1 Our starting point and where we will go

- We assumed that plural expressions like the two boys have ‘special’ denotations – they denote pluralities of individuals (Link 1983 a.o.)

Reminder:

- Pluralities stand in a one-to-one correspondence to nonempty sets of atomic individuals, but we will assume they are distinct from such sets.

1. There is a set $A \subseteq D_e$ of atomic individuals, a binary operation $+$ on $D_e$ and a function $f : (\mathcal{P}(A) \setminus \{\emptyset\}) \to D_e$ such that: 1) $f(\{x\}) = x$ for any $x \in A$ and 2) $f$ is an isomorphism between the structures $(\mathcal{P}(A) \setminus \{\emptyset\}, \cup)$ and $(D_e, +)$.

- We will use the notions in (2), following much of the literature.

2. For any $x, y \in D_e$, $S \subseteq D_e$:
   a. $x \leq y \Leftrightarrow x + y = y$ (“$x$ is a part of $y$”)
   b. $x \leq_a y \Leftrightarrow x \leq y \land x \in A$ (“$x$ is an atomic part of $y$”)
   c. $\sum S = f(\bigcup \{f^{-1}(x) \mid x \in S\})$ (the sum of all individuals in $S$)

3. a. $[[\text{the girls}]] = +[[\text{girl}]]$
   - If $[[\text{girl}]] = \{\text{Ada, Bea}\}$, then $[[\text{the girls}]] = \text{Ada} + \text{Bea}$
   b. $[[\text{Ada and Bea}]] = \text{Ada} + \text{Bea}$

- Which empirical phenomena motivate this move?

One of them is the particular behavior of semantically plural expressions in cumulative sentences (Link 1983, Krifka 1986 a.o.)

   b. The two girls fed the two cats.

- In other words: Sentences with two or more plural expressions can have ‘cumulative truth conditions’ (Langendoen 1978 a.o.)

5. a. The two girls fed the two cats.
   b. scenario: Ada fed Carl. Bea fed Dido. (5-a true)
• This is usually accounted for by assuming that we enrich predicate denotations by cumulation operators. Basic idea: ‘Add up’ the individuals in the basic extension.

• We need different operators for predicates of different arity (Sternefeld 1998).

• One place predicates: Closure under sum by operator *

(6) The two girls slept.

One-place predicates (type \(\langle e, t \rangle\)): Cumulation by *

(7) For any \(P \in D_{\langle e, t \rangle}\), \(*P\) is the smallest function \(f\) such that
a. for all \(x \in D_e\), if \(P(x) = 1\), then \(f(x) = 1\)

b. and for any \(S \subseteq D_e\), if for all \(y \in S, f(y) = 1\), then \(f(+S) = 1\).

(8) \([slept]\) = \{a, b, c\}

(9) \(*[slept]\) = \{a, b, c, a + b, b + c, a + c, a + b + c\}

• If we use the cumulated version of the predicate, (10) will come out as true if both girls slept, and as false otherwise

(10) The two girls slept.

• Two-place predicates: Closure under point-wise sum

(11) The two girls fed the cats.

Two-place predicates (type \(\langle e, \langle e, t \rangle \rangle\)): Cumulation by **

(12) For any \(P \in D_{\langle e, \langle e, t \rangle \rangle}\), \(**P\) is the smallest function \(f\) such that
a. for all \(x, y \in D_e\), if \(P(x)(y) = 1\), then \(f(x)(y) = 1\)

b. and for all \(S, S' \subseteq D_e\), if
   (i) for every \(x \in S\) there is a \(y \in S'\) such that \(f(x)(y) = 1\)
   (ii) and for every \(y \in S'\) there is an \(x \in S\) such that \(f(x)(y) = 1\),
   then \(f(+S)(+S') = 1\).

(13) \([fed]\) = \{(a, c), (b, d)\}

(14) \(**[fed]\) = \{(a, c), (b, d), (a+b, c+d)\}

• This gives us the ‘weak’ (= cumulative) truth-conditions for the sentence in (15).

(15) The two girls fed the cats.

• analogously for three-place predicates etc.

(16) The two girls gave the two cats the three sausages.

• Sometimes the required relation does not correspond to a surface constituent (Beck & Sauerland 2000).

(17) a. The two girls wanted to feed the two dogs. adapted from Beck & Sauerland (2000)

b. Ada wanted to feed Carl. Bea wanted to feed Dido. (17-a) true
(18) **required relation**: \( \lambda x.\lambda y.e \) *wanted to feed* \( x \)

- **Beck & Sauerland** (2000): In these cases, we must derive the require relation syntactically (by covert movement) and then affix it with the cumulation operator

(19) \([\text{the two girls}][[\text{the two dogs}][**[2[1[t_1 \text{ wanted to feed } t_2]]]]]]\)

**Therefore**: In this type of analysis – which we call the *predicate analysis*: Whenever we find cumulativity w.r.t. \( n \) pluralities, we need an \( n \)-ary relation in the object language that we can cumulate. If don’t find a surface constituent that corresponds to this relation, we have to derive one in covert syntax.

**Questions/Comments:**

Q: If the operators \( *, **, \) etc. are responsible for cumulation – do we find overt realizations of such operators in any language?
We haven’t found any – which will be relevant for us later. (But see Beck (2012) for potentially relevant data.)

Q: Do we have to cumulate relations between individuals? Can’t we cumulate relations between individuals and events?
Yes. There are several proposals of this type (e.g. Kratzer 2000). We will come back to them later.

- Hence, we have a system where we enrich (at least some of) the primitive domains by pluralities and cumulativity is the result of operations on the predicate. This is the background for what we will discuss in the following two classes.

- In the following, we will try to motivate a departure from the ‘standard’ view of cumulativity just discussed by giving evidence for the following claims:

**Claims**

(i) All semantic domains contain pluralities (i.e. objects corresponding to non-empty subsets of the ‘atomic’ domain): We do not only have pluralities of individuals, but also pluralities of predicates, propositions etc. (Schmitt 2013, 2018)

(ii) Cumulativity is not the result of enriching predicate denotations. Rather, it has to be built into the compositional system itself, since it also occurs in configurations where there is no way of deriving a cumulative predicate in the syntax. (Schmitt 2018, Haslinger & Schmitt 2018b, 2019)

- embedded pluralities ‘project’: If a node \( \alpha \) dominates a plurality-denoting expression, then \( \alpha \) itself denotes a plurality
- projection encodes cumulativity: Cumulativity is derived in a step-by-step process, following the independently motivated syntactic structure (i.e. without any LF movement steps that are specific to cumulative sentences)

(iii) This system allows us to deal with the behavior of plurals and conjunctions, but also with ‘asymmetrically distributive universals’ like every DPs or so-called distributive conjunctions
2 Motivation for higher-order pluralities

**Claim:** All semantic domains contain pluralities. We have pluralities of individuals, but also pluralities of predicates, propositions etc.

**What we will do:**

(i) Show that we find one of the hallmarks of plurality – cumulativity – also for expressions of higher types: Conjunctions with conjuncts of functional types

(ii) Show that existing accounts of cumulative readings of conjunction cannot account for these data

(iii) Provide further evidence for the assumption that conjunctions with functional types denote pluralities (of the conjuncts’ denotations)

2.1 Cumulativity as a symptom for plurality

We will use this schema to test for cumulativity:

\[(20)\]  
\[a. \ A \ R \ B \]
\[b. \ 1 \text{ iff } \forall x \in S_A (\exists y \in S_B (R(y)(x) = 1)) \land \forall y \in S_B (\exists x \in S_A (R(y)(x) = 1)) \]

where \(S_A, S_B\) are sets of objects that are intuitively

2.1.1 Predicate conjunction

• cumulativity for predicate conjunction

\[(21)\]  
\[a. \ Good \ Lord! \ The \ farm \ is \ on \ fire, [A \the \ ten \ animals] \ are [B \crowing\text{P} \ and \ barking\text{Q}]! \]
\[\text{And the farmer is singing Auld Lang Syne!} \]
\[b. \ [S \ [A \ The \ children \ in \ my \ class] \ are \ [B \blond\text{P} \ and \ brunette\text{Q}]], \text{ but the ones in Sue’s class all have red hair … how strange!} \]

\[(22)\]  
\[a. \ What \ a \ party! \ [S \ [A \ The \ ten \ teenagers \ I \ invited] \ are \ [B \smoking\text{P} \ and \ dancing\text{Q}] \ in \ the \ street] \text{ and the adults are getting drunk in the living room.} \]
\[b. \ Absurd! \ [S \ [A \ The \ people \ in \ this \ village] \ [B \smoke\text{P} \ and \ drink\text{Q}], \text{ but none of them has ever eaten a steak!} \]

• Winter (2001b) claims that cumulative readings are not generally available, using examples like (23).

\[(23)\]  
\[a. \ The \ birds \ are \ above \ the \ house \ and \ below \ the \ cloud. \]  
\[\text{Winter} \text{[2001b]} 340, (11-a)] \]
\[b. \ The \ ducks \ are \ swimming \ and \ quacking. \]  
\[\text{Winter} \text{[2001b]} 340, (12-b)] \]

Winter: (23-a) judged false in a scenario where some birds are above the cloud, even if all birds are above the house. His claim: Cumulative reading is only available if the distributive reading would be absurd.

\[(24)\]  
Winter’s generalization

A cumulative reading of a predicate conjunction \(P \ and \ Q\) is only available if the possibility that some individual satisfies both \(P\) and \(Q\) is ruled out by logic or world knowledge.

• Experimental evidence from Tieu et al. (2017) suggests that disjointness indeed has an effect. Both children and adults typically accept a cumulative reading for (25-c), but not (25-b), in scenario (25-a):
a. scenario: There are four bears. Two of them are big and brown. The others are small and white.

b. The bears are big and white. **not accepted**
c. The bears are big and small. **accepted**

- But Schmitt (2013, 2018) argues that (25) is too strong, on the basis of examples like (22), which are acceptable in cumulative scenarios.

  Such examples are often odd out of the blue, but improve when presented with a context that provides explicit alternatives to the conjuncts of the predicate conjunction, e.g. getting drunk in (22-a) or eat meat in (22-b).

- Roszkowski et al. (2019) provide experimental evidence that both children and adults access the cumulative reading for non-disjoint predicates as well, but there is a preference for distributive scenarios in cases where both readings are possible.

**Questions:**

- Even if the generalization in Winter (2001b) is too strong, we need to account for the effect in (25) – how? Generalization about (under)informativity of conjunctive statements in context?
- Is this reading a quirk of English or something we find cross-linguistically?
  - Schmitt (2018) reports that predicate conjuncts are accepted in cumulative scenario in several languages, but there is no broad cross-linguistic study yet.
  - Phenomenon not restricted to Indo-European languages – also in Hebrew, Iraqi Arabic, Hungarian (Schmitt 2018), Japanese (Kazuko Yatsushiro, p.c.), Estonian (data elicited by Magdalena Roszkowski, p.c.).
  - We are currently collecting judgments from various languages at http://test.terraling.com/groups/8/ properties 1A02, 1B02 and 1C02.

2.1.2 Propositional conjunction

- At least in English and German, conjunctions of embedded clauses exhibit cumulativity relative to plurals in the matrix clause.

- Again, the cumulative reading is usually not the most salient one out of the blue, but improves if we add a context where the propositions expressed by the conjuncts contrast with other alternatives.

(26) The agencies from Paris called and the one from Berlin. The conversations were useless. [S [A The agencies] claimed [B [that Macron was considering his resignation]]p and [((that) Merkel hired 10 new bodyguards]]q], but neither of them had anything to say about the Brexit negotiations.

(27) Abe may be into raw food and Bert into homeopathy, but they are not as crazy as you think – okay, [S [A they] believe [B [that cooked food causes headaches]]p and [((that) antibiotics will kill you]]q]], but neither of them would maintain that all drinking water in the US is poisoned – as your friend Gina does.

- In a suitable context, we can even get this reading when the classical conjunction of the propositions expressed by the conjuncts would be a contextual contradiction.

(28) a. scenario: Roy hasn’t been seen for a while. His friends have different theories about what happened, based on their individual past encounters with Roy. Ada believes that he lives
in a trailer in Alabama, trying to become a country singer. Bea believes that he built a cottage in Alaska and started making chainsaw art. Gene believes that he lives on a yacht and works for a drug cartel. Sue hears about this and tells me: Well, Ada and Bea certainly have weird ideas . . .

b. They believe that Roy lives in a trailer in Alabama and (that) he is holed up in a cottage in Alaska. But neither of them is as crazy as Gene: He believes that Roy resides on a yacht!

• **Pasternak (2018)** challenges the idea that ‘cumulative belief’ is related to semantic plurality. He introduces a notion of ‘collective/cumulative belief’ that applies regardless of whether the complement of the attitude predicate contains a conjunction/plural, motivated by data like (29).

(29) a. **scenario:** Paul just got married, and his cousins Arnie and Beatrice, who have never met, just caught wind of it. Arnie suspects that Paul’s husband is rich, and has no other relevant opinions. Beatrice thinks he’s a New Yorker, and has no other relevant opinions.

b. **Paul’s cousins think he married a rich New Yorker.**

true in (29-a) (Pasternak 2018:548, (6))

– To evaluate which propositions count as cumulative beliefs of a plural individual \(x\), he considers a set of propositions that are ‘relevant’ beliefs of the atomic parts of \(x\).

– A proposition \(p\) counts as a cumulative belief of \(x\) if it is true in every possible world that is compatible with as many as possible of these ‘relevant’ beliefs.

– In (29-a), the propositions \([\text{Paul married a rich man}]\) and \([\text{Paul married a New Yorker}]\) are ‘relevant’ beliefs of Arnie and Beatrice, respectively. Since these are compatible, all the highest-ranked worlds are worlds where there are both true. So, \([\text{Paul married a rich New Yorker}]\) is a cumulative belief of the plurality \(\text{arnie + beatrice}\).

• This analysis makes the following prediction:

(30) **Prediction of Pasternak (2018)**

Whenever two propositions \(p\) and \(q\) are contextually relevant, an individual \(a\) believes \(p\) and another individual \(b\) believes \(q\), and \(p\) and \(q\) are compatible, then \(a + b\) can be said to believe anything jointly entailed by \(p\) and \(q\).

• **Schmitt (2019a)** points out that this prediction is too strong. (31-b) does not seem true in scenario (31-a) (see Marty 2019 for a similar point).

(31) a. **scenario:** Ada is looking forward to Sue’s party: She is certain that every man at the party will fall in love with her. Bea is also looking forward to the party: She hates men and is certain that only one man will attend: Roy. Sue tells me: Ada and Bea are really looking forward to the party:

b. \(\text{Sie glauben, dass Roy sich in Ada verlieben wird! Die spinnen!} \)

they belief that Roy refl in Ada fall.in.love will. they are.crazy

‘They believe that Roy will fall in love with Ada. They are crazy!' (German, Schmitt 2019a)

• **Schmitt’s 2019a** generalization is that cumulative belief ascriptions usually involve either a clausal conjunction in the complement position of the attitude verb or a plurality-denoting expression somewhere within the embedded clause.
Open questions:
• Given data like (31), and Schmitt’s 2019a generalization based on such data, cases like Pasternak’s (29), which lack plural expressions within the embedded clause, are surprising.
  Does this mean that (29) exemplifies a different phenomenon, so that embedded clauses participate in two different kinds of ‘cumulativity’?
• Do we find cumulative readings of clausal conjunctions cross-linguistically?
  No real study of this question yet. Typological literature uninformative (no attempt to distinguish readings; if “sentential conjunctions” are discussed at all, they are usually conjunctions of main clauses).

Interim summary
• Cumulativity is not restricted to plural expressions of type $e$. Predicate conjunctions and propositional conjunctions have cumulative readings as well.
• Given enough context, these cumulative readings are available even if the meanings of the conjuncts are mutually incompatible.

Next steps
• We will use these conjunction data to support the idea that there are pluralities of derived types (predicates, propositions . . .)
• But there are analyses of cumulative predicate conjunction that do not draw on higher-type pluralities. We will first address these and show that they do not account for all the data.

2.2 Why existing accounts of predicate/propositional conjunction fail

2.2.1 Distributive analyses
• Distributive analyses of conjunction take the classical truth-functional operation $\land$ as a starting point (e.g. von Stechow 1974, Partee & Rooth 1983, Gazdar 1980, Keenan & Faltz 1985).

  (32)  a. $[p \text{ Abe went to the office}] \land [q \text{ Bert went to the gym}]$.
  b. $\lambda w.[\text{ Abe went to the office}] (w) \land [\text{ Bert went to the gym}] (w)$

• Most such proposals (exception: Keenan & Faltz 1985) take $\land$ to be the basic meaning of conjunction. Conjunction operations for other types (e.g. type $(e, t)$) are derived from type $t$ conjunction by means of a recursive schema.1

  (33)  Cross-categorial schema for distributive conjunction
  a. The set $TC$ of $t$-conjoinable types is the smallest set of semantic types such that $t \in TC$ and if $b \in TC$, then for all $a$, $\langle a, b \rangle \in TC$. (cf. Partee & Rooth 1983)
  b. $X \sqcap Y = \begin{cases} X \land Y & \text{if } X, Y \in D_t \\ \lambda Z_a. X(Z) \sqcap Y(Z) & \text{if } X, Y \in D_{(a, b)} \text{ and } \langle a, b \rangle \text{ is } t\text{-conjoinable} \end{cases}$

  (34) $[[\text{ smoke and dance }]] = \lambda x_e. \text{ smoke}(x) \land \text{ dance}(x)$.

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1See Geach 1970 and van Benthem 1991 for the syntactic prerequisites.
Without additional assumptions (such as those in Winter 2001a, Champollion 2015), such analyses do not give us cumulative readings at all.

(34) is a distributive predicate, since smoke and dance are distributive. Its truth-conditional impact in sentences with a plural subject should be analogous to sentences with non-conjoined distributive predicates like (35).

(35) a. The ten teenagers are smoking.
   b. ∀y, y ≤ a [the ten teenagers] → smoke(x)

(36) Prediction
   a. The ten teenagers are smoking and dancing.
   b. ∀y, y ≤ a [the ten teenagers] → smoke(x) ∧ dance(x)

To derive the cumulative reading of (36-a), we need a different interpretation of and. (But it is possible that this interpretation is ultimately derived from the operation ∩ defined in (33) – see Winter 2001a, Champollion 2015 for relevant proposals.)

2.2.2 Non-distributive analyses


These fall into two classes:


– analyses based on cross-categorial plurality: The non-distributive interpretation of conjunction involves a new primitive notion of plurality that is defined for any type. (Schmitt 2013, 2018)

'c-based' analyses: The point of departure is the non-distributive reading of type e conjunctions (37). Here, one can assume that and expresses the sum operation +.

(37) Abe and Bert

The basic idea is now that [smoke and drink] applies to possibly plural individuals, and is true of exactly those individuals that can be split into a smoking and a drinking part. (The smoking and the drinking part might overlap, which gives us the distributive reading as a special case.)

This intuition can be extended to all predicate types that ‘start in e’ and ‘end in t’:

(38) a. The set EC of e-conjoinable types is the smallest set such that e ∈ EC and for all a1, . . . , an ∈ EC, ⟨a1, . . . , an, t⟩ ∈ EC.
   b. For any x, y of type e: x ⊔ y = x + y
   c. For any X, Y of type ⟨a1, . . . , an, t⟩, where a1, . . . , an are e-conjoinable:
      X ⊔⟨a1,...,an,t⟩Y = λz1,...,zn.∃x1,...,xn,y1,...,yn[rx1 ⊔a1,y1 = z1 ∧ · · · ∧ rxn ⊔an,yn = zn ∧ X(x1) . . . (xn) ∧ Y(y1) . . . (yn)]

(39) a. Mary and Sue smoked and drank.
   b. [smoked] ⊔⟨e,t⟩[drank] = λze.∃xe,ye[x ⊔e y = z ∧ [smoked](x) ∧ [drank](y)]

There are two problems with this approach (Schmitt 2018)

(i) As we saw above, conjunctions of embedded clauses (type t or ⟨s, t⟩) also have cumulative readings. But these types are not e-conjoinable, so (38) doesn’t apply.
(ii) It is motivated by data like (39), where a conjunction of a functional type directly combines with a plural argument. But we also find cumulative readings in sentences where the predicate conjunction takes only semantically singular arguments and cumulates with a plural that is not its argument.

- Let’s look at an example of problem (ii).

(40) a. Diplomacy is useless! The French ambassador called this morning and the German one this afternoon. \[ S_A \text{ The ambassadors} \] kept insisting that the president must \[ B \{P, \text{ take a walk in Versailles} \} \text{ and } Q \{\text{ build a golf club in Bavaria} \} \], but neither of them said anything about the really pressing issue – the trade agreement with the EU.

b. scenario: The French ambassador insisted that the president must take a walk in Versailles. The German ambassador insisted that he must build a golf club in Bavaria.

(41) \[ \forall x \in S_A (\exists Y \in \{P, Q\} (x \text{ kept insisting that the president must } Y)) \]
\[ \land \forall Y \in \{P, Q\} (\exists x \in S_A (x \text{ kept insisting that the president must } Y)) \]

We intuitively get cumulation between the individual plurality \( [\text{the French ambassador}] + [\text{the German ambassador}] \) and the two predicates \( P \) and \( Q \).

But the individual plurality is not semantically an argument of the predicate conjunction. The predicate conjunction combines with the singular argument \( [\text{the president}] \).

- Prediction of the e-based analyses:

(42) \[ [[\text{the president take a walk in Versailles and build a golf club in Bavaria}]] = [[\text{take a walk in Versailles and build a golf club in Bavaria}]] ([\text{the president}]) \]
\[ = (P \sqcup_{(e,t)} Q)([\text{the president}]) \]
\[ = \exists x_e, y_e [P(x) \land Q(y) \land [\text{the president}] = x + y] \]
\[ = P([\text{the president}]) \land Q([\text{the president}]) \] (assuming that the president has no proper parts that could satisfy \( P \) or \( Q \))

\[ \rightsquigarrow \] Combining the predicate conjunction with a singular argument gives us a simple proposition with nothing ‘plural’ about it.

- This case seems analogous to Beck & Sauerland’s [2000] examples of non-lexical cumulative relations, e.g. (43):

(43) The two girls wanted to feed the two dogs. adapted from Beck & Sauerland (2000)

We analyzed (43) by applying the cumulation operator ** to a complex relational expression derived via LF movement. This creates a relation between pluralities of individuals.

- We could extend this approach to (40-a) if we assume that the predicate conjunction denotes a plurality of predicates.

(44) \[ [[\text{take a walk in Versailles}]] +_{(e,t)} [[\text{build a golf club in Bavaria}]] \]

(45) \[ [[\text{the ambassadors}]] [[\text{take a walk in Versailles and build a golf club in Bavaria}]] [[1_e[t_{1,e} \text{ kept insisting that the president must } t_{2,(e,t)}]]]] \]

- Before we show what this means in detail, let us discuss other phenomena that point towards a parallel treatment of type e conjunctions and higher-type conjunctions.
2.3 Additional evidence

- **Homogeneity**: ‘grey area’ in terms of truth-value judgements for sentences with definite plurals (Fodor [1970], Löbner [1987, 2000], Schwarzschild [1994] a.o.)

  (46) a. Dido bit the two boys.
  b. Dido didn’t bite the two boys.
  c. **scenario**: Dido bit only one of the two boys. (46-a) **false**, (46-b) ??

- (46-b) conveys that Dido bit none of the boys. This is different from the behavior of (semantically singular) universal quantifiers:

  (47) a. Dido bit every boy.
  b. Dido didn’t bite every boy.

  Judgements relating to homogeneity can be blurry (Križ & Chemla [2015]), but similar effects seem to be observable for individual conjunctions (Schwarzschild [1994], Szabolcsi & Haddican [2004]).

  (48) a. Dido bit Abe and Bert.
  b. Dido didn’t bite Abe and Bert.
  c. **scenario**: Dido bit Abe, but not Bert. (48-a) **false**, (48-b) ??

  **Note**: For this effect, it is crucial that **and** is unstressed.

- It seems to us that the same effect can be witnessed with predicate and propositional conjunction.

  (49-a) expresses that Yasu did both P and Q, but (49-b) conveys that he did neither (see Geurts [2005] for similar data and judgements).

  (49) a. **We shared an apartment with Yasu during the trip . . .**
  b. He [[drank]P and [smoked]Q], but at least there were no hard narcotics involved.
  b. He didn’t [[drink]P and [smoke]Q], but later the police found hard narcotics in his room.

- Relatedly, the continuation in (50-a) seems slightly odd unless we stress **and** or add a modifier like **both** that removes homogeneity.

  (50) **The party had already been going on for a couple of hours when Bert arrived.**
  a. He didn’t [dance]P and [smoke]Q. ?/?? He only smoked.
  b. He didn’t both [dance]P and [smoke]Q. He only smoked.

- Propositional conjunction is analogous: While (51-a) conveys that Abe claimed both p and q, (51-b) seems to prominently convey that he claimed neither.

  (51) **I just talked to Abe. We discussed his old enemies, Bert, Gina and Joe...**
  a. He said [p that Bert is in jail] and [q (that) Gina is in rehab]. But he didn’t say anything about Joe being in hiding.
  b. He didn’t say [p that Bert is in jail] and [q (that) Gina is in rehab]. But he did say something about Joe being in hiding.

- **Selectional restrictions**: Some lexical elements, e.g. respectively (Gawron & Kehler [2004]), combine with plural DPs as well as conjunctions of various categories. Their semantic contribution appears to be the same in each case.

  (52) a. **The two first amendments refer to the first two commandments, respectively.**

      (adapted from Munn [1993] (17c))
b. Abe and Bert fed Carl and Dido, respectively.
c. Abe and Bert smoked and danced, respectively.

• Similar pattern for antecedents of ‘internal’ reading of different (Sigrid Beck, p.c.)

(53) a. Different people fed Carl and Dido.
b. Different people sang and danced.

• Collective predication Finally, another reason to posit special denotations for plurals is that collective predicates like meet select for them (e.g. Link 1983).

von Stechow (1980) discusses potential cases of collectivity with predicate and propositional conjunction.

(54) a. That Major was good-minded and that Napoleon was evil-minded are two distinct facts.
b. To be drunk and to be sober are incompatible properties.

• Higher-type indefinites: It can be argued that DPs like something or the same thing may express quantification over properties or propositions (Zimmermann 2006, Moltmann 2008, Elliott 2017).

The arguments extend to plural indefinites like two things, which can plausibly be interpreted as quantifiers over pluralities of predicates or propositions (Haslinger 2019).

• Finally, Beck & Sharvit (2002) provide data that might provide arguments for extending the notion of plurality to question denotations. In particular, the denotations of questions that contain conjunctions or plural definites appear to have a ‘part-whole’ structure reminiscent of plurals.

– Quantificational variability effects

(55) Luise knows partly/for the most part which students took Semantics III and want to attend Acquisition of Semantics. (Beck & Sharvit 2002:121, (52))

– Cumulative readings of questions

(56) a. Which criminals will be convicted depends (exclusively) on these witnesses.
b. Scenario: The testimony of witness 1 determines whether criminal 1 will be convicted, the testimony of witness 2 determines whether criminal 2 will be convicted . . .

Open questions:

• How do we generalize existing treatments of these phenomena (e.g. homogeneity)?

• Can you think of other phenomena where conjunctions of various categories and plural DPs pattern analogously?
Interim summary

- We saw some evidence that predicate and propositional conjunctions can have cumulative readings relative to another plural expression.
- These readings are outside the scope of the ‘classical’ distributive treatment of such conjunctions.
- These readings are available even if the conjunction and the other plural don’t stand in a function-argument relation. Therefore existing non-distributive analyses can’t account for them.
- We suggested that Beck & Sauerland’s 2000 analysis can be generalized if we introduce pluralities of predicates, propositions, etc.
- We mentioned some other phenomena where predicate/propositional conjunctions pattern with definite plurals and individual conjunctions.

2.4 Higher-type pluralities: The basic idea

- We will adopt a maximally simple way of adding higher-type pluralities to the ontology: They will stand in a one-to-one correspondence with nonempty sets of ‘atomic’ domain elements.
- For instance, for type \(\langle e, t \rangle\), the atomic domain elements will be arbitrary one-place predicates of individuals. Predicate pluralities correspond to sets of those.

\[(57)\]

- a. For each type \(a\), there is an atomic domain \(A_a\) and a full domain \(D_a\) with the following properties:
  (i) \(D_a\) is a set such that \(A_a \subseteq D_a\) and there is an operation \(+_a : \mathcal{P}(D_a) \setminus \{\emptyset\} \rightarrow D_a\).
  (ii) There is a function \(pl_a : \mathcal{P}(A_a) \setminus \{\emptyset\} \rightarrow D_a\) such that: \(pl_a(\{x\}) = x\) for each \(x \in A_a\) and \(pl_a\) is an isomorphism from \((\mathcal{P}(A_a) \setminus \{\emptyset\}, \cup)\) to \((D_a, +)\).
- b. We define functional types on the basis of the full domains: For any types \(a, b\) :
  \[A_{\langle a, b \rangle} = D_b D_a,\]
  the set of partial functions from \(D_a\) to \(D_b\).
- c. For any type \(b \neq a\), \(D_a\) and \(D_b\) are disjoint.\(^2\)

- For each type \(a\), we assume that conjunctions with conjuncts of type \(a\) denote pluralities with the conjunct denotations as their atomic parts.

\[(58)\] 

- \([\text{smoke and dance}] = [\text{smoke}] +_{\langle e, t \rangle} [\text{dance}]\) corresponds to (but is not identical to) \([\text{smoke}], [\text{dance}]\)

- This gives us a way of interpreting (59), if we generalize \(*\) to higher-type pluralities:

\[(59)\] 

- \([[\text{the ambassadors} \ [\text{take a walk in Versailles and build a golf club in Bavaria}] [2_{\langle e, t \rangle} \ [t_1, t_2, \langle e, t \rangle]]]]\]

\[(60)\] 

- \(* (\lambda P_{\langle e, t \rangle} \cdot A_{\langle e, t \rangle} . x . x \text{ kept insisting that the president must } P)([[\text{take a walk in Versailles}] +_{\langle e, t \rangle} [[\text{build a golf club in Bavaria}]])[[[\text{the two ambassadors}]]] \)

\(^2\)The empty partial function should be exempt from the disjointness conditions.
Next step

- We will see that there are cases of cumulative predicate conjunction that cannot be captured by generalizing the ** operator.
- We will develop an alternative approach to composition in cumulative sentences, which crucially relies on higher-type pluralities.

3 What’s wrong with the predicate analysis?

Recall: The predicate analysis predicts that, whenever we have cumulation w.r.t. n pluralities, we need to derive a corresponding n-ary relation in covert syntax.

3.1 Locality

- We expect to find cumulative readings only in syntactic configurations that are independently known to permit LF movement.
- At first sight, this seems to predict that they are clause-bounded (cf. Beck & Sauerland 2000).
  But we’ve already seen some examples of cumulation across a clause boundary (61):

  (61) *Diplomacy is useless! The French ambassador called this morning and the German one this afternoon. [S [A The ambassadors] kept insisting that the president must [P take a walk in Versailles] and [Q build a golf club in Bavaria]], but neither of them said anything about the really pressing issue – the trade agreement with the EU.*

- Some of the ‘cumulative belief’ examples discussed in Schmitt (2019a) and Pasternak (2018) make the same point:

  (62) a. scenario: Sam owns a construction company and has six clients, none of whom know of the others’ existence. She has convinced each client that she would build a house for him. In reality, she is a con artist and built no houses at all.
  b. (In total,) Sam’s six clients think she built six houses for them. (Pasternak 2018:548, (4))

- But then, there are arguments against a categorical ban on LF movement across a clause boundary (Wurmbrand 2018).
- Or else, we could assume that whatever gives us the relevant LF-configuration with indefinites displaying ‘exceptional scope’ (see e.g. Reinhart 1997) is at work here.
- So, it is not clear whether locality considerations give us a good argument for/against the predicate analysis.
**But note:** Some of these examples actually give us evidence against ‘event-based’ approaches (see e.g. Kratzer 2000, Ferreira 2005, Zweig 2008).

- In such analyses, we assume that there are pluralities of events
- The cumulation operator ** applies to thematic relations, i.e. relations between individuals and events
- Conjunction as forming sums of events ([Lasersohn 1995](#))

\[(63)\]

a. Ada and Bea smoked and drank.

b. 1 iff Ada + Bea is the cumulative agent of a event that consists of a smoking part and a drinking part.

- Broadly speaking, such analyses allow for cumulativity between two pluralities X and Y only if X and Y are connected by a ‘chain of events’
- Since in some of our examples the conjunction/lower plural is part of the complement clause of an attitude predicate, we find cases of cumulativity where there is no such ‘chain of events’ linking the two pluralities

\[(64)\] [S [A The ambassadors] kept insisting that the president must [P take a walk in Versailles] and [Q build a golf club in Bavaria]]

- Embedded clauses could be interpreted as denoting sets/predicates of events, but not single (possible plural) events, since events are anchored to particular possible worlds

### 3.2 Flattening

- A better counterargument is provided by cases where it seems to be impossible to derive a single cumulative relation in the syntax.
- This is the case in examples with a higher-type conjunction that contains a second plural:

\[(65)\]

a. **scenario:** Ada owns a dog, Carl. Bea owns another dog, Dean, and a cat, Eric. Ada and Bea went on a trip and made Gene take care of their pets: Ada made Gene feed Carl, and Bea made Gene feed Dean and brush Eric.

b. *The two girls made Gene [[P feed the two dogs] and [Q brush Eric]]* when all he wanted to do was take care of his hamster. **true in (65-a)**

- There is no way to capture scenario (65-a) in terms of a single cumulative relation:

(i) It is not the case that ada + bea and carl + dean cumulatively satisfy the predicate \(\lambda x.\lambda y. x \text{ made Gene feed } y \text{ and brush Eric}: \) Ada didn’t make Gene brush Eric.

(ii) It is not the case that ada + bea and \(P + Q\) cumulatively satisfy the predicate \(\lambda x.\lambda P. x \text{ made Gene do } P: \) No girl made Gene feed both of the dogs.

(iii) There is no way to derive this reading by moving all three plural at LF: Both possible orders of these movements yield incorrect results

\[(66)\]

a. [[[the two girls] [[[P feed the two dogs] and [Q brush Eric]] [2 [1 [t1 made Gene t2]]]]]

b. [the two dogs [[[the two girls] [[[P feed t3] and [Q brush Eric]] [2 [1 [t1 made Gene t2]]]]]]

\(\Rightarrow\) doesn’t result in a 3-place cumulative relation
c. \([\text{the two girls} [\text{the two dogs} [[P \text{ feed } t_3] \text{ and } Q \text{ brush Eric}]] [2 [3 [1 [t_1 \text{ made } Gene t_2]]]]]]\)
\(\leadsto\) unbound trace in the predicate conjunction + vacuous \(\lambda\)-abstraction

- Intuitively, the adequate description of the truth-conditions is one where \([\text{the two girls}]\) is in a cumulative relation with a predicate plurality of the following form:

\[(67) \quad \text{feed Carl} + \text{feed Dean} + \text{brush Eric}\]

- So, intuitively, the part structure introduced by the embedded plurality (\([\text{the two dogs}]\)) must be accessible at the level of the VP-conjunction

- \emph{Carl and Dean} ‘projects’ its part-structure to the VP \emph{feed Carl and Dean}, so \emph{feed Carl and Dean} is a plural expression as well

- This will be the crucial idea behind our own analysis

### 3.3 Interim summary

**This is where we are at:**

- We argued for higher-order pluralities: Conjunctions with conjuncts of various semantic categories seem to denote pluralities of the conjuncts’ denotations
- We then considered the possibility to simply extend the predicate analysis to such higher order pluralities
- But then we noticed two flaws with the predicate analysis:
  
  (i) Cumulativity doesn’t seem to be subject to \textbf{syntactic locality} (but this problem can potentially be solved)
  
  (ii) The predicate analysis cannot deal with the \textbf{flattening problem}, where one plural expression is embedded in another one and both seem to cumulate with a higher plural. (This problem is serious – there is no obvious solution)

### 4 Plural projection
Plot:

- We will use our ‘higher order’ pluralities to formulate an alternative approach to cumulativity.
- It won’t use cumulation operators that target predicate extensions.

$\Rightarrow$ It therefore accounts for the earlier observation that we didn’t find morpho-syntactic evidence for these operators in the languages we looked at.

$\Rightarrow$ We don’t have to syntactically derive predicates that form the input for cumulation operators.

\[(68)\] The two girls wanted to feed the two dogs.

- Rather, cumulation is encoded by a compositional rule:

(i) The part-structure of a plural expression is transferred to its mother node. This means that any expression containing a plurality will itself denote a plurality. (Analogy with Alternative Semantics, see e.g. Rooth [1985], Kratzer & Shimoyama [2002], Simons [2005])

(ii) This encodes our earlier intuition w.r.t. the flattening problem:

\[(69)\] feed the two dogs and brush Eric $\Rightarrow$ feed Carl + feed Dean + feed Eric

\[(70)\]
\[
f(a) + f(b) \quad f(a) + g(a)
\]
\[
f + a + b \quad f + g + a
\]

(iii) In order to encode cumulativity, we won’t only have to work with higher-order pluralities, but with sets of such pluralities, which we call plural sets.

This allows us to encode various ‘combinations’

\[(71)\] feed and brush the two dogs $\Rightarrow$ \{ feed Carl + brush Dean, feed Dean + brush Carl, … \}

\[(72)\]
\[
\{ f(a) + g(b), f(b) + g(a), f(a) + g(a) + g(b), f(b) + g(a) + g(b), \ldots \}
\]
\[
\{ f + g \} \quad \{ a + b \}
\]

- Cases of non-local cumulation \[(68)\] involve iterated application of this rule.

4.1 Ontology

- We stick to the idea that every semantic domain contains pluralities, which correspond to nonempty sets of atomic domain elements

\[(73)\]

a. ada +, bea
b. smoke + ⟨e, t⟩ dance

- We introduce special types for plural sets into the type system. A plural set with elements of type \(a\) has type \(a^*\)

\[(74)\] The set \(T\) of semantic types is the smallest set such that \(e \in T, t \in T,\) for any \(a, b \in T,\)
\[
⟨a, b⟩ \in T,\) and for any \(a \in T, a^* \in T.
\]

Motivation:

- we don’t want the composition rules to treat them in the same way as ordinary type \(⟨e, t⟩\) predicates
- we will see that there are operators that select for a plural set
• Plural sets of type \( a^* \) will be distinct from ordinary unary predicates of type \( (a, t) \), but have the same algebraic structure.

We add the following to our definition of our semantic domains

**Reminder:** \( A_a \) . . . set of atomic denotations of type \( a; D_a \) . . . full domain including pluralities

(75) For any type \( a \), \( A_{a^*} \) is a set that is disjoint from \( \mathcal{P}(D_a) \) and on which the operations \( \cup, \cap \) and \( \setminus \) are defined. Further, there is a function \( p_{a^*} : \mathcal{P}(D_a) \to A_{a^*} \), that is an isomorphism wrt. \( \cup, \cap \) and \( \setminus \).

• We generalize the basic notions of plural semantics to arbitrary types

(76) For any type \( b \) and \( x, y \in D_b \):
   a. \( x +_b y =_{\text{def}} +_b(\{x, y\}) \) (binary sum operation)
   b. \( x \leq_b y \iff_{\text{def}} x +_b y = y \) (parthood)
   c. \( x \leq_b y \iff_{\text{def}} x \leq y \land x \in A_b \) (atomic parthood)

• Some notational conventions

(77) a. We use ‘starred’ variables like \( x^*, P^* \) etc. for types of the form \( a^* \).
   b. We sometimes omit type subscripts on cross-categorial operations like \( +_a \) or \( p_{a^*} \).
   c. For variables \( x, x_1, \ldots, x_n \) of any type, we write \( [x_1, \ldots, x_n] \) for the plural set \( p_{a^*}(\{x_1, \ldots, x_n\}) \) with elements \( x_1, \ldots, x_n \), and \( [x \mid \phi] \) for the plural set \( p_{a^*}(\lambda x.\phi) \).

• Some toy examples of how the system works

(78) a. \( A_e = \{\mathbf{A}, \mathbf{B}\} \), \( D_e = \{\mathbf{A}, \mathbf{B}, \mathbf{A} + \mathbf{B}\} \)
   b. \( A_{e^*} = \{[\ ], [\mathbf{A}], [\mathbf{B}], [\mathbf{A} + \mathbf{B}], [\mathbf{A}, \mathbf{B}], [\mathbf{A}, \mathbf{A} + \mathbf{B}], [\mathbf{B}, \mathbf{A} + \mathbf{B}], [\mathbf{A}, \mathbf{B}, \mathbf{A} + \mathbf{B}]\} \)

(79) a. \( A_{(e,t)} = \{\text{smoke}_{(e,t)}, \text{dance}_{(e,t)}, (\lambda x.\text{smoke}_{(e,t)}(x) \lor \text{dance}_{(e,t)}(x)), \ldots\} \)
   b. \( D_{(e,t)} = \{\text{smoke}_{(e,t)}, \text{dance}_{(e,t)}, (\lambda x.\text{smoke}_{(e,t)}(x) \lor \text{dance}_{(e,t)}(x)), \text{smoke}_{(e,t)} + \text{dance}_{(e,t)}, \text{smoke}_{(e,t)} + (\lambda x.\text{smoke}_{(e,t)}(x) \lor \text{dance}_{(e,t)}(x)), \text{dance}_{(e,t)} + (\lambda x.\text{smoke}_{(e,t)}(x) \lor \text{dance}_{(e,t)}(x)), \ldots\} \)

Next steps

- We will translate our semantics of conjunction and plural (in)definites into this system
- Then we will introduce a notion of cumulativity for arbitrary types

(i) The operators \( * , ** \) etc. require an argument of a type ending in \( t \)
(ii) This was the motivation for the problematic LF movement of plural expressions
(iii) Once we have a way of defining ‘cumulative versions’ of arbitrary binary operations, this LF movement is no longer needed – we can interpret all plurals *in situ* if we want to

4.2 Semantics of plurals and conjunction

• Plural definites denote singleton plural sets (→ the move to plural sets does not add anything substantial here)

   b. \([\text{the girls}] = [\text{ada} + \text{bea} + \text{claire}]\)
• Indefinites denote non-singleton plural sets (generalization of Kratzer & Shimoyama’s 2002 Alternative Semantics treatment of indefinites)

(81)  
  a. \([a \text{ girl}] = [\text{ada, bea, claire}]\)  
  b. \([\text{two girls}] = [\text{ada + bea, bea + claire, ada + claire}]\)

• Conjunction will be identified with a recursive sum operation \(\oplus\) that can ‘look inside’ plural sets. When applied to a collection of plural sets, this operation

  (i) considers all possible ways of choosing one element from each of the sets
  (ii) forms the sum of the chosen element for each of these ‘assignments’
  (iii) and collects all the sums formed in this way into a single plural set

(82) For any type \(a\), the operation \(\bigoplus_a : \mathcal{P}(D_a) \setminus \{\emptyset\} \to D_a\) is defined as follows:  
For any nonempty \(S \subseteq D_a\):
  a. If \(a\) is a non-plural type (i.e. \(a\) is not of the form \(b^*\)): \(\bigoplus_a S = \{a\} S\)  

\(\leadsto \text{ada} \oplus \text{bea} = \text{ada + bea}\)

  b. If \(a = b^*\) for some type \(b\): \(\bigoplus_a S = \{\bigoplus_b (\{f(X^*) \mid X^* \in S\}) \mid f\text{ is a function from } S \to D_b \land \forall X^* \in S: f(X^*) \in p^{\#-1}(X^*)\}\)  

\(\leadsto [\text{A, B}] \oplus [\text{C, D}] = [\text{A + C, B + C, A + D, B + D}]\)

(83)  
  a. \([\text{two girls}] = [\text{ada + bea, bea + claire, ada + claire}]\)  
  b. \([\text{a pet}] = [\text{dean, eric}]\)  
  c. \([\text{two girls and a pet}] = [\text{two girls}] \oplus [\text{a pet}]\)  

\(= [\text{ada + bea + dean, ada + bea + eric, bea + claire + dean, bea + claire + eric, ada + claire + dean, ada + claire + eric}] \leadsto \text{set of all pluralities consisting of two girls and a pet}\)

(84) \([\text{smoke and drink}] = [\text{smoke}] \oplus [\text{drink}] = [\text{smoke + drink}]\)

(85) Notational convention: For any type \(a\) and any \(x, y \in D_a\): \(x \oplus a y = \bigoplus_{\{x, y\}}\).

• This gives us a new analysis of conjunction: Conjunction denotes a cross-categorial operation related to sum formation, but is not identified with sum formation

(86) \([\text{and}(a^*, (a^* , a^* )))] = \lambda x a^*. \lambda y a^*. x \oplus a^* y\) for any type \(a\)
Interim summary & outlook

• We introduced a notion of ‘plural sets’ – sets of pluralities for every domain
• New analysis of plural indefinites: two girls denotes the set of all sums of two girls
• New analysis of conjunction: operation that combines two or more plural sets and returns the set of all pluralities formed by choosing an element of each set and summing them up

• Why do these sets help us with cumulativity?

(i) Cumulative truth conditions intuitively involve different ways of ‘matching up’ parts of one plurality with parts of another plurality.

\[(\text{Mary and Sue drank and smoked})\]

\[([\text{drink(Mary)} + \text{smoke(Sue)}], \text{smoke(Mary)} + \text{drink(Sue)}, \text{drink(Mary)} + \text{drink(Sue)} + \text{smoke(Sue)}])\]

(ii) We can express these different ‘matchings’ in terms of a plural set as in (87). Our composition rules will produce plural sets and combine them.

(iii) Declarative sentences involving plurals will denote a plural set of propositions as in (87). To obtain truth conditions, we quantify existentially over this set:

\[\text{Given a plural set } p^* \in A^* \text{ of propositions and a world } w \ldots\]

\[a. \ p^* \text{ is true in } w \text{ iff there is a plurality } p \in p^{*-1}(p^*) \text{ such that for all } q \leq_a p, q(w) = 1\]

\[b. \ p^* \text{ is false in } w \text{ iff for all pluralities } p \in p^{*-1}(p^*), \text{ there is a } q \leq_a p \text{ such that } q(w) = 0.\]

4.3 The projection rule

• We will now explain the composition rule which will replace the cumulation operators **, *** etc. This rule applies directly to plural sets and encodes cumulativity. We will first illustrate it for the sentence in (89).

\[\text{Ada, Bea and Claire smoked and drank.}\]

\[\text{scenario: Ada smoked. Bea drank. Claire drank. } (89-a) \text{ true}\]

• In order to define this rule, we first need to define the notion of a cover. (Note: There is a second, more widespread use of the term cover in plural semantics that goes back to Schwarzschild [1996]. Unlike Schwarzschild’s notion, our notion of a cover is unrelated to the context-dependency of sentences involving plurals.)

• Informally, a cover C of two pluralities P and x is a relation between the atomic parts of P and the atomic parts of x, such that

(i) every atomic part of P occurs at least once as a first element of a pair in C

(ii) and every atomic part of x occurs at least once as a second element of a pair in C.

\[\text{Let } P \in D_a, x \in D_b. \text{ A relation } R \subseteq A_a \times A_b \text{ is a cover of } (P, x) \text{ iff } +((\{P' \mid \exists x' : (P', x') \in R\}) = P \text{ and } +((\{x' \mid \exists P' : (P', x') \in R\}) = x.\]

• For illustration, consider what we get for this relation when considering the two pluralities A+B+C and smoke+drink
(91) a. \( P = \text{smoke} + \text{drink}, \ x = A + B + C \)

b. Covers: \( \{ (\text{smoke}, A), (\text{drink}, B), (\text{drink}, C) \}, \{ (\text{smoke}, A), (\text{smoke}, B), (\text{drink}, C) \}, \{ (\text{smoke}, A), (\text{smoke}, C), (\text{drink}, B), (\text{smoke}, B) \} \ldots \)

c. Not a cover: \( \{ (\text{smoke}, A), (\text{smoke}, B), (\text{smoke}, C) \} \)

On the basis of the covers, we can now define the operation \( C \) which will be added to our system as a new composition rule.

a. It applies at any node with one daughter denoting a plural set of a functional type \( (a, b)^* \) and the other daughter denoting a plural set of the matching argument type \( a^* \).

b. We consider all relations that are covers of a plurality from the function set and a plurality from the argument set.

c. For each cover, we perform functional application for the pairs related by the cover and sum up the results.

d. We collect these sums into a plural set of type \( b^* \).

For our example, \( C \) yields the following denotation, a plural set of type \( t^* \):

\[
C([\text{smoke} + \text{drink}, \ [\text{ada} + \text{bea} + \text{claire}]] = [\text{smoke(ada)} + \text{drink(bea)} + \text{drink(claire)}], \text{drink(ada)} + \text{drink(bea)} + \text{smoke(claire)}], \text{smoke(ada)} + \text{drink(bea)} + \text{smoke(claire)}], \ldots]
\]

In (93), the operation \( C \) is formally defined for functions and arguments of arbitrary type.

(93) Cumulative Composition (CC)

a. For any \( P^* \in D_{(a,b)^*} \) and \( x^* \in D_{a^*} \):

\[
C(P^*, x^*) = \left( \bigoplus \{ P'(x') \ | \ (P', x') \in R \} \right) \ | \ \exists P \in p^*(P^*), x \in p^*(x^*) : R \text{ is a cover of } (P, x)
\]

b. For any meaningful expressions \( \phi \) of type \( (a, b)^* \) and \( \psi \) of type \( a^* \), \( [\phi \ \psi] \) is a meaningful expression of type \( b^* \), and \( [\phi \ \psi] = C([\phi], \ [\psi]) \).

4.4 Applications

To see how this rule allows us to get rid of the ** operator in the general case, consider the simple plural sentence in (94).

\[
(94) \quad \text{The two girls fed the two cats.}
\]

\[
(95) \quad [\text{the two girls}] = [A + B], \ [\text{the two cats}] = [C + D]
\]

Basic idea: The part-whole structure of the plural set \( [C + D] \) ‘projects’ up in the tree. In other words, fed the two cats will itself denote a plural set with an analogous structure.

\[
(96) \quad [\text{fed the two cats}] = C([\text{fed}], [C + D]) = [\text{fed}(C) + \text{fed}(D)]
\]

Note that in this case, there is only one cover to consider: \( \{ (\text{fed}, C), (\text{fed}, D) \} \)

To combine this VP denotation with the meaning of the subject, we apply \( C \) yet again:

\[
(97) \quad [\text{the two girls fed the two cats}] = C([\text{fed}(C) + \text{fed}(D)], [A + B]) = [\text{fed}(C)(A) + \text{fed}(D)(B)]
\]
\[
\begin{align*}
\text{fed}(C)(B) + \text{fed}(D)(A), \\
\text{fed}(C)(A) + \text{fed}(C)(B) + \text{fed}(D)(B), \ldots
\end{align*}
\]

- (97) is true iff it contains at least one predicate plurality all parts of which are true.
  This is the case iff there is a cover of \((\text{fed}(C) + \text{fed}(D), A + B)\) such that each of the two predicates is true of each individual related to it by the cover.
  This, in turn, is the case if \(A\) and \(B\) cumulatively fed \(C\) and \(D\).

- In sum, the CC rule is a generalization of the \textit{**} operator that applies to functions and arguments of any type.
  It can apply at any step of the derivation provided that the daughter nodes are of the right type. This removes the need for \textit{**} operators in the syntax.

- Recall the **problem with the original \textit{e}-based analysis of predicate conjunction**: It only worked for cases where the predicate conjunction cumulates with another plural that is its semantic argument.
  We can now avoid this problem: Since the CC rule always returns a plural set, the resulting denotation at any step will have ‘parts’ corresponding to the parts of the predicate conjunction.

\begin{enumerate}
  \item \([\text{the two cats}]\) gives us access to the parts \(C\) and \(D\)
  \item \([\text{feed the two cats}]\) gives us access to the parts \(\text{feed}(C)\) and \(\text{feed}(D)\)
\end{enumerate}

- By applying the CC rule at every node dominating a plural, we can now analyze examples like (99) without any special LF movement.

(99) \textit{The two girls wanted to feed the two cats.}

\([\text{The two girls}]\) ultimately cumulates with the plural set \([\lambda x.\text{want}(\text{feed}(C))(x)] + (\lambda x.\text{want}(\text{feed}(D))(x))\].

**Interim summary & outlook**

- We introduced a new composition rule that combines a plural set of functions with a plural set of matching arguments. Intuitively, this is a ‘cumulative version’ of functional application.

- This rule lets the part-whole structure associated with a plural expression ‘project’ to larger constituents containing it.

- In case the two plurals standing in the cumulative relation are not in a function-argument configuration, we apply this rule at each of the intervening nodes. This removes the need for syntactically derived cumulative relations.

- **Next step**: What about our original criticism of the predicate analysis – the flattening effect (100)?

(100) \textit{The two girls made Gene \([P \text{ feed the two dogs}]\ and \([Q \text{ brush Eric}]\) when all he wanted to do was take care of his hamster.}

We will now show how plural sets and the CC rule allow us to account for such data.
The nodes labeled (i)-(iv) in the following tree all involve applications of the CC rule.

\[ (101) \]

\[ \lambda x. \text{made}(\text{feed}(C)(G))(x) + \lambda x. \text{made}(\text{feed}(D)(G))(x) + \lambda x. \text{made}(\text{brush}(E)(G))(x) \]

\[ \text{(made)} \]

\[ \text{(iii) } [\text{feed}(C)(G) + \text{feed}(D)(G) + \text{brush}(E)(G)] \]

\[ \text{(ii) } [\text{feed}(C) + \text{feed}(D) + \text{brush}(E)] \]

\[ [\text{G}] \]

\[ \text{Gene} \]

\[ \text{(i) } [\text{feed}(C) + \text{feed}(D)] \]

\[ \lambda Q, [\text{brush}(E)] \oplus Q \]

\[ \lambda Q, [\text{brush}(E)] \oplus Q \]

\[ \lambda P, \lambda Q, P \oplus Q \]

\[ \text{the two dogs} \]

\[ \text{and} \]

\[ \text{brush Eric} \]

\[ \text{feed} \]

\[ \text{[C + D]} \]

- Since our system contains higher-type pluralities, the operation \( \oplus \) allows us to form the sum of the predicates corresponding to \textit{feed the two dogs} and \textit{brush Eric}.

- Crucially, since \textit{feed the two dogs} has a plural denotation, we end up with a single ‘flat’ plurality consisting of 3 (!) atomic parts.

- This plurality ‘projects’ up in the structure via further applications of the CC rule, until we end up with a predicate sum that can cumulate with \([\text{the two girls}]\).
Summary

(i) We argued that all semantic domains contain pluralities

**why?** The behavior of conjunctions that couldn’t be accounted for otherwise.

(ii) We then proposed a new system for deriving cumulativity that makes crucial use of this
generalized notion of plurality

**why?** Tradional analyses run into problems for which there are no obvious solutions (lack of syntactic
locality, flattening problem)

(iii) This system differs from existing analyses in a number of ways

- cumulativity is not derived by operators targeting predicate denotations
- In cases of non-local cumulation, we don’t need to form the relevant relations syntactically
- cumulativity is encoded by a compositonal rule, which
  a. lets the ‘part’-structure of embedded pluralities be reflected by the denotations of
     the nodes containing them
  b. operates ‘step-by-step’ along the lines of the syntactic structure

(iv) This let us account for a number of empirical observations

- The behavior of conjunctions (qua the assumption that every semantic domain contains
  pluralities)
- The lack of syntactic plurality and the flattening problem: We don’t have to derive predi-
cates syntactically that can then act as the input for cumulation operators

What we will do now....

- Our proposal also differs from existing proposals w.r.t. **scope**
- Predicate analysis (also based on proposals like Sher 1990): In order to derive cumulativity w.r.t.
  n-many pluralities, we require an n-ary relation.
  ~~~ syntactic asymmetries are ‘levelled out’.
- Since our analysis involves a rule that applies step by step, along the lines of the hierarchical
  structure, our system does not ‘flatten out’ syntactic asymmetries
- So, if we found that scope/syntactic asymmetries matter in cumulative sentences, then our system
  should in principle be suited to account for this type of phenomenon
- But are there any such phenomena where scope matters in cumulative sentences?
  **Yes!**

  (i) The behavior of every DP, [Schein 1993, Kratzer 2000] a.o.: Syntactic asymmetries matter,
      (Champollion 2010, Haslinger & Schmitt 2018b). Analogously for other asymmetrically
      distributive universals, (Haslinger & Schmitt 2019). **This is what we will look at
      next!**

  (ii) The behavior of modified numerals, (Buccola & Spector 2016, Haslinger & Schmitt 2018a)

  (iii) The behavior of constructions where we seem to cumulate ‘across’ an intensional predicate,
       (Schmitt 2019b)
References


