Scope-related cumulativity asymmetries and cumulative composition*

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Today's talk:

(i) Empirical problem: Cumulative readings of *every* DPs, distributive conjunction

(ii) Novel analysis: cumulation is built into composition rules

(iii) Independent motivation: Behavior of plural expressions in conjunctions

(iv) Comparison to existing analyses (time allowing)

1 Singular universals and distributive conjunctions

1.1 Cumulativity asymmetries: English singular universals

Distributivity wrt. lower plural expressions

(1) *Every girl in this town* fed *(the) two dogs.*

SCENARIOS: girls Ada and Bea, dogs Carl and Dean

'distributive' scenario: Ada fed Carl and Dean. Bea fed Carl and Dean. TRUE
'cumulative' scenario: Ada fed Carl. Bea fed Dean. FALSE

Cumulativity wrt. higher plural expressions

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(2) *(The) two girls fed every dog in this town.*

SCENARIOS:

'distributive' scenario: Ada fed Carl and Dean. Bea fed Carl and Dean. TRUE

'cumulative' scenario: Ada fed Carl. Bea fed Dean. TRUE

Schein (1993), Kratzer (2000), Zweig (2008), Champollion (2010)

1.2 Cumulativity asymmetries: German distributive conjunction

CONTEXT: There are two skiing World Cup races this weekend. Ada and Bea are the only Austrian participants. Ada is competing in the downhill and Bea in the slalom.

Cumulativity wrt. higher plural expressions

(3) *Zum Glück haben die zwei Österreicherinnen sowohl die Abfahrt als auch den Slalom gewonnen!*
to-the fortune have the two Austrians PRT the downhill PRT also the slalom won
'Fortunately, the two Austrians won both the downhill and the slalom.'

'cumulative' scenario: Ada won the downhill. Bea won the slalom. TRUE

Distributivity wrt. lower plural expressions

(4) *Zum Glück haben sowohl die Ada als auch die Bea die zwei Rennen gewonnen!*
to-the fortune have PRT the Ada PRT also the Bea the two races won
'Fortunately, both Ada and Bea won the two races.'

'cumulative' scenario: Ada won the downhill. Bea won the slalom. FALSE

1.3 Interim summary: Asymmetrically distributive universals (ADUs)

(i) always have a distributive reading wrt. semantically plural expressions in their scope

(ii) allow for cumulative readings if they occur in the scope of a semantically plural expression

(iii) Assumption here: asymmetry tied to scope (following Champollion (2010), further research needed)

ADUs cross-linguistically

- singular universals: English *every* DPs, German *jed-* DPs
- German distributive conjunction: *sowohl A als auch B* 'A as well as B'
- possibly other distributive conjunctions: Hungarian *A is és B is*, Polish *i A i B* (preliminary data)

Next point: Why ADUs represent a problem for a theory of cumulativity.

1.4 Schein sentences

(5) *The two girls wanted to buy the two dogs.*

adapted from Beck and Sauerland (2000)

- **Cumulative truth conditions:**
  Each of the two girls wanted to buy at least one of the two dogs &
  for each of the two dogs, at least one of the two girls wanted to buy it
- $\Rightarrow$ Relation $[\lambda x.\lambda y.y$ wanted to buy $x]$ applies **cumulatively** to the girls and the dogs
- Cumulative relation may be derived by LF-movement Beck and Sauerland (2000)

Schein sentences: ADUs 'sandwiched' between two other plural expressions.

Schein (1993), Kratzer (2000), Champollion (2010)

(6) *Ada and Bea taught every dog two new tricks.* adapted from Schein (1993)

SCENARIO: There are two dogs, Carl and Dean.
Ada taught Carl trick 1 &
Ada taught Carl trick 2 &
Ada taught Dean trick 3 &
Bea taught Dean trick 2 TRUE

(i) it is not the case that for every dog each of the girls taught it two tricks
$\Rightarrow$ every dog **cumulative** wrt. Ada and Bea

(ii) every dog was taught two tricks, tricks can be different
$\Rightarrow$ every dog **distributive** wrt. two tricks

Why a cumulative relation between individuals isn't enough

- Cumulative relation R1, which takes the arguments Ada+Bea and Carl+Dean:

  (7) $\mathrm{R1} = \lambda x_e.\lambda y_e.y$ taught x two new tricks

  No cumulation with two tricks $\rightsquigarrow$ each girl taught two tricks to some dog.

  predicted FALSE

- Cumulative relation R2, which takes the arguments a+b and two tricks:

  (8) $\mathrm{R2} = \lambda x_e.\lambda y_e.y$ taught x to every dog

  No cumulation with every dog $\rightsquigarrow$ The two tricks must be the same for each dog.

  predicted FALSE

1.5 Our approach: Predicate pluralities

(9) *Ada and Bea taught every dog two new tricks.* adapted from Schein (1993)

Existing approaches

- Cumulative relations between events and individuals
  Schein (1993), Kratzer (2000), Zweig (2008)
- Cumulative relations between individuals plus more complex LF Champollion (2010)

Our basic idea

- Cumulation between individuals and predicate pluralities
- Ada+Bea must be in a cumulative relation with one of the elements of this set:

  (10) {**taught C T1** + **taught C T2** + **taught D T1** + **taught D T2**,
  taught C T1 + **taught C T2** + **taught D T2** + **taught D T3**,
  taught C T3 + **taught C T2** + **taught D T1** + **taught D T2**, ...}

- We only consider those pluralities of predicates that assign two tricks to each dog.

2 Independent motivation

2.1 Flattening effect

(11) Ada owns a dog, Carl. Bea owns another dog, Dean, and a cat, Eric. Now they went on a trip and guess what ...
The two girls made Gene [[feed the two dogs]$_P$ and [brush Eric]$_Q$] when all he wanted to do was take care of his hamster. Schmitt (2017)

SCENARIO: A made G feed C, B made G feed D, B made G brush E. TRUE

What happens in this scenario:

- Cumulativity between the two girls and P and Q : No girl satisfies both P and Q.
- Cumulativity between the two girls and the two dogs: No girl made Gene feed both of the dogs
- We cannot derive a 3-place cumulative relation between the two girls, the two dogs and P and Q, because P and Q contains the two dogs
- Intuitively, we want binary cumulation between a+b and the following predicate plurality:

  (12) **feed Carl** + **feed Dean** + **brush Eric**

- 'Flattening': two plural expressions (P+Q and Carl+Dean) correspond to only one plurality in the semantics.

Interim summary

- Traditional approach to cumulative truth-conditions: Binary relations between individuals apply cumulatively. Relations may be syntactically derived.
- Schein-sentences problematic for this approach
- Our idea: use cumulation with pluralities of predicates.
- Independent motivation: Flattening effects

3 Analysis, part 1: Plural projection

3.1 Basic ideas

- The part structure of lower pluralities 'projects' up to higher pluralities (cf. focus projection / Hamblin sets)

(13) *feed Carl and Dean*

$\text{feed}(\text{carl})_{\langle et\rangle} + \text{feed}(\text{dean})_{\langle et\rangle}$

$\text{feed}_{\langle e\langle et\rangle\rangle}$ $\quad$ $\text{carl}_e + \text{dean}_e$

(14) *feed and brush Dean*

$\text{feed}(\text{dean})_{\langle et\rangle} + \text{brush}(\text{dean})_{\langle et\rangle}$

$\text{feed}_{\langle e\langle et\rangle\rangle} + \text{brush}_{\langle e\langle et\rangle\rangle}$ $\quad$ dean_e

- Crucial step: Cumulativity encoded in projection mechanism: Compositional rule
- For this rule to be generalizable – one more level of complexity: Plural sets

(15) *feed and brush Carl and Dean*

$\{\ \text{feed}(\text{carl})+\text{brush}(\text{dean}),\ \text{feed}(\text{dean})+\text{brush}(\text{carl}),\ \ldots\}$

$\{\ \text{feed}_{\langle e\langle et\rangle\rangle} + \text{brush}_{\langle e\langle et\rangle\rangle}\ \}$ $\quad$ $\{\ \text{carl}_e + \text{dean}_e\ \}$

- No syntactically derived predicates needed; in cases of 'non-lexical cumulation', the composition rule applies at each intervening node

3.2 Ontology, informally

- All domains contain pluralities (including domains for complex types).
- We define a sum-operation $+$ for any type: Isomorphic to union of sets of atoms.

(16) $D_e = \{\ \textbf{Ada, Bea, Ada+Bea}\ \}$,
$D_{\langle e,t\rangle} = \{\lambda x.\text{smoke}(x), \lambda x.\text{dance}(x), \lambda x.\text{smoke}(x) + \lambda x.\text{dance}(x) \ldots\}$

- For every type a there is a type a^* of 'plural sets'.
- The domains $D_{\langle a,t\rangle}$ and D_{a^*} are disjoint, but have the same algebraic structure. We write [ ] instead of { } for plural sets.

(17) D_{e^*} = { [ ], [**Ada**], [**Bea**], [**Ada+Bea**], [**Ada, Bea**], [**Ada, Ada+Bea**], [**Bea, Ada+Bea**], [**Ada, Bea, Ada+Bea**] }

3.3 Semantic background

- We employ some 'trivial' **type shifts** between domains D_a, D_{a^*} that we don't indicate.
- Plural definites and indefinites denote plural sets of type e^*

(18) ⟦*the girls*⟧ = [**Ada+Bea**]

(19) ⟦*two pets*⟧ = [**Carl+Dean**, **Carl+Eric**, **Dean+Eric**]

- Conjunction involves 'recursive' sum $\bigoplus$

(20) ⟦*Ada and two pets*⟧ = [**Ada**] $\oplus$ [**Carl+Dean**, **Carl+Eric**, **Dean+Eric**]
= [**Ada+Carl+Dean**, **Ada+Carl+Eric**, **Ada+Dean+Eric**]

- A plural set S of propositions is **true** iff S contains **at least one** element p such that **all** atomic parts of p are true.

3.4 Cumulative composition

- A **cover** of (P, x) is a relation between atomic parts of P and atomic parts of x in which each atomic part of P and each atomic part of x occurs at least once.

(21) P = **smoke+dance**, x = **Ada+Bea**
a. {⟨ **smoke**, **Ada** ⟩, ⟨ **dance**, **Bea** ⟩}
b. {⟨ **smoke**, **Bea** ⟩, ⟨ **dance**, **Ada** ⟩, ⟨ **dance**, **Bea** ⟩} ...

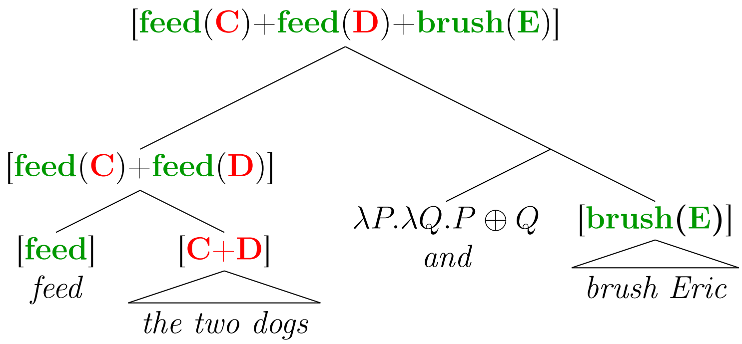
- Compositional rule for cumulation: $\mathcal{C}$
  - It takes two plural sets $P^*_{\langle a,b\rangle^*}$ and $x^*_{a^*}$ and gives us a plural set of type b^*.
  - We take all covers of some plurality from $P^*_{\langle a,b\rangle^*}$ and some plurality from $x^*_{a^*}$.
  - For each cover R, we form the sum of values $+\{P(x) \mid (P, x) \in R\}$. (actually more complex: we use the 'recursive sum' $\bigoplus$ when P, x are themselves plural sets)
- Example:

(22) a. *Two children are smoking and dancing.*
b. $\mathcal{C}$([**smoke+dance**]) ([**A+B**, **A+G**, **B+G**]) = [**S**(**A**)+**D**(**B**), **S**(**A**)+**D**(**G**), **S**(**B**)+**D**(**G**), **D**(**A**)+**S**(**B**), **D**(**A**)+**S** (**G**), **D**(**B**)+**S**(**G**) **S**(**A**)+**D**(**A**+ **D**(**B**), ...]

3.5 Deriving the flattening effect for conjunction

(23) *The two girls made Gene [[feed the two dogs] and [brush Eric]]*

(24)



Interim summary: Plural projection

- Semantic plurality 'projects' by means of a cross-categorial operation $\mathcal{C}$ which also encodes cumulativity.
- Syntactically derived cumulative relations and the corresponding LF movement are not needed: In the case of non-lexical cumulation $\mathcal{C}$ applies at every intervening node.
- This is made possible by assuming pluralities and plural sets of any semantic type.
- Unlike earlier approaches to cumulativity, the present theory naturally accounts for the flattening effect.

4 Analysis, part 2: Cumulativity asymmetries

What we will do:

- We will give a new meaning for *every* that capture cumulativity asymmetries:

  (25) a. *Every girl fed (the) two dogs.*
  b. *(The) two girls fed every dog in this town.*

- Rationale based on Schein sentences: We want predicate pluralities that 'cover' every dog and assign two tricks to each dog.

  (26) *Ada and Bea taught every dog two new tricks.*

  (27) {**taught C T1** + **taught C T2** + **taught D T1** + **taught D T2**,
  taught C T1 + **taught C T2** + **taught D T2** + **taught D T3**,
  taught C T3 + **taught C T2** + **taught D T1** + **taught D T2**, ...}

4.1 *every* DPs, informally

- Function of type $\langle\langle e, a\rangle^*, a^*\rangle$ – directly manipulates plural sets of predicates.

(28) *every girl fed two pets*
⟦*every girl*⟧([**feed**(**C**) + **feed**(**D**), **feed**(**C**)+**feed**(**E**), **feed**(**D**)+**feed**(**E**)])

- For each atomic individual x in the restrictor, we choose a predicate-plurality P from the scope, apply each $P' \leq_a P$ to x and take the sum (P applies 'distributively' to x)

(29) **feed**(**C**)(**A**)+ **feed**(**D**)(**A**), **feed**(**C**)(**B**)+ **feed**(**E**)(**B**), ...

- For each such assignment of predicate-pluralities, we take the sum over all individuals and form the plural set of all such sums

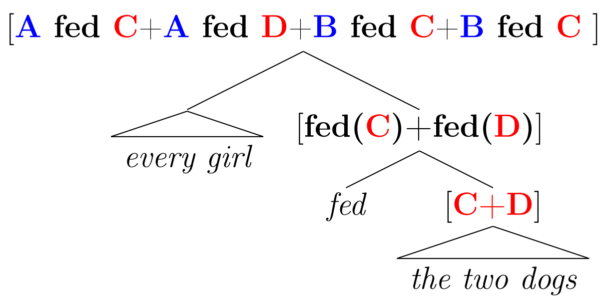
(30) ⟦*every girl*⟧([**feed**(**C**) + **feed**(**D**), **feed**(**C**)+**feed**(**E**), **feed**(**D**)+**feed**(**E**)]) =
[**feed**(**C**)(**A**)+ **feed**(**D**)(**A**) + **feed**(**C**)(**B**)+**feed**(**E**)(**B**),
feed(**C**)(**A**)+ **feed**(**E**)(**A**) + **feed**(**C**)(**B**)+ **feed**(**D**)(**B**),
feed(**C**)(**A**)+ **feed**(**E**)(**A**) + **feed**(**D**)(**B**)+**feed**(**E**)(**B**), ...]

- The resulting value is a plural set containing predicates/propositions

4.2 Deriving cumulativity asymmetries

(31) *Every girl in this town fed the two dogs.* **only distributive**

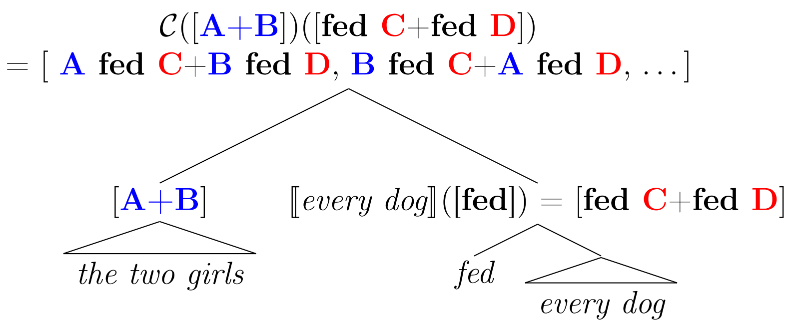
(32)



Prediction: Singular universals always distributive wrt. material in their scope

(33) *The two girls fed every dog in this town.* **cumulative possible**

(34)



Prediction: Cumulation with material outscoping *every* possible, since *every P Q* returns a plurality

Schein sentences:

(35) *Ada and Bea taught every dog two new tricks.*

[**A taught C T1+A taught C T2+B taught D T2+A taught D T3,**
A taught D T1+B taught D T2 + B taught C T2+B taught C T3, ...]

[**A+B**]
Ada and Bea

[**taught C T1+taught C T2 + taught D T2+taught D T3,**
taught D T1+taught D T2 + taught C T2+taught C T3, ...]

every dog

[**taught T1+taught T2,**
taught T2+taught T3, **taught T1+taught T3**]

taught two new tricks

Interim Summary: Plural projection and cumulativity asymmetries

- *every* DPs take plural sets as their argument. They 'distributively' apply elements to atoms in the restrictor.
- The result is a plural set, which can be cumulated with higher pluralities.
- This analysis can be generalized to distributive conjunction (see handout)

5 Comparison with existing theories

5.1 Event-based analyses (Schein 1993, Kratzer 2000, Zweig 2008)

Basic idea: Cumulation targets relations between events and individuals.

(36) *The two girls taught every dog two new tricks.*

(37) $\exists e[\mathbf{teach}(e) \wedge \textsc{agent}(e)(\mathbf{A}+\mathbf{B}) \wedge \textsc{ben}(e)(+(\mathbf{dog})) \wedge \forall y \leq_A +(\mathbf{dog})[\exists Z \in [\![\textit{two tricks}]\!].\exists e' \leq e[\textsc{theme}(e')(Z) \wedge \textsc{ben}(e')(y)]]]$
adapted from Zweig (2008)

Differences to our proposal

- We don't **require** events, so we can maintain that some predicates that allow for cumulativity don't have an event/state argument.
- No special story needed for cumulation across predicates where arguments are neither individuals nor events, such as (some) attitude verbs (38).

(38) *The Georgian ambassador called this morning, the Russian one at noon.*
They think that Trump should take a walk with Putin and build a hotel in Tbilisi, but neither addressed the Caucasus conflict!

5.2 Individual-based analysis (Champollion 2010)

- No appeal to events
- *every* DPs denote pluralities of individuals

(39) $[\![$*every dog*$]\!] = [\![$*the dogs*$]\!] =$ **C+D**

- *every* must directly c-command a distributivity or cumulation operator (*, **, ...)
- traces of *every* DPs must range over atoms

(40) *The two girls taught every dog two new tricks.*

(41) [[*the two girls*][[*every dog*] ** [2 1 [[*two new tricks* [3 [t_1 ****taught* t_2 t_3]]]]]]]

(42) $[\![$[2 1 [[*two new tricks* [3 [t_1 ****taught* t_2 t_3]]]]]$]\!] =$
$\lambda x_e : x$ atomic.λy_e.there is a plurality Z of two tricks such that y cumulatively taught Z to x

Differences to our proposal:

- Champollion (2010) *must* assume that traces of ADUs range over atoms: No straightforward account for distribution to non-atomic subpluralities.

(43) *Sowohl die Mädchen als auch die Buben haben zwei Hunde gefüttert*
PRT the girls PRT also the boys have two dogs fed
'Both the girls and the boys fed two dogs.' (German)

SCENARIO: The girls fed two dogs between them and the boys fed two dogs between them. TRUE

- Our account, as opposed to theories working with syntactically derived predicates, generalizes to flattening effects with conjunction.

6 Conclusion

- We presented a system that derives cumulativity without syntactically derived cumulative relations
- This system derives cumulative truth-conditions step-by-step, along the lines of the hierarchical structure
- This system accounts for the 'flattening' effects with conjunction
- We looked at cumulative asymmetries and 'flattening' effects with ADUs
- We showed how our system accounts for these data

Questions/problems

- Some technical issues (see appendix below)
- Expansion to collective predicates?
- Expansion to non-upward-monotone DPs (*less than five, exactly five …*)
- Cross-linguistic differences concerning conditions on cumulative reading – scope vs. grammatical function (Flor 2017)

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A Technical details

A.1 Ontology

- For every type a, there is a set A_a of atomic elements and the domain D_a contains the elements from A_a and pluralities thereof (cf. Schmitt 2013 for independent motivation).
- **Pluralities** correspond to nonempty subsets of A_a and are formed via a sum operation $+$.
- In addition, we introduce separate semantic types for '**plural sets**' that form the input to cumulation. In particular, a plural set with elements of type a is itself a possible denotation of type a^*. We assume that D_{a^*} and $A_{\langle a,t\rangle}$ are isomorphic, but disjoint. This allows us to define semantic operations that are sensitive to whether their argument is a plural set.

Definition 1 (Types)**.** *The set T of semantic types is the smallest set such that:*

(i) $e \in T, t \in T$

(ii) for any $a, b \in T$, $\langle a, b\rangle \in T$

(iii) for any $a \in T$, $a^* \in T$

Definition 2 (Semantic domains)**.** *Let A be the (nonempty) set of atomic individuals. For each type a, there is an 'atomic domain' A_a and a full domain D_a with the following properties:*

(i) $A_e = A; A_t = \{0, 1\}$

(ii) For any type a:

(a) D_a is a set such that $A_a \subseteq D_a$ and there is an operation $+\!\!\!\!+_a : \mathcal{P}(D_a)\backslash\{\emptyset\} \to D_a$.

(b) There is a bijection $pl_a : \mathcal{P}(A_a)\backslash\{\emptyset\} \to D_a$ such that $pl_a(\bigcup \mathcal{S}) = +\!\!\!\!+(\{pl_a(S) : S \in \mathcal{S}\})$ for any $\mathcal{S} \subseteq (\mathcal{P}(A_a)\backslash\{\emptyset\})$ and $pl_a(\{x\}) = x$ for each $x \in A_a$.

(c) D_a and $\mathcal{P}(A_a)$ are disjoint.

(iii) For any types a, b: $A_{\langle a,b\rangle} = D_b^{D_a}$, the set of all partial functions from D_a to D_b.

(iv) For any type a, there is a set D_a^* disjoint from $\mathcal{P}(D_a)$ on which the operations $\cup$, $\cap$ and $\backslash$ are defined and a function $pl_a^* : \mathcal{P}(D_a) \to D_a^*$ that is an isomorphism wrt. $\cup$, $\cap$ and $\backslash$.

(v) For any type a, $A_{a^*} = D_a^*$.

(vi) For any types a, b, D_a and D_b are disjoint. [issue with empty set?]

Notational conventions:

- In the metalanguage, we use natural language expressions in boldface to denote the meanings of lexical elements (e.g. **girl**, **anna**, **show**).
- We use 'starred' variables like x^*, P^* etc. for types of the form a^*.
- We occasionally omit type subscripts on operations that are defined cross-categorially, like pl_a^*, pl_a, $\oplus_a$ etc.
- If we apply $\cup, \cap, \in$, cardinality etc. to plural sets, these operations are defined via the isomorphism between $\mathcal{P}(D_a)$ and D_a^*.
- For any type b and $x, y \in D_b$:
  - $x +_b y = +\!\!\!\!+_b(\{x, y\})$
  - $x \leq y \Leftrightarrow x +_b y = y$
  - $x \leq_a y \Leftrightarrow x \leq y \wedge y \in A_b$
- We occasionally write $[x_1, \ldots, x_n]$ for $pl^*(\{x_1, \ldots, x_n\})$.

A.2 Building cumulativity into the compositional system

- We define a cross-categorial operation $\bigoplus$ that occurs both in the semantics of conjunction and in cumulativity.
- In most cases $\bigoplus$ is equivalent to the usual sum operation $+$, but if the elements to be summed up are plural sets, it has a 'distributive' effect: Intuitively it produces the set of all pluralities that can be obtained by applying some choice function to the plural sets and summing up all the values.
- In the case of nested plural sets (i.e. type $(a^*)^*$), this is done 'recursively'. This stipulation is needed for the analysis of conjunction particles.

Definition 3 (Recursive sum)**.** *The operation* $\bigoplus_a : \mathcal{P}(D_a)\backslash\{\emptyset\} \rightarrow D_a$ *is defined for any type a in the following way:*

(i) For any type a that is not of the form b^**, and any nonempty* $S \subseteq D_a$, $\bigoplus S = +S$.

(ii) For any type b^* *and any nonempty* $S \subseteq D_{b^*}$, $\bigoplus_{b^*} S = pl^*(\{X \mid \exists f : f$ *is a function from* S *to* $D_b \wedge \forall X^* \in S : f(X^*) \in pl^{*-1}(X^*) \wedge X = \bigoplus_b(\{f(X^*) \mid X^* \in S\})\})$

Notational convention: For any type a and any $x, y \in D_a$: $x \oplus_a y = \bigoplus_a(\{x, y\})$.

The notion of a 'cover' formalizes the weak truth conditions of cumulative sentences.

Definition 4 (Cover)**.** *Let* $P \in D_a, x \in D_b$*. A relation* $R \subseteq A_a \times A_b$ *is a* ***cover*** *of* (P, x) *iff* $+(\{P' \mid \exists x' : (P', x') \in R\}) = P$ *and* $+(\{x' \mid \exists P' : (P', x') \in R\}) = x$.

We now define a way of combining plural sets. Intuitively, we take *any* cover of some plurality in the functor set and some plurality in the argument set, and then sum up the values for all function-argument pairs in the cover.

Definition 5 (Recursive Cumulation)**.** *For any* $P^* \in D_{\langle a,b\rangle^*}$ *and* $x^* \in D_{a^*}$*:*
$\mathcal{C}(P^*, x^*) = pl^*(\{\bigoplus(\{P(x) \mid (P, x) \in R\}) \mid \exists P \in pl^{*-1}(P^*), x \in pl^{*-1}(x^*) : R$ *is a cover of* $(P, x)\})$

Definition 6 (Composition rules)**.** *We assume that for each type a, there is a (possibly empty) set* L_a *of lexical expressions of type a, and there is a function M that assigns any* $\alpha \in L_a$ *some denotation in* D_a*. For any type a, we can then define the meaningful expressions of type a and their denotations as follows:*

(i) For any lexical expression $\alpha \in L_a$, α *is a meaningful expression of type a and* $[\![\alpha]\!] = M(\alpha)$.

(ii) For any meaningful expressions α *of type* $\langle a, b\rangle$ *and* β *of type a,* $[\alpha\ \beta]$ *is a meaningful expression of type b. In this case,*
$[\![[\alpha\ \beta]]\!] = [\![\alpha]\!]([\![\beta]\!])$
if $[\![\alpha]\!]$ *and* $[\![\beta]\!]$ *are both defined,* $[\![\alpha]\!] \in A_{\langle a,b\rangle}$ *(i.e.* $[\![\alpha]\!]$ *is not a proper plurality) and* $[\![\beta]\!] \in dom([\![\alpha]\!])$*. Otherwise,* $[\![[\alpha\ \beta]]\!]$ *is undefined.*

(iii) For any meaningful expressions α *of type* $\langle a, b\rangle^*$ *and* β *of type* a^*, $[\alpha\ \beta]$ *is a meaningful expression of type* b^**. In this case,*
$[\![[\alpha\ \beta]]\!] = \mathcal{C}([\![\alpha]\!], [\![\beta]\!])$ *(see Definition 5)*
if $[\![\alpha]\!]$ *and* $[\![\beta]\!]$ *are both defined. Otherwise,* $[\![[\alpha\ \beta]]\!]$ *is undefined.*

(iv) Nothing else is a meaningful expression of type a.

Note: This definition is vague in that it does not say anything about what $\mathcal{C}(P^*, x^*)$ should look like in cases where functional application fails 'within' the cumulation rule (i.e. some atomic part of some $P \in P^*$ is undefined for some atomic part of some $x \in x^*$). We do not attempt a serious treatment of presupposition projection here. The empirical issue of how presuppositions – and other cases of partiality in semantics – actually interact with cumulation is presently not clear to us.

We also did not include variable binding as there are technical problems involved in combining variable binding with a semantics based on alternative sets. These problems arise independently in the semantics of questions and focus.

A.3 Lexicon

Note: Here, we assume that open-class elements always denote atomic objects of types that do not contain any *, i.e. only the functional lexicon and the composition rules can manipulate plural sets. Lexically collective predicates are not analyzed correctly within the present system.

Type-shifts mediate between 'singular' denotations and plural sets. Here, we represent these type-shifts in the syntax for simplicity, but ultimately we do not think this is necessary (or even well motivated).

Definition 7 (Type shifts)**.** *For any type a:*

(i) $[\![\Uparrow_{\langle a,a^*\rangle}]\!] = \lambda x_a.pl^*(\{x\})$

(ii) $[\![\Downarrow_{\langle a^*,a\rangle}]\!] = \lambda S^*_{a^{**}}.pl^*(\bigcup\{pl^{*-1}(S) \mid S \in S^*\})$

We take non-distributive conjunction to be the natural-language counterpart of the cumulation operation $\oplus$.

Definition 8 (Coordination)**.** *For any type a:*

- $[\![\text{AND}_{\langle a,\langle a,a\rangle\rangle}]\!] = \lambda x_a.\lambda y_a.x \oplus_a y$
- $[\![\text{OR}_{\langle a^*,\langle a^*,a^*\rangle\rangle}]\!] = \lambda X^*_{a^*}.\lambda Y^*_{a^*}.pl^*(pl^{*-1}(X^*) \cup pl^{*-1}(Y^*))$

Note: The operation $\oplus_{a^*}$ as defined here is problematic as a potential meaning for conjunction because it is idempotent and commutative, but not associative. So $(D_{a^*}, \oplus)$ does not form a semilattice even though $(D_{a^*}, +)$ does. However, $(D_a, \oplus)$ is a semilattice for any type a that is not of the form b^*, due to the isomorphism with subsets. The cases where associativity fails are all such that two or more conjuncts are identical in meaning. At present, it is unclear to us how the cumulation operation $\bigoplus$ could be redefined in such a way that conjunction always comes out as associative.

We assume that definite and indefinite plural DPs are of type e^*. In particular, a plural indefinite denotes a set of pluralities of individuals. Note that this approach will not extend to non-upward-monotonic indefinites such as *exactly ten people* or *at most ten people*.

Definition 9 (Plural definites and upward-monotonic indefinites)**.**

- $\mathcal{A}(P^*_{\langle e,t\rangle^*}) = \lambda x_e.[\exists P_{\langle e,t\rangle}.P \in pl^{*-1}(P^*) \wedge \exists P'_{\langle e,t\rangle}.P' \leq_a P \wedge P'(x)]$
- $[\![\text{PL}_{\langle\langle e,t\rangle^*,e^*\rangle}]\!] = \lambda P^*_{\langle e,t\rangle^*}.pl^*(\lambda x_e.\forall y_e[y \leq_a x \rightarrow \mathcal{A}(P^*)(y)])$
- $[\![\text{DEF}_{\langle e^*,e^*\rangle}]\!] = \lambda x^*_{e^*} : \exists x \in pl^{*-1}(x^*).[\forall y \in pl^{*-1}(x^*).y \leq x].pl^*(\{\iota x \in pl^{*-1}(x^*).\forall y \in pl^{*-1}(x^*).y \leq x\})$
- $[\![\text{TWO}_{\langle e^*,e^*\rangle}]\!] = \lambda x^*_{e^*}.pl^*(\lambda x.pl^{*-1}(x^*)(x) \wedge |x| = 2)$ *where* $|x|$ *is the number of atomic parts of* x.

Notes:

- Conjunction and disjunction in the restrictor of a determiner are treated analogously to avoid generating plurality inferences for conjunctions in the restrictor. This is done by the operation $\mathcal{A}$, which forms the set of all atomic parts of some plurality in the plural set. However, since there are empirical differences between conjunctive and disjunctive NP-coordination, this analysis will have to be modified.
- The pluralization operator PL intuitively forms the union of all the atomic parts of all elements in a plural set. Dissociating this operation from the determiner allows us to analyze phrases like *two girls* and *the two girls* without positing silent existential quantifiers and without type mismatches.

The basic idea behind the analysis of *every/jeder* is the following: When an *every* DP combines with a plural set of predicates, we take those sums that can be formed by choosing some predicate plurality in the plural set for each individual, combining all individuals with their respective predicate pluralities and summing up all the values.

Definition 10 (Singular universals)**.**

- *for any* $P_{\langle a,b\rangle}, x_a$*:* $\mathcal{D}(P,x) = \mathbin{+\!\!\!\!/}(\{Q(x) : Q \leq_a P\})$
- $[\![\text{EVERY}_{\langle\langle e,t\rangle^*,\langle\langle e,a\rangle^*,a^*\rangle\rangle}]\!] = \lambda P^*_{\langle e,t\rangle^*}.\lambda R^*_{\langle e,a\rangle^*}.pl^*\{\mathbin{+\!\!\!\!/}(\{\mathcal{D}(f(x),x) : x \in \mathcal{A}(P^*)\}) : f$ *is a function from* $\mathcal{A}(P^*)$ *to* $pl^{*-1}(R^*)\}$

In the case of conjunction particles, cumulation is built into the lexical entry. This licenses cumulative readings of the plural conjuncts within a distributive conjunction of the type *sowohl A als auch B* 'A as well as B'. The distributive effects of these conjunctions are derived from the effects of $\bigoplus$ and the fact that such conjunctions have to be type-shifted in order to combine with the predicate via cumulation (see examples below).

Definition 11 (Conjunction particles)**.**
$[\![\mu_{\langle e^*,\langle\langle e,a\rangle^*,a^*\rangle\rangle}]\!] = \lambda x^*_{e^*}.\lambda P^*_{\langle e,a\rangle^*}.\mathcal{C}(P^*, x^*)$

In principle, our approach can also account for distributive readings of predicates that combine with plural arguments.

Definition 12 (Predicate-level distributivity operators)**.**
$[\![\text{DIST}_{\langle\langle a,b\rangle^*,\langle a^*,b^*\rangle\rangle}]\!] = \lambda P^*_{\langle a,b\rangle^*}.\lambda x_a.pl^*(\{Q \mid \exists f : f$ *is a function from* $\{y \mid y \leq_a x\}$ *to* $pl^{*-1}(P^*) \wedge Q = \mathbin{+\!\!\!\!/}(\{\mathcal{D}(f(y),y) \mid y \leq_a x\})\})$

A problem: In the present system, distributive predicates with DIST and cumulative predicates differ in semantic type ($\langle e^*, t^* \rangle$ vs. $\langle e, t \rangle^*$). This means that we can't get 'mixed' readings for predicate conjunctions like (44) where only one conjunct has a distributive interpretation (such readings are discussed by Dowty (1987)). At a technical level this could be addressed by replacing the cumulation rule with a cumulation operator that applies freely in the syntax (of type $\langle \langle a, b \rangle^*, \langle a^*, b^* \rangle \rangle$ like DIST), but we do not want to do this since we have not seen an overt counterpart of this operator in any language.

(44) Ada and Bea ate two pizzas and had a glass of wine.

B Examples

Note: In the following examples we gloss over the problematic behavior of $\bigoplus$ with conjuncts identical in meaning that was mentioned above.

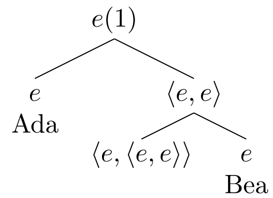
(45) A toy model

a. $M(\text{dog}) = \{\textbf{carl}, \textbf{dean}, \textbf{eric}\}$
b. $M(\text{girl}) = \{\textbf{ada}, \textbf{bea}\}$
c. $M(\text{boy}) = \{\textbf{che}, \textbf{ed}\}$
d. $M(\text{trick}) = \{t_1, t_2, t_3\}$

DP meanings

(46) Ada and Bea

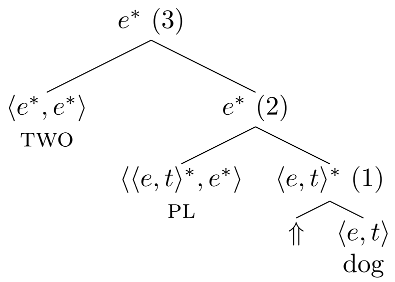
a.



b. $[\![(1)]\!] = \textbf{ada} \oplus \textbf{bea} = \textbf{ada} + \textbf{bea}$

(47) two dogs

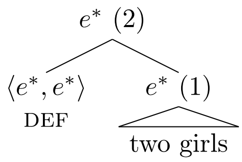
a.



b. $[\![(1)]\!] = [\textbf{dog}] = \text{PL}^*(\{\textbf{dog}\})$
c. $[\![(2)]\!] = pl^*(\{x_e \mid \forall y \leq_a x : \textbf{dog}(y)\}) = [\textbf{carl}, \textbf{dean}, \textbf{eric}, \textbf{carl} + \textbf{dean}, \textbf{carl} + \textbf{eric}, \textbf{dean} + \textbf{eric}, \textbf{carl} + \textbf{dean} + \textbf{eric}]$
d. $[\![(3)]\!] = pl^*(\{x_e \mid x \in pl^{-1}([\![(2)]\!]) \wedge |x| = 2\}) = [\textbf{carl}{+}\textbf{dean}, \textbf{carl}{+}\textbf{eric}, \textbf{dean}{+}\textbf{eric}]$

(48) the two girls

a.

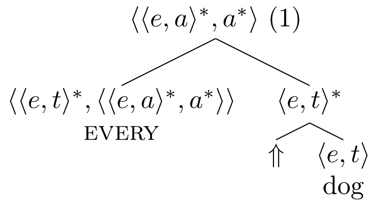


b. $[\![(1)]\!] = [\mathbf{ada} + \mathbf{bea}]$

c. $[\![(2)]\!] = pl^*(\{\iota x \in pl^{*-1}([\![(1)]\!]).\forall y \in pl^{*-1}([\![(1)]\!]).y \leq x\})$ if $\exists x \in pl^{*-1}([\![(1)]\!]).[\forall y \in pl^{*-1}([\![(1)]\!]).y \leq x]$, otherwise undefined
$= pl^*(\{\iota x \in \{\mathbf{ada} + \mathbf{bea}\}.\forall y \in \{\mathbf{ada} + \mathbf{bea}\}.y \leq x\})$ if $\exists x \in \{\mathbf{ada} + \mathbf{bea}\}.[\forall y \in \{\mathbf{ada} + \mathbf{bea}\}.y \leq x]$, otherwise undefined
$= [\mathbf{ada} + \mathbf{bea}]$

(49) every dog

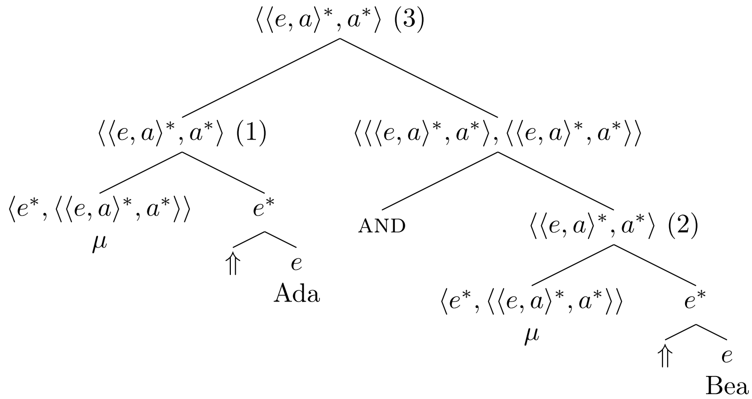
a.



b. $[\![(1)]\!] = \lambda R^*_{\langle e,a\rangle^*}.pl^*\{+(\{\mathcal{D}(f(x), x) : x \in \mathcal{A}([\mathbf{dog}])\}) : f$ is a function from $\mathcal{A}([\mathbf{dog}])$ to $pl^{*-1}(R^*)\}$
$= \lambda R^*_{\langle e,a\rangle^*}.pl^*\{+(\{\mathcal{D}(f(x), x) : x \in A_e \wedge \mathbf{dog}(x)\}) : f$ is a function from $\{x \mid x \in A_e \wedge \mathbf{dog}(x)\}$ to $pl^{*-1}(R^*)\}$

(50) sowohl Ada als auch Bea
PRT Ada PRT also Bea
'Ada as well as Bea' (German)

a.



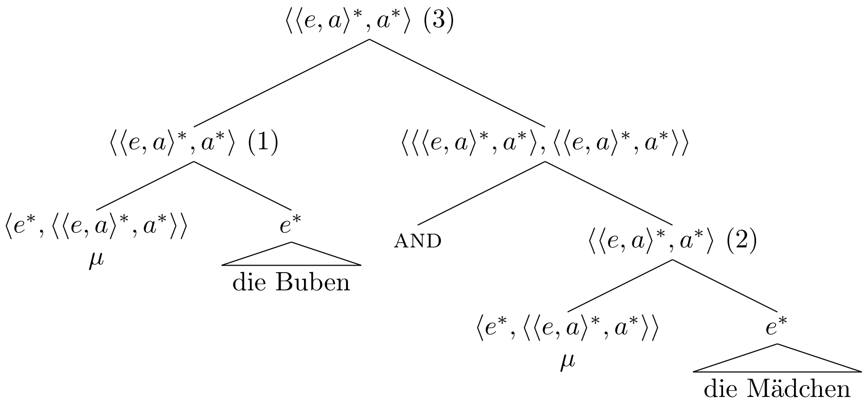
b. $[\![(1)]\!] = \lambda R^*_{\langle e,a\rangle^*}\mathcal{C}(R^*, [\mathbf{ada}])$

c. $[\![(2)]\!] = \lambda R^*_{\langle e,a\rangle^*}\mathcal{C}(R^*, [\mathbf{bea}])$

d. $[\![(3)]\!] = (\lambda R^*_{\langle e,a\rangle^*}\mathcal{C}(R^*, [\mathbf{ada}])) + (\lambda R^*_{\langle e,a\rangle^*}\mathcal{C}(R^*, [\mathbf{bea}]))$

(51) sowohl die Buben als auch die Mädchen
PRT the boys PRT also the girls
'the boys as well as the girls' (German)

a.



b. $[\![(1)]\!] = \lambda R^*_{\langle e,a\rangle^*}\mathcal{C}(R^*, pl^*(\{\bigoplus\{x \mid \mathbf{boy}(x)\}\}))$ if $\exists x.\mathbf{boy}(x)$, otherwise undefined $= \lambda R^*_{\langle e,a\rangle^*}\mathcal{C}(R^*, [\mathbf{che} + \mathbf{ed}])$

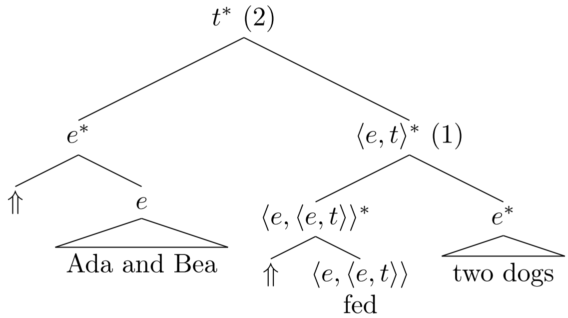
c. $[\![(2)]\!] = \lambda R^*_{\langle e,a\rangle^*}\mathcal{C}(R^*, pl^*(\{\bigoplus\{x \mid \mathbf{girl}(x)\}\}))$ if $\exists x.\mathbf{girl}(x)$, otherwise undefined $= \lambda R^*_{\langle e,a\rangle^*}\mathcal{C}(R^*, [\mathbf{ada} + \mathbf{bea}])$

d. $[\![(3)]\!] = (\lambda R^*_{\langle e,a\rangle^*}\mathcal{C}(R^*, [\mathbf{che} + \mathbf{ed}])) + (\lambda R^*_{\langle e,a\rangle^*}\mathcal{C}(R^*, [\mathbf{ada} + \mathbf{bea}]))$

Simple examples with cumulation

(52) Ada and Bea fed two dogs.

a.

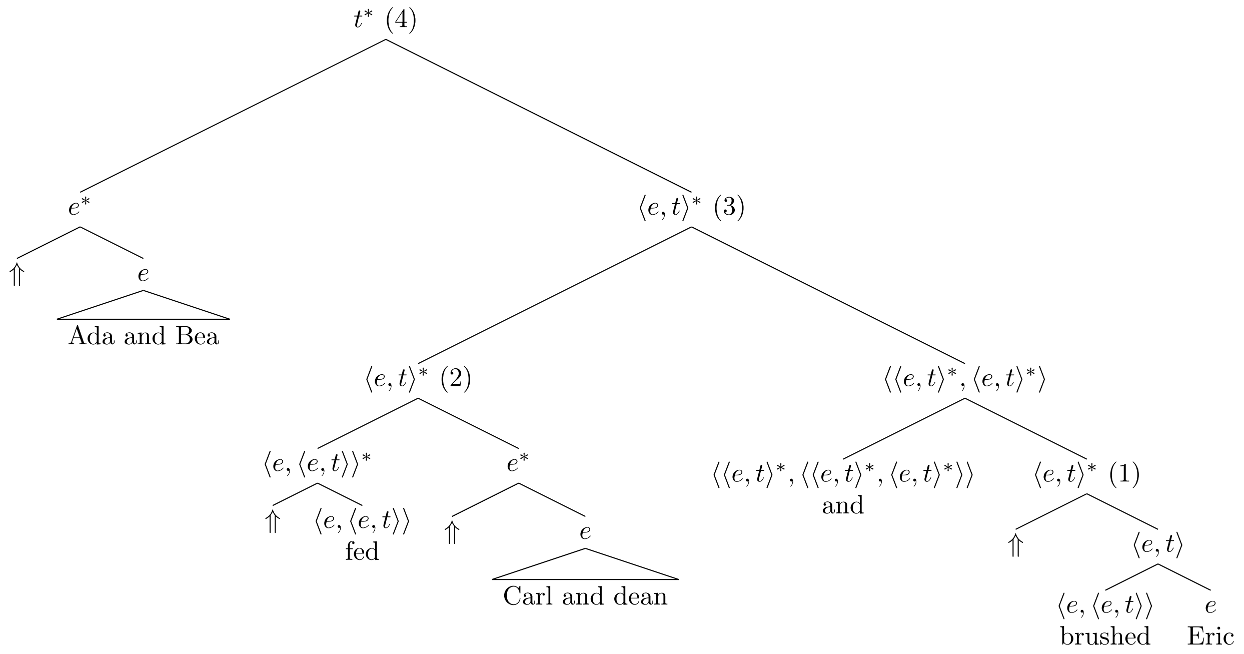


b. $[\![(1)]\!] = \mathcal{C}([\mathbf{fed}], [\![\text{two dogs}]\!]) = \mathcal{C}([\mathbf{fed}], [\mathbf{carl} + \mathbf{dean}, \mathbf{carl} + \mathbf{eric}, \mathbf{dean} + \mathbf{eric}]) = [\mathbf{fed}(\mathbf{carl}) + \mathbf{fed}(\mathbf{dean}), \mathbf{fed}(\mathbf{carl}) + \mathbf{fed}(\mathbf{eric}), \mathbf{fed}(\mathbf{dean}) + \mathbf{fed}(\mathbf{eric})]$

c. $[\![(2)]\!] = \mathcal{C}([\![(1)]\!], [\mathbf{ada}{+}\mathbf{bea}]) = [\mathbf{fed}(\mathbf{carl})(\mathbf{ada}){+}\mathbf{fed}(\mathbf{dean})(\mathbf{bea}), \mathbf{fed}(\mathbf{carl})(\mathbf{bea}){+}\mathbf{fed}(\mathbf{dean})(\mathbf{ada}),$
$\mathbf{fed}(\mathbf{carl})(\mathbf{ada}) + \mathbf{fed}(\mathbf{eric})(\mathbf{bea}), \mathbf{fed}(\mathbf{carl})(\mathbf{bea}) + \mathbf{fed}(\mathbf{eric})(\mathbf{ada}),$
$\mathbf{fed}(\mathbf{dean})(\mathbf{ada}){+}\mathbf{fed}(\mathbf{eric})(\mathbf{bea}), \mathbf{fed}(\mathbf{dean})(\mathbf{bea}){+}\mathbf{fed}(\mathbf{eric})(\mathbf{ada}), \mathbf{fed}(\mathbf{carl})(\mathbf{ada}){+}$
$\mathbf{fed}(\mathbf{carl})(\mathbf{bea}) + \mathbf{fed}(\mathbf{dean})(\mathbf{bea}), \ldots]$

(53) Ada and Bea fed Carl and dean and brushed Eric.

a.



b. $[\![(1)]\!] = [\mathbf{brushed}(\mathbf{eric})]$

c. $[\![(2)]\!] = \mathcal{C}([\mathbf{fed}],[\mathbf{carl}+\mathbf{dean}]) = [\mathbf{fed}(\mathbf{carl})+\mathbf{fed}(\mathbf{dean})]$

d. $[\![(3)]\!] = \bigoplus\{[\mathbf{brushed}(\mathbf{eric})],[\mathbf{fed}(\mathbf{carl})+\mathbf{fed}(\mathbf{dean})]\} = [\mathbf{brushed}(\mathbf{eric})+\mathbf{fed}(\mathbf{carl})+\mathbf{fed}(\mathbf{dean})]$

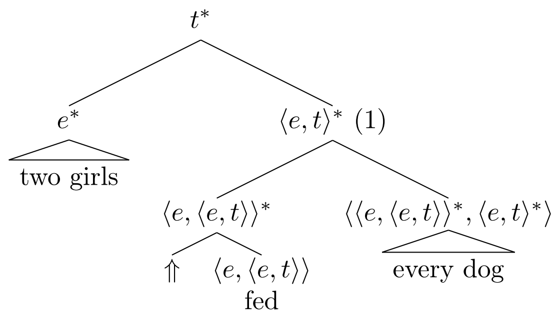
e. $[\![(4)]\!] = \mathcal{C}([\mathbf{brushed}(\mathbf{eric})+\mathbf{fed}(\mathbf{carl})+\mathbf{fed}(\mathbf{dean})],[\mathbf{ada}+\mathbf{bea}])$
$= [\mathbf{brushed}(\mathbf{eric})(\mathbf{ada})+\mathbf{fed}(\mathbf{carl})(\mathbf{bea})+\mathbf{fed}(\mathbf{dean})(\mathbf{bea}), \mathbf{brushed}(\mathbf{eric})(\mathbf{bea})+\mathbf{fed}(\mathbf{carl})(\mathbf{ada})+\mathbf{fed}(\mathbf{dean})(\mathbf{bea}), \mathbf{brushed}(\mathbf{eric})(\mathbf{bea})+\mathbf{fed}(\mathbf{carl})(\mathbf{bea})+\mathbf{fed}(\mathbf{dean})(\mathbf{ada}), \mathbf{brushed}(\mathbf{eric})(\mathbf{ada})+\mathbf{fed}(\mathbf{carl})(\mathbf{ada})+\mathbf{fed}(\mathbf{dean})(\mathbf{bea}), \mathbf{brushed}(\mathbf{eric})(\mathbf{bea})+\mathbf{fed}(\mathbf{carl})(\mathbf{ada})+\mathbf{fed}(\mathbf{dean})(\mathbf{ada}),$
$\mathbf{brushed}(\mathbf{eric})(\mathbf{ada})+\mathbf{fed}(\mathbf{carl})(\mathbf{bea})+\mathbf{fed}(\mathbf{dean})(\mathbf{ada}),\ldots]$

Note: We are not discussing the examples with *made x do P* here as they would require a generalized version of the cumulation rule that works with intensional predicates.

Distributive items

(54) Two girls fed every dog. (cf. Schein 1993, Kratzer 2000)

a.



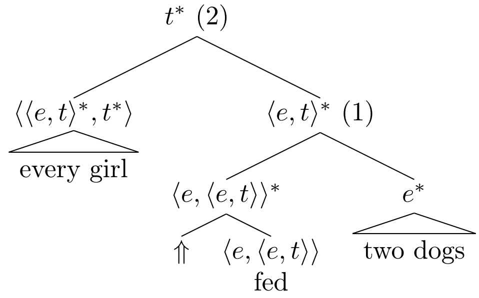
b. $[\![(1)]\!] = (\lambda R^*_{\langle e,\langle e,t\rangle\rangle^*}.pl^*\{+(\{\mathcal{D}(f(x),x) : x\in A_e\wedge\mathbf{dog}(x)\}) : f \text{ is a function from } \{x \mid x\in A_e\wedge\mathbf{dog}(x)\} \text{ to } pl^{*-1}(R^*)\})([\mathbf{fed}])$
$= pl^*\{+(\{\mathcal{D}(\mathbf{fed},x) : x\in A_e\wedge\mathbf{dog}(x)\})\} = pl^*\{+(\{\mathbf{fed}(x) : x\in A_e\wedge\mathbf{dog}(x)\})\} = [\mathbf{fed}(\mathbf{carl})+\mathbf{fed}(\mathbf{dean})+\mathbf{fed}(\mathbf{eric})]$

c. $[\![(2)]\!] = \mathcal{C}([\mathbf{fed}(\mathbf{carl})+\mathbf{fed}(\mathbf{dean})+\mathbf{fed}(\mathbf{eric})],[\mathbf{ada}+\mathbf{bea}])$
$= [\mathbf{fed}(\mathbf{carl})(\mathbf{ada})+\mathbf{fed}(\mathbf{dean})(\mathbf{ada})+\mathbf{fed}(\mathbf{eric})(\mathbf{bea}), \mathbf{fed}(\mathbf{carl})(\mathbf{bea})+\mathbf{fed}(\mathbf{dean})(\mathbf{ada})+\mathbf{fed}(\mathbf{eric})(\mathbf{ada}), \mathbf{fed}(\mathbf{carl})(\mathbf{ada})+\mathbf{fed}(\mathbf{dean})(\mathbf{bea})+\mathbf{fed}(\mathbf{eric})(\mathbf{ada}),$

$\mathbf{fed}(\mathbf{carl})(\mathbf{ada}) + \mathbf{fed}(\mathbf{dean})(\mathbf{bea}) + \mathbf{fed}(\mathbf{eric})(\mathbf{bea}), \ldots]$

(55) Every girl fed two dogs.

a.

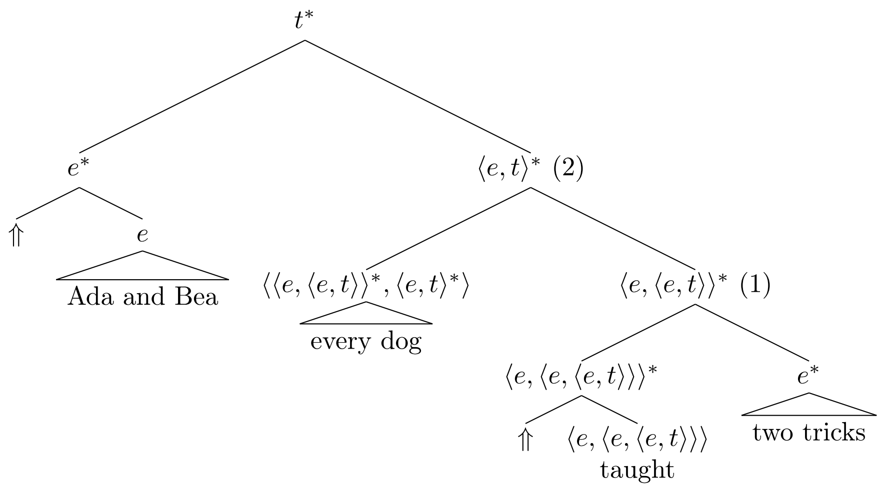


b. $[\![(1)]\!] = [\mathbf{fed}(\mathbf{carl}) + \mathbf{fed}(\mathbf{dean}), \mathbf{fed}(\mathbf{carl}) + \mathbf{fed}(\mathbf{eric}), \mathbf{fed}(\mathbf{dean}) + \mathbf{fed}(\mathbf{eric})]$

c. $[\![(2)]\!] = (\lambda R^*_{\langle e,a\rangle^*}.pl^*\{\textstyle +(\{\mathcal{D}(f(x),x) : x \in A_e \wedge \mathbf{girl}(x)\}) : f$ is a function from $\{x \mid x \in A_e \wedge \mathbf{girl}(x)\}$ to $pl^{*-1}(R^*)\})([\![(1)]\!])$
$= pl^*\{+(\{\mathcal{D}(f(x),x) : x \in \{\mathbf{ada}, \mathbf{bea}\}\}) : f$ is a function from $\{\mathbf{ada}, \mathbf{bea}\}$ to $pl^{*-1}([\![(1)]\!])\}$
$= pl^*\{\mathcal{D}(f(\mathbf{ada}), \mathbf{ada}) + \mathcal{D}(f(\mathbf{bea}), \mathbf{bea}) : f$ is a function from $\{\mathbf{ada}, \mathbf{bea}\}$ to $\{\mathbf{fed}(\mathbf{carl}) + \mathbf{fed}(\mathbf{dean}), \mathbf{fed}(\mathbf{carl}) + \mathbf{fed}(\mathbf{eric}), \mathbf{fed}(\mathbf{dean}) + \mathbf{fed}(\mathbf{eric})\}\}$
$= [\mathbf{fed}(\mathbf{carl})(\mathbf{ada}) + \mathbf{fed}(\mathbf{dean})(\mathbf{ada}) + \mathbf{fed}(\mathbf{carl})(\mathbf{bea}) + \mathbf{fed}(\mathbf{dean})(\mathbf{bea}),$
$\mathbf{fed}(\mathbf{carl})(\mathbf{ada}) + \mathbf{fed}(\mathbf{dean})(\mathbf{ada}) + \mathbf{fed}(\mathbf{carl})(\mathbf{bea}) + \mathbf{fed}(\mathbf{eric})(\mathbf{bea}),$
$\mathbf{fed}(\mathbf{carl})(\mathbf{ada}) + \mathbf{fed}(\mathbf{dean})(\mathbf{ada}) + \mathbf{fed}(\mathbf{dean})(\mathbf{bea}) + \mathbf{fed}(\mathbf{eric})(\mathbf{bea}),$
$\mathbf{fed}(\mathbf{carl})(\mathbf{ada}) + \mathbf{fed}(\mathbf{eric})(\mathbf{ada}) + \mathbf{fed}(\mathbf{dean})(\mathbf{bea}) + \mathbf{fed}(\mathbf{eric})(\mathbf{bea}), \ldots]$

(56) Ada and Bea taught every dog two tricks. (cf. Schein 1993)

a.



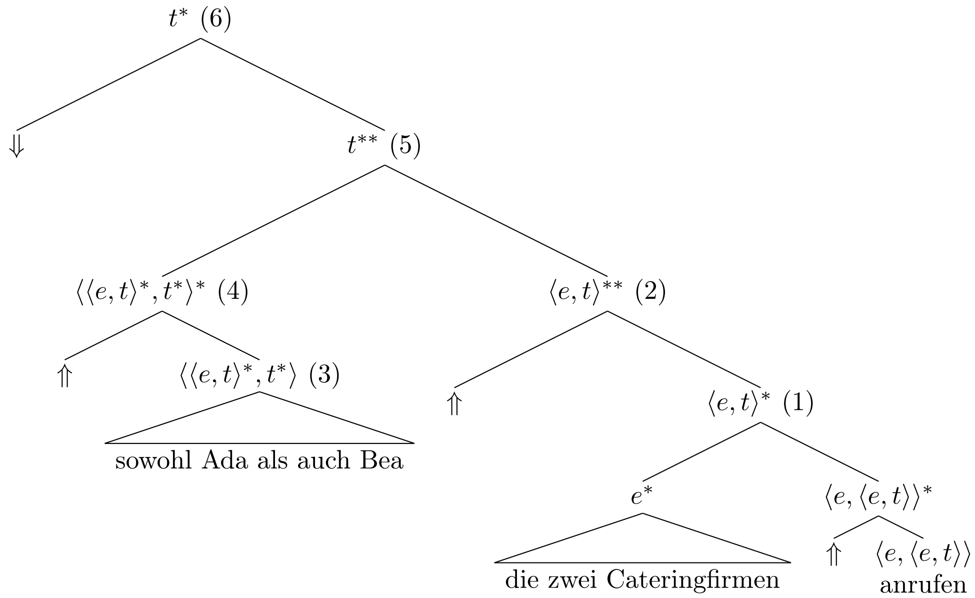
b. $[\![(1)]\!] = \mathcal{C}([\mathbf{taught}], [t_1{+}t_2, t_1{+}t_3, t_2{+}t_3]) = [\mathbf{taught}(t_1){+}\mathbf{taught}(t_2), \mathbf{taught}(t_1){+}\mathbf{taught}(t_3), \mathbf{taught}(t_2) + \mathbf{taught}(t_3)]$

c. $[\![(2)]\!] = (\lambda R^*_{\langle e,a\rangle^*}.pl^*\{+(\{\mathcal{D}(f(x),x) : x \in A_e \wedge \mathbf{dog}(x)\}) : f$ is a function from $\{x \mid x \in A_e \wedge \mathbf{dog}(x)\}$ to $pl^{*-1}(R^*)\})([\![(1)]\!])$
$= pl^*\{+(\{\mathcal{D}(f(x),x) : x \in \{\mathbf{carl}, \mathbf{dean}, \mathbf{eric}\}\}) : f$ is a function from $\{\mathbf{carl}, \mathbf{dean}, \mathbf{eric}\}$ to $\{\mathbf{taught}(t_1) + \mathbf{taught}(t_2), \mathbf{taught}(t_1) + \mathbf{taught}(t_3), \mathbf{taught}(t_2) + \mathbf{taught}(t_3)\}\}$
$= pl^*\{\mathcal{D}(f(\mathbf{carl}), \mathbf{carl}) + \mathcal{D}(f(\mathbf{dean}), \mathbf{dean}) + \mathcal{D}(f(\mathbf{eric}), \mathbf{eric}) : f$ is a function from $\{\mathbf{carl}, \mathbf{dean}, \mathbf{eric}\}$ to
$\{\mathbf{taught}(t_1) + \mathbf{taught}(t_2), \mathbf{taught}(t_1) + \mathbf{taught}(t_3), \mathbf{taught}(t_2) + \mathbf{taught}(t_3)\}\}$
$= [\mathbf{taught}(t_1)(\mathbf{carl}) + \mathbf{taught}(t_2)(\mathbf{carl}) + \mathbf{taught}(t_1)(\mathbf{dean}) + \mathbf{taught}(t_2)(\mathbf{dean}) + \mathbf{taught}(t_1)(\mathbf{eric}) + \mathbf{taught}(t_2)(\mathbf{eric}),$
$\mathbf{taught}(t_1)(\mathbf{carl}) + \mathbf{taught}(t_2)(\mathbf{carl}) + \mathbf{taught}(t_1)(\mathbf{dean}) + \mathbf{taught}(t_2)(\mathbf{dean}) + \mathbf{taught}(t_1)(\mathbf{eric}) + \mathbf{taught}(t_3)(\mathbf{eric}),$
$\mathbf{taught}(t_1)(\mathbf{carl}) + \mathbf{taught}(t_2)(\mathbf{carl}) + \mathbf{taught}(t_1)(\mathbf{dean}) + \mathbf{taught}(t_3)(\mathbf{dean}) +$

$\textbf{taught}(t_2)(\textbf{eric}) + \textbf{taught}(t_3)(\textbf{eric})$,
$\textbf{taught}(t_1)(\textbf{carl}) + \textbf{taught}(t_2)(\textbf{carl}) + \textbf{taught}(t_1)(\textbf{dean}) + \textbf{taught}(t_3)(\textbf{dean}) +$
$\textbf{taught}(t_1)(\textbf{eric}) + \textbf{taught}(t_3)(\textbf{eric}), \ldots]$

(57) Sowohl Ada als auch Bea haben die zwei Cateringfirmen angerufen.
PRT Ada PRT also Bea have.3PL the two catering-companies called
'Both Ada and Bea called the two catering companies.' (German)

a.

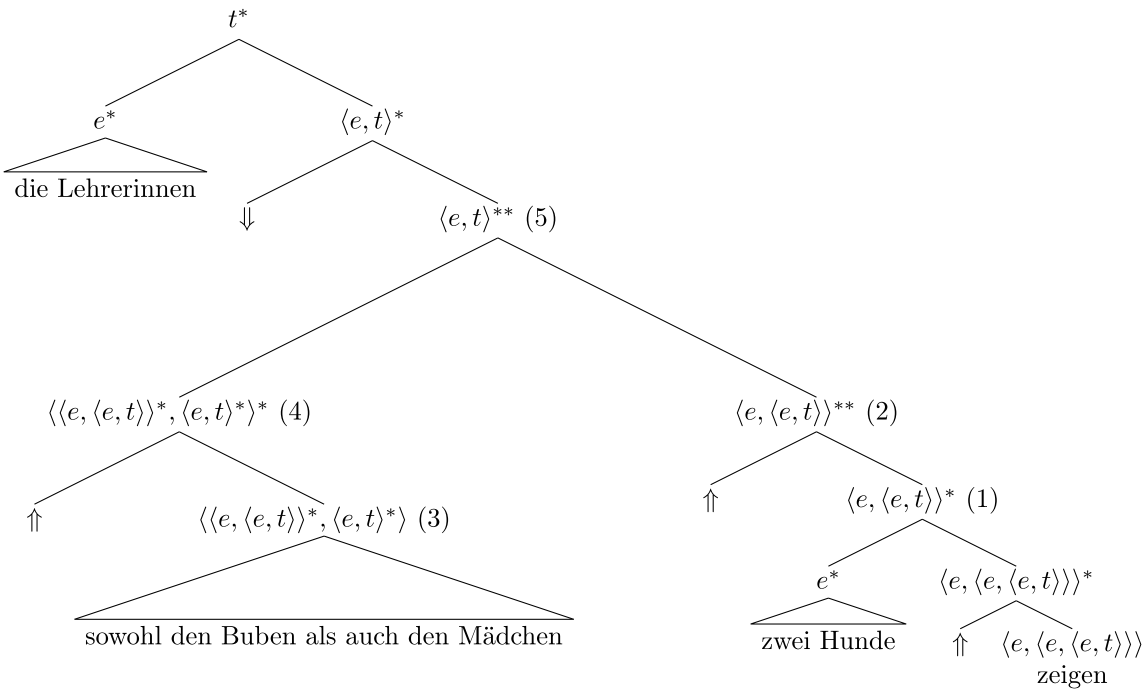


b. $[\![(1)]\!] = [\textbf{call}(f_1) + \textbf{call}(f_2)]$
c. $[\![(2)]\!] = [[\textbf{call}(f_1) + \textbf{call}(f_2)]]$
d. $[\![(3)]\!] = (\lambda R^*_{\langle e,t\rangle^*}\mathcal{C}(R^*, [\textbf{ada}])) + (\lambda R^*_{\langle e,t\rangle^*}\mathcal{C}(R^*, [\textbf{bea}]))$
e. $[\![(4)]\!] = [(\lambda R^*_{\langle e,t\rangle^*}\mathcal{C}(R^*, [\textbf{ada}])) + (\lambda R^*_{\langle e,t\rangle^*}\mathcal{C}(R^*, [\textbf{bea}]))]$
f. $[\![(5)]\!] = \mathcal{C}([(\lambda R^*_{\langle e,t\rangle^*}\mathcal{C}(R^*, [\textbf{ada}])) + (\lambda R^*_{\langle e,t\rangle^*}\mathcal{C}(R^*, [\textbf{bea}]))], [[\textbf{call}(f_1) + \textbf{call}(f_2)]])$
$= [(\lambda R^*_{\langle e,t\rangle^*}\mathcal{C}(R^*, [\textbf{ada}]))([\textbf{call}(f_1) + \textbf{call}(f_2)]) \oplus (\lambda R^*_{\langle e,t\rangle^*}\mathcal{C}(R^*, [\textbf{bea}]))([\textbf{call}(f_1) +$
$\textbf{call}(f_2)])]$
$= [\mathcal{C}([\textbf{call}(f_1) + \textbf{call}(f_2)], [\textbf{ada}]) \oplus \mathcal{C}([\textbf{call}(f_1) + \textbf{call}(f_2)], [\textbf{bea}])]$
$= [[\textbf{call}(f_1)(\textbf{ada}) + \textbf{call}(f_2)(\textbf{ada})] \oplus [\textbf{call}(f_1)(\textbf{bea}) + \textbf{call}(f_2)(\textbf{bea})]]$
$= [[\textbf{call}(f_1)(\textbf{ada}) + \textbf{call}(f_2)(\textbf{ada}) + \textbf{call}(f_1)(\textbf{bea}) + \textbf{call}(f_2)(\textbf{bea})]]$
g. $[\![(6)]\!] = [\textbf{call}(f_1)(\textbf{ada}) + \textbf{call}(f_2)(\textbf{ada}) + \textbf{call}(f_1)(\textbf{bea}) + \textbf{call}(f_2)(\textbf{bea})]$

Note: The additional $\Uparrow$ shifts on the subject and the VP are necessary here since we have a plurality of quantifiers which can't directly combine with the predicate via functional application. After the shift, the subject and the predicate can combine via cumulation. The additional 'level' of plural sets created by these shifts accounts for the distributivity of quantifier conjunctions.

(58) Die Lehrerinnen haben sowohl den Buben als auch den Mädchen zwei Hunde gezeigt.
the teachers have PRT the boys PRT also the girls two dogs showed
'The teachers showed both the boys and the girls two dogs.' (German)

a.



b. $[\![(1)]\!] = \mathcal{C}([\mathbf{show}],[\mathbf{carl}+\mathbf{dean},\mathbf{carl}+\mathbf{eric},\mathbf{dean}+\mathbf{eric}])$
$= [\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{dean}),\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{eric}),\mathbf{show}(\mathbf{dean})+\mathbf{show}(\mathbf{eric})]$

c. $[\![(2)]\!] = [[\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{dean}),\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{eric}),\mathbf{show}(\mathbf{dean})+\mathbf{show}(\mathbf{eric})]]$

d. $[\![(3)]\!] = (\lambda R^*_{\langle e,\langle e,t\rangle\rangle^*}\mathcal{C}(R^*,[\mathbf{che}+\mathbf{ed}])) + (\lambda R^*_{\langle e,\langle e,t\rangle\rangle^*}\mathcal{C}(R^*,[\mathbf{ada}+\mathbf{bea}]))$

e. $[\![(4)]\!] = [(\lambda R^*_{\langle e,\langle e,t\rangle\rangle^*}\mathcal{C}(R^*,[\mathbf{che}+\mathbf{ed}])) + (\lambda R^*_{\langle e,\langle e,t\rangle\rangle^*}\mathcal{C}(R^*,[\mathbf{ada}+\mathbf{bea}]))]$

f. $[\![(5)]\!] = \mathcal{C}([(\lambda R^*_{\langle e,\langle e,t\rangle\rangle^*}\mathcal{C}(R^*,[\mathbf{che}+\mathbf{ed}]))+(\lambda R^*_{\langle e,\langle e,t\rangle\rangle^*}\mathcal{C}(R^*,[\mathbf{ada}+\mathbf{bea}]))],[[\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{dean}),\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{eric}),\mathbf{show}(\mathbf{dean})+\mathbf{show}(\mathbf{eric})]])$
$= [(\lambda R^*_{\langle e,\langle e,t\rangle\rangle^*}\mathcal{C}(R^*,[\mathbf{che}+\mathbf{ed}]))([\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{dean}),\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{eric}),\mathbf{show}(\mathbf{dean})+\mathbf{show}(\mathbf{eric})])$
$\oplus(\lambda R^*_{\langle e,\langle e,t\rangle\rangle^*}\mathcal{C}(R^*,[\mathbf{ada}+\mathbf{bea}]))([\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{dean}),\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{eric}),\mathbf{show}(\mathbf{dean})+\mathbf{show}(\mathbf{eric})])]$
$= [\mathcal{C}([\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{dean}),\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{eric}),\mathbf{show}(\mathbf{dean})+\mathbf{show}(\mathbf{eric})],[\mathbf{che}+\mathbf{ed}])\oplus\mathcal{C}([\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{dean}),\mathbf{show}(\mathbf{carl})+\mathbf{show}(\mathbf{eric}),\mathbf{show}(\mathbf{dean})+\mathbf{show}(\mathbf{eric})],[\mathbf{ada}+\mathbf{bea}])]$
$= [[\mathbf{show}(\mathbf{carl})(\mathbf{che})+\mathbf{show}(\mathbf{dean})(\mathbf{ed}),\mathbf{show}(\mathbf{carl})(\mathbf{ed})+\mathbf{show}(\mathbf{dean})(\mathbf{che}),$
$\mathbf{show}(\mathbf{carl})(\mathbf{che})+\mathbf{show}(\mathbf{eric})(\mathbf{ed}),\mathbf{show}(\mathbf{carl})(\mathbf{ed})+\mathbf{show}(\mathbf{eric})(\mathbf{che}),$
$\mathbf{show}(\mathbf{dean})(\mathbf{che})+\mathbf{show}(\mathbf{eric})(\mathbf{ed}),\mathbf{show}(\mathbf{dean})(\mathbf{ed})+\mathbf{show}(\mathbf{eric})(\mathbf{che}),\ldots]$
$\oplus[\mathbf{show}(\mathbf{carl})(\mathbf{ada})+\mathbf{show}(\mathbf{dean})(\mathbf{bea}),\mathbf{show}(\mathbf{carl})(\mathbf{bea})+\mathbf{show}(\mathbf{dean})(\mathbf{ada}),$
$\mathbf{show}(\mathbf{carl})(\mathbf{ada})+\mathbf{show}(\mathbf{eric})(\mathbf{bea}),\mathbf{show}(\mathbf{carl})(\mathbf{bea})+\mathbf{show}(\mathbf{eric})(\mathbf{ada}),\ldots]]$
$= [[\mathbf{show}(\mathbf{carl})(\mathbf{che})+\mathbf{show}(\mathbf{dean})(\mathbf{ed})+\mathbf{show}(\mathbf{carl})(\mathbf{ada})+\mathbf{show}(\mathbf{dean})(\mathbf{bea}),$
$\mathbf{show}(\mathbf{carl})(\mathbf{che})+\mathbf{show}(\mathbf{dean})(\mathbf{ed})+\mathbf{show}(\mathbf{carl})(\mathbf{ada})+\mathbf{show}(\mathbf{dean})(\mathbf{bea}),\ldots,$
$\mathbf{show}(\mathbf{carl})(\mathbf{ed})+\mathbf{show}(\mathbf{eric})(\mathbf{che})+\mathbf{show}(\mathbf{carl})(\mathbf{ada})+\mathbf{show}(\mathbf{dean})(\mathbf{bea}),$
$\mathbf{show}(\mathbf{carl})(\mathbf{ed})+\mathbf{show}(\mathbf{eric})(\mathbf{che})+\mathbf{show}(\mathbf{carl})(\mathbf{ada})+\mathbf{show}(\mathbf{dean})(\mathbf{bea}),\ldots]]$