs, I spelled out a new analysis for plural denotations and plural composition with three crucial features: (i) All semantic domains contain pluralities (actual pluralities and plural sets). There is thus no difference between a domain of primitives, like the domain of individuals, and a domain of complex objects, e.g. that of functions from individuals to truth-values. (ii) Conjunctions with conjuncts of any semantic category are treated on a par with other plural expressions, denoting sets containing the sum of all the conjuncts’ denotations. (iii) Cumulative truth-conditions are not due to the semantic enrichment of predicates by operators, but rather to a composition rule that lets pluralities project up to the nodes dominating them. The current proposal fares better than existing ones wrt. all the data discussed so far: It derives cumulative truth-conditions without requiring semantic locality (as non-intersective theories of conjunction do), and without giving rise to either the syntactic problem or the projection problem (as the predicate analysis does).

5 Discussion and outlook

This paper made two main points: First, the class of expressions denoting pluralities is much bigger than previously thought. Namely, conjunctions with conjuncts of several semantic categories also denote pluralities (of the objects their conjuncts denote); therefore, the respective semantic domains must contain pluralities. This point was motivated by clear parallels between plural DPs and conjunctions and by the fact that no existing theory of conjunction can derive them. Second, plural composition – the way in which pluralities combine with other elements in the sentence – does not happen via cumulation operations on predicate denotations, but rather in a step-by-step fashion, via a compositional rule that essentially encodes cumulation and lets pluralities ‘project up the tree’. This claim resulted from the observation that alternative theories – where predicate denotations are cumulated – face both syntactic and semantic problems.

I already mentioned a number of open questions in the text above and there are certainly many more. Here I only want to briefly comment on the most obvious shortcoming: In several configurations, pluralities don’t project in the sense above. Put differently, some elements seem to ‘eat up’ plural sets or ‘intervene’ in plural projection.23 The most obvious case are collective predicates like the lexically object-collective compare in (118-a). The system fails, as it derives the sentence meaning in (118-b) and incorrectly predicts that (118-a) can never be true, because compare cannot be attributed to atomic objects.

(118) a. Abe compared ‘The Iliad’ and ‘War and Peace’
   b. {A compared I ⊕ A compared W+P}

Furthermore, a number of ‘quantificational’ expressions block plural projection from some positions. One particularly obvious case are NP-conjunctions in the restrictor of a determiner, as in (119-a). (119-a) has two readings: What I call the ‘internally distributive’ reading, which is identical to having the determiner apply to each conjunct separately, (119-b), and the ‘internally non-distributive’ reading, which is not, (119-c).24 Given the rule of cumulative composition from section 4 above, my account will only yield us the internally distributive reading for any...
determiner (essentially, each NP-conjunct will combine individually with the determiner) and hence fails to derive internally non-distributive readings such as (119-b) for (119-a).

(119)  
   a. Ten \{pl. cats and dogs\} attacked Abe.
   b. Ten cats and ten dogs attacked Abe.
   c. A plurality of ten animals attacked Abe, which consisted only of cats and dogs.

To tackle these problems, the system could be expanded by treating all ‘interveners’ for plural projection as lexically specified to take plural sets as their arguments. Take for instance the definite determiner, the treatment of which I omitted in section 4 above. (120) gives a lexical entry that will derive the correct result not only for simple cases (like the cats) but also for more complex cases, like the cats and dogs: It takes a plural set of predicates as its argument and then maps it to plural set of individuals – namely that set, which contains the maximal plural individual consisting exclusively of individuals that each either a cat or a dog. (I use \(C'\) in (120) as shorthand for what is, essentially, weaker version of the cumulativity relation \(C\), except that is a relation in \(PL \times S\) rather than in \(S \times S\).)

(120) \(\lbrack \text{the} \rbrack = \lambda P \langle e, t \rangle : \exists x (C'(x)(P)) \cdot \{x : C'(x)(P) \land \neg \exists y > x C'(y)(P)\}\)

where \(C'(a)(P)\) holds for any \(a\), \(P\) iff for every atomic part \(a'\) of \(a\), there is \(Q \in P\), such that there is a part of \(Q\) that holds of \(a'\).

Determiners like ten could be treated in analogy, e.g. as in (121), in order to derive the reading in (119-c). According to (121), ten cats and dogs yields us the set of all pluralities consisting of ten individuals that each are either a cat or a dog. Of course, this will not derive the internally distributive reading, for which we would need additional assumptions.

(121) \(\lbrack \text{ten} \rbrack = \lambda P_{(\text{en})} : \exists x, (C'(x)(P)) \cdot \{x : C'(x)(P) \land |x| = 10\}\)

I leave a full expansion of the system along these lines to future research, as the discussion of the empirical facts and the existing literature would go beyond the scope of this paper (but cf. Haslinger and Schmitt (2018)). Not only would we have to describe and account for the observation that determiners differ wrt. whether they allow for internally non-distributive readings (cf. Heycock and Zamparelli 2005, Champollion 2015 a.o.), but we would also have to make sure that the determiner denotations we give are compatible with how full non-referential DPs interact with other material in the sentence: This includes the behavior of DP-conjunction (like most boys and most cats, cf. Krifka 1990, Champollion 2015 a.o.), but also the observation that some non-referential DPs can display cumulative construals wrt. other plural material in the sentence (cf. Kroch 1974, Scha 1981, Link 1987, Krifka 1990, Sher 1990, Schein 1993, Landman 2000 a.o.).

References
Amsterdam/New York: Elservier.

(33) determiners are causing the problems). Cf. Heycock and Zamparelli (2005), Champollion (2015) for discussion.


Winter, Yoad. 2007. Multiple coordination: meaning composition vs. the syntax-semantics interface. Ms.


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