Intensional interveners in plural predication and plural projection *

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Abstract

This paper investigates cumulative readings of sentences in which some, but not all of the plural expressions have a _de dicto_ reading, i.e. sentences where the lower plural is interpreted in the scope of an attitude verb like _believe_. I argue that such cases represent a problem for accounts where cumulativity is derived by cumulating relations between the pluralities involved, because the required input relation cannot be formed. I then motivate and propose an alternative analysis where all plural expressions are interpreted _in situ_. I expand the ‘plural projection’ framework put forth by [Schmitt (2018)] and [Haslinger & Schmitt (2018b)], where embedded pluralities ‘project’ to the denotations of higher nodes in the sense that the latter reflect the part-structure of the former and where cumulativity is derived via a compositional rule in a step-by-step fashion. I show that if the denotations of the plurals with the _de dicto_ construal are analyzed as pluralities of individual concepts, which project in the afore-mentioned sense to pluralities of propositions, the data can be explained straightforwardly. This proposal differs from treatments in terms of collective belief that don’t appeal to pluralities of propositions ([Pasternak 2018b,a]), in that it (i) arguably generalizes to a larger number of examples and (ii) links grammatical plurality in the embedded clause to the availability of cumulative readings.

1 Introduction

Pluralities – the denotations of plural expressions such as _Ada and Bea, the two cats, two cats_ etc. – seem to be able to ‘inherit’ properties of their parts (cf. [Scha 1981], [Link 1983], [Krißka 1986], [Schwarzschil 1996] a.o.). This means that intuitively, a property of a plurality can be derived by ‘adding up’ properties of its parts: The sentence in (1-a) is true in the scenario in (1-b) – neither Ada nor Bea fed two cats, but if we ‘add up’ the cats fed by Ada and the cats fed by Bea, we end up with a plurality of two cats. (We could also speak of ‘adding up’ properties of the plural object, of course – e.g. in (1-a) the individual cats are not each fed by both Ada and Bea.)

(1) a. _Ada and Bea fed two cats._
   b. _scenario:_ Ada fed cat Ivo, Bea fed cat Joe.

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These facts – commonly known as ‘cumulativity’ – are usually seen as the result of predicate cumulation (see e.g. Sternefeld (1998)): Cumulating the denotation of feed, for instance, means that we end up with the set of all pairs of individuals that are either in the basic extension or derived by forming pluralities of feeders, while simultaneously forming pluralities of their respective feedees, and vice versa.

### 1.1 The empirical focus of this paper: Cumulativity across intensional interveners

In this paper, I present data from German and English that represent a special case of cumulativity. (Analogous data are mentioned by Schmitt (2018) and discussed in detail by Pasternak (2018a,b). I return to Pasternak’s position in Section 5.2.)

As schematized in (2), the relevant configurations will involve two plural expressions, just as (1) above, but in addition an intensional operator – an attitude predicate – that syntactically intervenes between the two plurals. The relevant reading for my purposes here is one where the lower plural (plural 2) has a de dicto construal relative to the intensional operator and where, at the same time, we seem to find cumulativity w.r.t. plural 1 and plural 2. For the purposes of this paper, I will refer to such cases as ‘DDC-sentences’ (for ‘de-dicto cumulative’).

(2) [plural 1 .... [INT .... [ plural 2 ]]]

More concretely, I will be looking at sentences like (3-b) in scenarios like (3-a). The sentence can be true in the scenario and thus exhibits a reading that is analogous to what we witnessed in (1) above, as it seems to involve an ‘adding up’ of properties: Neither Ada nor Bea individually believe that two monsters were roaming the castle – but their beliefs, intuitively speaking, seem to ‘add up’ to a belief about two monsters. In other words, we seem to witness a cumulative relation between two pluralities – those denoted by Ada and Bea and two monsters, respectively. Crucially, however, Ada’s and Bea’s belief is de dicto – no monsters exist in the scenario.

(3) a. scenario Ada believes in zombies, Bea believes in griffins. Neither exist. Last week, Ada and Bea spent the night at Roy’s castle. Around midnight, Ada thought she heard a zombie walking around in her room. A little later, Bea believed she saw a griffin sitting on her bed. They didn’t talk about it with each other but they each took Roy aside and told him what they believed was going on. Later, Roy tells me: Well, I had invited Ada and Bea to the castle. Bad idea... I know that people find it a little spooky here, but...

b. [Ada and Bea] pl 1 actually believed that [two monsters] pl 2 were roaming the castle!

### 1.2 Point 1: A problem for the ‘predicate-analysis’ of cumulativity

My first analytical point will be to show that DDC-sentences represent a problem for the ‘traditional’ analysis of cumulativity mentioned above, which I will henceforth refer to as the ‘predicate analysis’. This type of analysis takes cumulativity to be result of enriching – i.e. ‘cumulating’ – predicates of individuals (see e.g. Beck & Sauerland (2000)). (I will also show that this problem extends to analyses where we enrich predicates of individuals and events, as in e.g. Kratzer (2003). Informally speaking, the predicate analysis predicts that whenever we find cumulativity w.r.t n-many pluralities, we must identify a constituent denoting an n-ary relation that can act as the input to cumulation. For cumulative readings of sentences like (3), where we seem to have a cumulative relation between Ada and Bea and two monsters, the only such relation is the one in (4). However, two monsters will then be outside the scope of the intensional operator – which means we don’t derive the de dicto reading.

(4) \(\lambda x_e.\lambda y_e.\lambda y.\) believed that \(x\) was roaming the castle.
In principle, this problem could be circumvented by positing a non-standard meaning of the DP *two monsters*, such that it lacks existential import even when it occurs outside of the scope of other operators (see e.g. Condoravdi et al. [2001] for a related proposal). Nevertheless, I argue that such an approach is not tenable: Not only does it force us to assume ambiguity for all kinds of indefinites, it also faces a much more general problem for the predicate analysis *per se*. Namely, Schmitt (2018), Haslinger & Schmitt (2018b) argue that there is a particular configuration – which essentially involves one plural expression embedded in another one and which they call the ‘flattening’-case, for reasons that will become clear below – where the predicate analysis of cumulation cannot derive the correct truth-conditions, simply because it is impossible to derive the adequate input-relation. I will show that such flattening cases can be reproduced for DDC-sentences and that thus the predicate analysis does not generalize to all instances of DDC-sentences, irrespective of our assumptions about the denotation of the indefinite.

1.3 Point 2: An analysis in terms of plural projection

Having identified this problem, I will propose an alternative analysis. It expands the ‘plural projection’ approach by Schmitt (2018), Haslinger & Schmitt (2018b, 2019a), which was originally motivated (amongst other things) by the afore-mentioned ‘flattening’-problem. Broadly speaking, what my expansion of this system will yield for DDC-sentences is that the subject plurality denoted by *Ada and Bea* in (3-b) will never stand in a cumulative relation with a plurality denoted by *two monsters*; rather, Ada and Bea will cumulatively have to believe a plurality of propositions, the ‘parts’ of which are determined by the ‘parts’ of pluralities of individual concepts of monsters. Put informally, Ada and Bea will end up in a cumulative belief-relation with one of the propositional pluralities in (5) (where ‘+’ indicates plurality-formation, to be defined later on).

(5) \{ that a griffin is roaming the castle + that a zombie is roaming the castle, \\
    that a werewolf is roaming the castle + that a zombie is roaming the castle, \ldots \}

The original plural projection mechanism that my analysis is based on involves two core ideas (see Schmitt 2018, Haslinger & Schmitt 2018b, 2019a): First, *all* semantic domains contain pluralities, i.e. objects corresponding to non-singleton sets of ‘atoms’ of the domain. Thus we do not only have pluralities of individuals, but also pluralities of predicates, propositions etc. Second, pluralities ‘project’: If a syntactic node \( \alpha \) denotes a plurality, so will the node immediately dominating it, namely, a plurality of values obtained by combining the atomic parts of \( \llbracket \alpha \rrbracket \) with the meaning of its sister. (In this respect, the system is analogous to uses of Hamblin-style alternative semantics, e.g. Rooth 1985, Kratzer & Shimoyama 2002, Simons 2005) This is schematized in (6) where \( f, g \) stand for functions that have \( a \) and \( b \) in their domains, and ‘+’ again represents plurality formation.

\[
\begin{align*}
  f(a) + f(b) & \quad f(a) + g(a) \\
  \text{\scriptsize{\( a + b \)}} & \quad \text{\scriptsize{\( f + g \)}}
\end{align*}
\]

Crucially, the ‘projection’ rule, i.e. the rule by means of which pluralities combine with the denotations of their sisters, encodes cumulativity. This feature makes the system slightly more complex than the sketch in (6) might suggest, as it will work with *sets* of pluralities: Whenever two pluralities combine with each other, the node immediately dominating them will denote a set of value pluralities that each satisfy the following: Each atomic part of the function is applied to at least one atomic part of the argument, and for each atomic part of the argument, at least one atomic part of the function is applied to it. This is schematized in (7). Hence, if a sentence contains any plurality-denoting expression, the mechanism will yield us a set of pluralities of propositions. The truth-definition maps such a set to true just in case at least one of the elements in this set consists exclusively of true atoms.
Since the system uses sets of pluralities of arbitrary type, the composition mechanism goes beyond the notion of cumulativity found in the preceding literature: While the ‘traditional’ notion of cumulativity only applies to relations being true or false of a sequence of pluralities, plural projection defines a ‘cumulative version’ of almost any semantic operation, regardless of what the type of its result is. We thus do not have to syntactically derive relations that are subsequently cumulated.

For DDC-sentences, this means that the lower plural – two monsters in (3) can be interpreted in situ, i.e. in the scope of the intensional operator. I will essentially propose that two monsters denotes a set of pluralities of individual concepts, roughly as in (8). Each plurality in the set consists of two individual concepts f, g, s.th. in every world where f is defined, it yields us a monster, and likewise for g. Building on a proposal by Condoravdi et al. (2001), I assume that we can only ‘pluralize’ or ‘count’ distinct individual concepts, encoding this idea by requiring that f and g cannot ‘overlap’ – there is no world where both are defined and where they yield us the same individual.

(8) \{f + g | f, g are individual concepts & monster(f) & monster(g) & \forall w \in \text{dom}(f) \cap \text{dom}(g)(f(w) \neq g(w))\}

The plural set sketched in (8) then ‘projects’ in the sense sketched above, yielding a plural set of propositions roughly equivalent to (5) as the denotation of the embedded clause. Further application of the projection rule will require that in order for the sentence to be true, Ada and Bea must cumulatively believe at least one of the pluralities in the set – which are the correct truth-conditions.

1.4 Point 3: Consequences of the proposal

While my initial motivation for an analysis in terms of plural projection is technical – alternative analyses like the predicate analysis don’t consistently derive the correct truth-conditions – I will also provide independent motivation for such a proposal. This motivation will concern two points: First, the system I propose assumes an ‘arbitrary’ notion of propositional pluralities in the sense that any two or more propositions can be ‘summed up’ to form such a plurality. Furthermore, the system predicts that in principle any such propositional plurality can be cumulatively believed by a plurality of individuals. In other words, there are no prima facie constraints in terms of logical compatibility etc. on the formation of propositional pluralities and on cumulative belief, and I will argue that this assumption is empirically warranted. This also means that the data I will discuss are not compatible with a notion of collective belief that draws on notions like compatibility. The second point is that my proposal ties the availability of plural propositions – and thus the availability of cumulative belief – to grammatical means inducing plurality formation: In the system I employ, plural propositions are always the result of ‘projection’ of a plural expression embedded in the structure (such as two monsters in (3-b) above) or ‘direct’ sum-formation via conjunction. Thus, if no such expression is present in the structure, we should not find cumulative belief. I will argue that this prediction is correct: I will show contra (Pasternak 2018a) that ‘cumulative inferences’ are not based on entailment and similar logical relations, but correlate with the presence of grammatical devices of plurality.

As a final (brief) point, I will raise a problem of my analysis which I believe to be of a very general nature (see Haslinger & Schmitt 2019b). Not only will it occur in any approach to DDC-sentences that views them as involving cumulativity, but it will also surface in a number of conceivable analyses of arguably related phenomena like Hob-Nob - sentences (Geach 1967 a.o.). It concerns the notion of distinctness in individual concepts: While many analyses employing individual concepts view them as total functions (see e.g. Aloni 2001, Condoravdi et al. 2001), I believe we need to use partial
functions (for instance, not all possible worlds will have monsters in them). This has the consequence that the non-overlap condition in (8) becomes too weak, but it is unclear how it could be strengthened.

1.5 Structure of the paper

The paper is structured as follows: In Section 2, after having provided the necessary background, I present the core data involving DDC-sentences and lay out the basic problem for the predicate analysis. Section 3 shows that this problem cannot be circumvented by assuming a different denotation for the indefinite, because of the independent problem of flattening cases. Section 4 introduces the plural projection mechanism and shows how it can be supplemented so as to derive our data. In Section 5 I discuss the consequences of such a proposal and compare it to potential alternatives. Section 6 concludes the paper.

2 The problem

After a brief sketch of what could be considered the standard view of plural denotations and plural predication, which I here call the ‘predicate analysis’, I present the initial problem that arises for this system once we consider DDC-sentences.

2.1 Background: The predicate analysis

For the moment, I draw on standard assumptions in plural semantics, following Link 1983 a.o.: The semantic domain \( D_e \) does not only contain atomic individuals like \( Ada \) or \( Bea \) but also plural individuals like the plural individual consisting of \( Ada \) and \( Bea \). Plural individuals are formed by a sum operation \( \oplus \) defined on \( D_e \): There is a non-empty set \( A \subseteq D_e \) of atomic individuals, a binary operation \( \oplus \) on \( D_e \) and a function \( f : (\mathcal{P}(A) \setminus \{\emptyset\}) \to D_e \) such that: 1) \( f(\{a\}) = a \) for any \( a \in A \) and 2) \( f \) is an isomorphism between \( (\mathcal{P}(A) \setminus \{\emptyset\}, \cup) \) and \( (D_e, \oplus) \). I furthermore use the notions in (9).

(9) For any \( a, b \in D_e, S \subseteq D_e \):
   a. \( a \leq b \iff a \oplus b = b \) ("\( a \) is a part of \( b \")
   b. \( a \preceq b \iff a \leq b \land a \in A \) ("\( a \) is an atomic part of \( b \")
   c. \( \bigoplus S = f(\bigcup \{f^{-1}(x) \mid x \in S\}) \) (the sum of all individuals in \( S \))

Conjunctions of individual denoting phrases denote the sum of their conjuncts’s denotations, i.e. \( [Ada \text{ and } Bea] = Ada \oplus Bea \), and definite plurals like the two cats the sum of all the individuals in the NP extension.\(^1\)

Cumulative inferences are derived via cumulation operations on predicate denotations (see Link (1983), Kripka (1986) a.o.). If (10-a) and (10-b) are true, so is (10-c) (and also (10-d), if \( Ada \) and \( Bea \) are the only salient girls and Ivo and Joe the only salient cats.) This is taken to suggest that if the feeding-relation ‘primitively’ holds of the pairs \( \langle A, I \rangle \) and \( \langle B, J \rangle \), then its cumulated version holds of the pair \( \langle A \oplus B, I \oplus J \rangle \).

(10) a. \( Ada \text{ fed Ivo.} \)
   b. \( Bea \text{ fed Joe.} \)
   c. \( Ada \text{ and } Bea \text{ fed Ivo and Joe.} \)
   d. \( The \text{ two girls fed the two cats.} \)

More precisely, for each arity of a predicate, we can schematically define a corresponding cumulation operator (cf. Sternefeld 1998). The one relevant for (10) is the operation \( ** \), (11), which targets the

\(^1\)For the moment, I stick to a purely extensional system.
extensions of transitive predicates (cf. Krifka (1986)). When combined with such an extension, it yields us the cumulated version thereof, as illustrated for feed in (12) below: Once the basic extension of feed in (12-a) – here represented as a set of ordered pairs of feeders and feedees – is combined with **, as in (12-b), we obtain the set of all pairs of individuals that are either in the basic extension or derived by forming pluralities of feeders, while simultaneously forming pluralities of their respective feedees, and vice versa, (12-b). 

(11) For any \( P \in D_{e, \epsilon(e, t)} \), **\( P \) is the smallest function \( f \) s.t. for all \( x, y \in D_e \), if \( P(x, y) = 1 \), then \( f(x, y) = 1 \) and for all \( S, S' \subseteq D_e \), s.t. for every \( x' \in S \) there is a \( y' \in S' \) and \( f(x', y') = 1 \) and for every \( y' \in S' \) there is an \( x' \in S \) and \( f(x', y') = 1 \).

(12) a. \( \llbracket \text{feed} \rrbracket = \{\langle A, I \rangle, \langle B, J \rangle\} \)

b. **\( \llbracket \text{feed} \rrbracket = \{\langle A, I \rangle, \langle B, J \rangle, \langle A \oplus B, I \oplus J \rangle\} \)

Hence, if both sentences in (10-a) and (10-b) are true, we predict (10-c) to be true: If we symbolize by ‘**‘, following much of the literature. (13) thus has the LF in (15-a) and we obtain the correct truth-conditions paraphrased in (15-b) (where I naively assume that want to marry primitively holds only of pairs of atomic individuals).

(13) The two women wanted to marry the two men.  

(14) \( \lambda x e . \lambda y e . y \) wanted to marry \( x \)

Beck & Sauerland (2000) therefore propose that predicates like (14) can be formed by covert (tucking-in) movement and are subsequently affixed with the operator introducing ** (which I here simply also symbolize by ‘**‘, following much of the literature). (13) thus has the LF in (15-a) and we obtain the correct truth-conditions paraphrased in (15-b) (where I naively assume that want to marry primitively holds only of pairs of atomic individuals).

(15) a. \( \llbracket \text{the two women} \rrbracket \llbracket \text{the two men} \rrbracket \llbracket \text{[2 [1 [t1 wanted to marry t2]]]]} \rrbracket \)

b. \( \forall x \leq_{AT} \llbracket \text{the two women} \rrbracket (\exists y \leq_{AT} \llbracket \text{the two men} \rrbracket (x \text{ wanted to marry } y)) \land \forall y \leq_{AT} \llbracket \text{the two men} \rrbracket (\exists x \leq_{AT} \llbracket \text{the two women} \rrbracket (x \text{ wanted to marry } y)) \)

This approach can be expanded to our original example in (11) above – which includes a plural indefinite (two cats) – by simply assuming that such indefinites involve existential quantification over variables ranging over plural individuals (see Link 1987), e.g. (16).

(16) \( \llbracket \text{two cats} \rrbracket = \lambda P(e). \exists x \leq \llbracket \text{cat} \rrbracket (|x| = 2 \land P(x)) \).

In summary, the standard view of pluralities and plural predication assumes that the intuitive ‘adding up’ of properties that we witnessed in (1) and (10-c) above results from ‘enriching’, i.e. cumulating the adequate predicate extension. Thus, if we find cumulativity w.r.t. \( n \)-many pluralities, we must find an adequate \( n \)-ary predicate that can act as the input to the corresponding cumulation operator. This predicate can correspond to a lexical item or a surface constituent, but may also derived syntactically, as Beck & Sauerland’s (2000) observations suggest.

\(^2\)Cf. Sternefeld (1998) for analogous operations for \( n \)-transitive predicates where \( n > 2 \).
2.2 Cumulative readings across intensional interveners

Just in (1) and (10-c) above, DDC-sentences involve an ‘adding up’ of properties, but their truth-conditions cannot straightforwardly be captured by the mechanism just outlined.

I will base my discussion on German data, but supplement it with English examples in the ‘basic’ cases that I will use for the theoretical discussion below.

Consider (17), which is analogous to (3) in the introduction above: For many speakers, the German sentence in (17-b) is an adequate description of the scenario in (17-a). Similar judgments are found for the parallel English sentence in (17-c).

(17) a. scenario Ada believes in zombies, Bea in griffins. Neither exist. Last week, Ada and Bea spent the night at Roy’s castle. Around midnight, Ada thought she heard a zombie walking around in her room. A little later, Bea believed she saw a griffin sitting on her bed. They didn’t talk about it with each other, but they each took Roy aside and told him what they believed. Later, Roy tells me: Well, I had invited Ada and Bea to the castle. Bad idea... I know it can be a little spooky here, but ....

b. diese Idioten haben echt geglaubt, dass da zwei Monster im Schloss unterwegs waren!

c. these idiots actually believed that there two monsters were roaming the castle!

At first sight, these examples seem parallel to our initial example in (1). The sentences are true even though neither Ada nor Bea believed individually that two monsters were roaming the castle, just as the sentence in (1) above is true although neither Ada nor Bea individually fed two cats. But there is a crucial difference: The embedded subject two monsters has a de dicto reading – no monsters exist in the world of evaluation. This gives rise to an analytical problem: What we witness looks like cumulativity wrt. two pluralities – the denotations of Ada and Bea and two monsters. But which transitive predicate forms the input to ** – which relation do we have to cumulate? The obvious answer is, of course, (18-a), which could be derived via the LF in (18-b). But if two monsters (in analogy to (16) above) denotes an existential quantifier over pluralities of individuals, this LF can only be the source of the de re-reading paraphrased in (18-c), and not of the required de dicto-reading: Since in order to obtain cumulativity the two pluralities denoted by Ada and Bea and two monsters must act as the arguments of the relation, neither can occur within the relation – which means that two monsters will not be in the scope of the intensional operator denoted by believe.

(18) a. $\lambda x, y [\lambda x, y \text{believes that } x \text{ were roaming the castle}]$

b. $[[A \text{ and } B][[\text{two monsters}][**[2[1[t_1 \text{ believes that } t_2 \text{ were roaming the castle ]]}]]]]$

c. There is a plurality $x$ of two monsters s.th. both Ada and Bea believed of some part $y$ of

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3Both German and English exhibit variation, which could be correlated with dialectal differences – while most speakers of Southern variants (including myself) judge the sentences fine, some speakers of Northern variants reject them. However, since there are exceptions on both sides, dialect does not seem to be the only differentiating factor. From now on, the German judgements I report will be based on the group that considers (17-b) true in the scenario in (19-a).

4von Stechow (1980) mentions similar examples, but only addresses the de re reading. Tiskin (2014) also addresses similar examples but concentrates on the partial de re-reading, i.e. a reading where there are two objects that are then described the property of being a monster. Such examples raise several issues, but not the ones I focus on here. The reading I am after here is discussed briefly by Schmitt (2018) and a greater length by Pasternak (2018a,b).

5There is an additional issue with this LF. Just as in (13) we have a case of non-lexical cumulativity. Beck & Sauerland (2000) argue that since the input relation in these cases is derived by covert movement, instances of non-lexical cumulativity must be – and are – constrained by syntactic restrictions of such movement, e.g. clause-boundedness. Schmitt (2018) contests this generalization. The movement (18-b) would clearly violate clause-boundedness. However, this issue is tangential to my main points here.
x that y was roaming the castle and each atomic part y of x, either Ada or Bea believed that y was roaming the castle.

Before we further investigate the problem, let me address three questions that might arise on the basis of (17). First, **DDC-sentences cannot be reduced to a hidden de re reading** involving e.g. real-world sounds or visual phenomena that we could identify with the source of Ada’s and Bea’s beliefs (cf. Zimmermann 1999 for related discussion). In order for (17-b) (17-c) to be true in the scenario, Ada and Bea can just imagine they heard something. Furthermore, examples like (19) show that such a hidden- de re analysis fails even in those cases where we do have real-world objects that could act as the source of the belief: (19-b) is true in (19-a) even though the number of ‘belief-objects’ (four) does not match the number of real-world objects connected to these belief-objects (two).

(19) a. **scenario** Ada believes in zombies, Bea in griffins. Neither exist. Last week, Ada and Bea spent the night at Roy’s castle. Around midnight, Ada heard a sound in her bedroom and thought it was caused by two zombies fighting. A little later, Bea heard a sound in her bedroom and was certain it was caused by two griffins chasing each other. They didn’t talk about it with each other, but they each took Roy aside and told him what they believed was going on. Later, Roy tells me: Well, I had invited Ada and Bea to the castle. Bad idea... it can be a little spooky here, but guess what....

6See e.g. Beck 2000 for the interaction of such modifiers and plurality.

b. **diese Idioten haben echt geglaubt, dass da vier Monster unterwegs waren!**

‘These idiots really believed that four monsters were roaming the castle!’

Second, **DDC-sentences can be reproduced with plural modifiers or determiners other than numerals.** We find similar facts with conjunctions (of bare plurals), (20-a), with plurals including modifiers like verschiedene/unterschiedliche (‘various’, ‘different’), (20-b), and with the modifier insgesamt(‘in total’), (20-c). All these sentences are true in the scenario in (19-a) above.

(20) a. **diese Idioten haben echt geglaubt, dass da Greife und Zombies unterwegs waren!**

‘These idiots have really believed that there are monsters and zombies roaming the castle!’

b. **diese Idioten haben echt geglaubt, dass da verschiedene/unterschiedliche Monster unterwegs waren!**

‘These idiots really believed that there were various/different monsters roaming the castle!’

c. **diese Idioten haben echt geglaubt, dass da insgesamt vier Monster unterwegs waren!**

‘These idiots really believed that there were four monsters in total roaming the castle!’

Finally, **DDC-sentences are not tied to idiosyncratic properties of the verb believe** (but I will use believe in my discussion below for purposes of simplicity). We find similar effects for e.g. erwarten (‘expect’), befürchten (‘fear’) and hoffen (‘hope’), as witnessed by the fact that the two variants of (21-b) are true in the scenario in (21-a) and (22-b) is true in the scenario in (22-a).

(21) a. **scenario** Ada believes in zombies, Bea in griffins. Neither exist. Ada and Bea will spend the night at Roy’s castle. Ada is worried that the castle will be attacked by the leader of the griffins. Bea is afraid that a large zombie is roaming the area and will...
attack at the castle. They didn’t talk to each other about it, but each talked to Roy. He
tells me: Well, you won’t believe it....

b. **diese Idioten erwarten/befürchten tatsächlich, dass zwei Monster heute nacht das
castle attack will
tatsächlich, dass zweiMonster angreifen werden!**

‘These idiots actually expect/fear that two monstes will be attacking the castle tonight.’

(22) a. **scenario Ada believes in zombies, Bea in griffins. Neither exist. Ada and Bea will**

spend the night at Roy’s castle. Ada is eager to meet the leader of the griffins who she
believes to live at the castle. Bea is looking forward to encountering the zombie king,
who he believes to roam the area. They didn’t talk to each other about it, but each
talked to Roy. Roy tells me: **Well, you won’t believe it....**

b. **diese Idioten hoffen tatsächlich, dass hier zwei Monster auftauchen werden.**

‘These idiots really hope that two monstes will show up here.’

2.3 Interim summary

In the preceding paragraphs I introduced DDC-sentences – sentences like (23) with a cumulative
reading, where the lower plural is interpreted de dicto. We saw why the predicate analysis, out-
lined at the very beginning of this section, runs into a problem: In this type of analysis, whenever
we find cumulativity w.r.t. *n*-many pluralities, we must identify an adequate *n*-ary relation that can
serve as the input for the corresponding cumulation operator. For sentences like (23) under their
DDC-reading, we must thus form a relation like (24), which, however, will give the indefinite (two
monsters) wide scope over the intensional operator. Accordingly, if we assume a ‘traditional’ semantics for the indefinite – one where we have existential quantification over pluralities of individuals as in (25) – we get existential import, i.e. we only derive the de re reading.

(23) **Ada and Bea believed that two monsters were roaming the castle!**

(24) **∀x.∀y. y believes that x were roaming the castle**

(25) **[two monsters] = AP_{(e)iera}x ≤ ∪ [monster](|x| = 2&P(x)).**

3 Can we maintain the predicate analysis after all?

In the following, I briefly discuss the possibility of giving an alternative denotation to the indefinite,
while maintaining the basic assumptions of the predicate analysis. We will find this road to be
blocked, however, by the existence of examples where we cannot form the adequate input relation
for cumulation, irrespective of the denotation of the indefinite.

3.1 Changing the denotation of the indefinite

DDC-sentences are not the only type of construction that gives rise to the problem that an indefinite
which we are apparently forced to put outside of the scope of the intensional (and any other) operator
doesn’t give rise to an existential entailment.

One well-known case are so-called ‘Hob-Nob sentences’ as in (26), first discussed by Geach [1967].
The sentence in (26-b) is true in the scenario in (26-a), which raises a problem concerning the ante-
cedent for the pronoun she in the second conjunct: Since Hob’s belief and Nob’s belief appear to
be about the same ‘object’ (in a pre-theoretical sense), it would seem we require an LF, where the
indefinite takes wide scope. But then the indefinite outscopes the intensional operator and thus the
sentence should only have a chance of being true if there is a witch in the world of evaluation - which is not the case in the scenario provided.

(26) a. scenario The Gotham City newspaper have reported that a witch, referred to as ‘Samantha’, has been on a rampage. According to the article, she has been blighting farm animals. In reality, there is no such person: the animals all died of natural causes. The reporters simply assumed that a witch was responsible for the mishaps, and dubbed her ‘Samantha’. Hob and Nob both read the Gotham Star and believe the stories about the witch. Hob thinks Samantha must have blighted Bob’s mare, which took ill yesterday. Nob thinks Samantha killed his friend Cob’s sow.

b. Hob thinks a witch blighted Bob’s mare, and Nob thinks she killed Cob’s sow.

Another potentially related case is found in sentences with prevent-type predicates and indefinite objects like (27). According to Condoravdi et al. (2001), (27) does not only have a ‘general’ reading (any combination of potential strikes that add up to three was prevented), but also a ‘specific’ reading - there are three potential strikes that were prevented by negotiations. Again, this seems to suggest that the indefinite outscopes the intensional predicate, but this in turn would give us existential import.

(27) Negotiation prevented three strikes.

Some (but not all) solutions that have been proposed for these problems assume that we should give the indefinite scope over the intensional predicate - but manipulate its meaning in a way that won’t entail existence. There are different implementations of this idea, ranging from existential quantification over so-called ‘objects of thought’ (i.e. objects that don’t have to exist, see e.g. Edelberg 1986, Cumming 2014 for critical discussion in the context of Hob-Nob sentences) to existential quantification over individual concepts (a variant of which is proposed by Condoravdi et al. (2001) for the cases in (27)). At this point, the implementation does not matter (but I will come back to Condoravdi et al.’s 2001 analysis in section 4), as it should be more or less clear how such a type of approach could be used for DDC-sentences: If we assume such an alternative denotation for the indefinite, we can in principle stick to the predicate analysis – that is, maintain the LF from (18-b) above, repeated in (28-a) and derive a binary relation like (28-b) (the particulars, like the type of the arguments, would hinge on the actual proposal). Since the indefinite does not give us existential import, we don’t run into the ‘only de-re reading’ problem from above anymore.

(28) a. [[A and B][two monsters][** [2 [1 [t1 believes that t2 were roaming the castle ]]]]]

b. **λx.λy. y believes that x were roaming the castle

Different technical questions will arise, of course, depending on the specific implementation adapted, but they are not my concern here. Furthermore, such a type of approach must assume ambiguity for indefinites – because the de re / de dicto distinction is no longer correlated with the relative scope of the indefinite and the intensional operator. It is unclear whether this is a welcome result, as it is unclear to whether any language ‘spells out’ this particular ambiguity formally. In any case, this is not my main motivation for discarding this type of proposal.

3.2 The flattening problem and DDC-sentences

The crucial problem of any variant of the predicate analysis of DDC-sentence is in fact unrelated to the denotation of the indefinite, but is connected to the relation that forms the input to cumulation:

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It is unclear whether formal distinction between specificity and non-specificity that some language make consistently reflects the de re / de dicto distinction in any such language (see von Heusinger 2011 for an overview).
Namely, not all relations required for cumulation can be derived syntactically (see Schmitt (2018), Haslinger & Schmitt (2018b, 2019a)).

For my description of the basic problem, which I will then show to extend to DDC-sentences, I make the assumption that VP-predicate conjunctions like *feed Ivo and brush Harry* in (29-b) denote pluralities of predicates (see Gawron & Kehler 2004, Schmitt 2013, 2018). (But note that the problem is independent of this hypothesis.) It is motivated by the observation that (29-b) can be true in the scenario (29-a) (see Schmitt 2013). This means that the predicate conjunction *P and Q* has a cumulative reading relative to *the two girls*, i.e. that the relation \[ \lambda P. \lambda x. x \text{ made Gene do } P \] applies cumulatively to the two girls and a predicate plurality ‘made up’ of *P* and *Q*. (See section 4.1 for more discussion.)


b. The two girls made Gene [[feed Ivo]\(_P\) and [brush Harry]\(_Q\)] when all he wanted to do was take care of his hamster.

Recall that according to the predicate analysis, whenever we observe a cumulative reading w.r.t. *n*-many pluralities, we must identify an *n*-ary relation between these pluralities in the syntax that the respective cumulation operator (e.g. \(*\)**) can attach to. If the cumulative relation does not correspond to a surface constituent, it must be derived syntactically. In the predicate analysis, cumulativity is thus always tied to relation-denoting LF constituents. In light of this, consider the sentence in (30-b), which is true in the scenario in (30-a).


b. The two girls made Gene [[feed the two cats]\(_P\) and [brush Harry]\(_Q\)] when all he wanted to do was take care of his hamster.  

Schmitt 2018

In (30), the predicate conjunction *P and Q* again has a cumulative reading relative to *the two girls* (as it is not the case that each girl made Gene brush Harry etc.). Thus, again the relation \[ \lambda P. \lambda x. x \text{ made Gene do } P \] applies cumulatively to the two girls and the two predicates *P* and *Q*. As there is no surface constituent denoting this relation, the obvious solution within the predicate analysis would be to assume an LF like (31) (This requires an extension of ** to higher types, cf. Schmitt 2013).

(31) \[ [[[\text{the two girls}][\text{feed the two cats and brush Harry}]][** [2 [1 [t\(_1\) made G t\(_2\)]]]]]] \]

Yet, in scenario (30), *the two cats* also has a cumulative reading relative to *the two girls*, since neither of the girls made Gene feed both of the cats. There is no obvious way of interpreting *feed the two cats* in (31) that accounts for this fact. Even if we interpret it as being true of plural individuals that cumulatively feed both of the dogs, the problem persists, since the semantic argument of *feed the two cats* in (30-b) is the singular individual Gene rather than any plurality of girls.

In Haslinger & Schmitt (2018b), we call this phenomenon the flattening effect (see section 4 for the reasons). The problem with this particular configuration is that three plural expressions participate in cumulation – *the two girls*, *the two dogs* and the predicate conjunction – but there is no way of deriving a relation that might form the input for **, since one plural expression syntactically contains another. If we move only two plural expressions in the syntax, as in (31), the resulting LF won’t give us the right semantics. But if we move *the two cats* out of the predicate conjunction, it will be unclear how to interpret the resulting structure since the predicate conjunction would contain an unbound trace. Therefore, the predicate analysis cannot account for these cases.

Crucially for my purposes here, the flattening effect also extends to DDC-sentences, as shown in...
The German sentence in (32-b) is structurally parallel to the sentence in (30-b) in the sense that the plural expression *two monsters* is embedded in another plural expression, the VP-conjunction *P and Q*. However, it also contains an attitude verb as the matrix predicate. The sentence is true in the scenario in (32-a). This means that we find the same effect as in (30) – the flattening effect – which means that we cannot syntactically derive the input relation for cumulation.

Furthermore *two monsters* in (32-b) has a *de dicto* reading in the scenario. Above, I considered the possibility of giving an alternative denotation to indefinites to be able to maintain the predicate analysis in ‘standard’ cases like (33-a) – i.e. an analysis where we form the binary relation in (33-b) which then forms the input to cumulation. What (32) shows is that we cannot use the predicate analysis consistently to derive DDC-sentences – irrespective of the denotation we assume for the indefinite.

As a final point, let me address a question whether there aren’t alternative versions of the predicate analysis that can account for the flattening effect. So far, I only ruled out the predicate analysis, which is based on predicates of individuals. A system where we cumulate relations between individuals and events (i.e. thematic relations), however, as proposed by e.g. [Kratzer 2003, 2007] (and, in a way, also by [Schein 1993]) could derive the facts in (30). Without going into details, such analyses could give us a plural event of making Gene do different things (feed Ivo, feed Joe, brush Harry), of which the girls could be the plural agent. Yet, as we point out in ([Haslinger & Schmitt 2018b, 2019a]), such an event-based story also does not offer an general account of cumulativity: It is unclear how such accounts deal with the fact that cumulative relations can ‘reach inside’ arguments that denote neither individuals nor events, like complements of attitude verbs – which, of course, includes ‘simple’ DDC-sentences like (33-a) or also complex cases like (32)\(^9\). In some such event-based analyses, attitude verbs combine with eventualities rather than propositions ([Kratzer 2006, Moulton 2013, Elliott 2017], but as we note in ([Haslinger & Schmitt 2018b]), such accounts still involve an operator within the complement clause that maps propositions to eventualities. As a purely event-based system for cumulativity cannot ‘reach’ below this operator, the problem remains.

### 3.3 Interim summary

The preceding paragraphs raised the question whether we can save the predicate analysis of DDC-sentences by attributing an alternative denotation to the indefinite – one that would allow it to scope

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9In fact (30-b) also contains a modal expression that I glossed over above.

9Such analyses also do not derive the *de re* readings of such sentences, which are discussed in ([Schmit 2013, 2018]).
above the intensional operator without existential import. I left the details of such a treatment open, because of a more general problem with the predicate analysis: It has no way to deal with flattening cases – cumulative readings of sentences with a particular syntactic configuration where one plurality-denoting expression is contained in another one. In such cases, the input relation for cumulation cannot be syntactically derived. Accordingly, flattening cases show that the predicate analysis cannot be a generalized treatment of cumulativity. Since flattening cases can be reproduced for DDC-sentences, the predicate analysis furthermore is not a potential general analysis for DDC-sentences – irrespective of our assumptions about the denotation of the indefinite. My final point was to argue that alternative versions of the predicate analysis, which cumulate relations between individuals and events, do not represent viable candidates, either, as they are generally unsuited to account for cumulative dependencies that ‘reach inside’ the complements of attitude predicates.

4 Analysis: Plural projection and pluralities of individual concepts

I will now propose a different account of DDC-sentences, which circumvents both of the problems alternative treatments are facing: First, it will let us interpret two monsters in the scope of the intensional operator, which means we don’t have to assign an alternative denotation to two monsters (and thus we will avoid having to assume ambiguity of indefinites). Second, it will let us deal with the flattening problem, i.e. it has no problem of accounting for the truth-conditions of sentences like (30) and (32) above. In order to achieve this, I will change my perspective on DDC-sentences as in (34): So far, I assumed that they involve a ‘direct’ cumulative relation between the denotations of the two plural expressions, Ada and Bea and two monsters. The direction I am taking now is that they will involve what boils down to a cumulative belief-relation between the matrix subject and a propositional plurality. Informally speaking, the complement sentence in (34-a) will denote a set of pluralities of propositions like the one in (34-b), and in order for (34) to be true, Ada and Bea will have to cumulatively believe at least one of its elements.

(34) Ada and Bea believed that two monsters were roaming the castle!

(35) a. that two monsters were roaming the castle.
   b. { that a griffin is roaming the castle + that a zombie is roaming the castle, that a werewolf is roaming the castle + that a zombie is roaming the castle, . . . }

In order to get to the set in (35), I will build on the ‘plural projection’ mechanism proposed by Schmitt (2018) and Haslinger & Schmitt (2018b, 2019a). This mechanism is motivated, among other things, by flattening cases like (30) and rests on two assumptions. The first assumption, motivated by the cross-categorial behavior of conjunctions (see section 4.1), is that all semantic domains contain pluralities: We do not only have pluralities of ‘primitives’ (like individuals), but also pluralities of predicate denotations, pluralities of propositions etc. More precisely, the system assumes a ‘flat’ plural semantics for each domain, where pluralities stand in a one-to-one correspondence with non-empty sets of the atomic elements of the domain, but are not identified with such sets. The second assumption is that pluralities ‘project’ in the sense that, if a node α dominates an expression denoting a plurality, the denotation of α will reflect the part-structure of that plurality. The formal mechanism implementing this idea is analogous to Alternative Semantics (cf. e.g. Hamblin 1973, Rooth 1985). Crucially, however, cumulativity is built into the projection rule. Here is a brief preview of how the system works. Whenever we combine a function and an argument and at least one of the two constituents denotes a plurality, we end up with a plurality of values. When an atomic (i.e. non-plural) function combines with a plural argument, the result will be the sum of all values obtained by applying the function to atomic parts of the argument. Likewise, if a function plurality combines with an atomic argument, we get the sum of all values obtained by applying
atomic parts of the function to the argument. This is sketched in (36), where \( f, g \) are functions that have \( a, b \) in their domain, and \( + \) indicates plurality formation:

\[
\begin{align*}
(36) \quad f(a) + f(b) & \quad f(a) + g(a) \\
\quad f + a + b & \quad f + g + a
\end{align*}
\]

However, this does not yet cover cases where we combine functor pluralities and argument pluralities. In this case, a single plurality of values would be insufficient: Cumulativity essentially means that there are multiple ways of combining the function parts with the argument parts. Accordingly, plural expressions are not simply assigned pluralities, but sets of pluralities – plural sets – as their denotations. The result of combining a set of function pluralities and a set of argument pluralities will be a set of pluralities of the values obtained by applying atomic function parts to atomic argument parts. This set will include exactly those pluralities that ‘cover’ all the parts of some function plurality and all the parts of some argument plurality, as schematized in (37).

\[
(37) \quad \{ f(a) + g(b), f(b) + g(a), f(a) + g(a) + g(b), f(b) + g(a) + g(b), f(a) + f(b) + g(a), f(a) + f(b) + g(b), f(a) + f(b) + g(a) + g(b) \} \quad \{ f + g \} \quad \{ a + b \}
\]

Importantly, this process is repeated at any syntactic node that dominates at least one plural expression. Thus, in cumulative sentences, the rule applies at each node intervening between the plural expressions that participate in cumulativity. Sentences will thus end up denoting plural sets of propositions, which count as true if at least one plurality in the set consists exclusively of true propositions. I will here adapt this basic framework so as to be able to extend it to DDC-sentences, making use of the idea that a plural expression like two monsters can in principle ‘project’ to a plural set of pluralities of propositions: What I will claim, essentially, is that two monsters denotes a plural set of individual concepts, roughly as in (38): For any plurality in this set, it will have to hold that (i) both ‘parts’ are individual concepts that yield us a monster in every world where they are defined and (ii) the two parts must not overlap – they cannot yield us the same value in any world where both are defined. (My final proposal will be slightly more complex, but rests on the same idea.)

\[
(38) \quad \{ f + g \mid f, g \text{ are individual concepts} & \text{monster}(f) \& \text{monster}(g) \& \forall w \in \text{dom}(f) \cap \text{dom}(g)(f(w) \neq g(w)) \}
\]

Like any other plurality, the denotation of two monsters will project in the sense sketched above, so that the embedded clause will end up with a denotation very similar to (35-b). Further application of the projection rule yields us something analogous to the set in (39) as the denotation for the whole sentence – which means, essentially, that in order for (34) to be true, Ada and Bea must cumulatively believe one of the propositions in (35-b). Hence, we will end up with the correct truth-conditions.

\[
(39) \quad \{ \text{Ada believes that a griffin is roaming the castle} + \text{Bea believes that a zombie is roaming the castle}, \ldots \}
\]

In section 4.1 I briefly present some of the motivation given by Schmitt (2018), Haslinger & Schmitt (2018b, 2019a) for the plural projection system, independently of DDC-sentences. In section 4.2 I outline the basic mechanism, following Haslinger & Schmitt 2018b and extend it to intensions. In section 4.3 I expand it to DDC-sentences.
4.1 Plural projection: Motivation for the basic system

Before we consider the actual system, I briefly provide some motivation for its two core assumptions. (For a more detailed discussion of these points, plus additional motivation, see Schmitt 2018, Haslinger & Schmitt 2018). The first assumption is that all semantic domains contain pluralities, including functional domains like the domain of one-place predicates or the domain of propositions. It is motivated by the behavior of conjunctions (and-coordinations) in German and English, Schmitt 2013 2018, 2018. Conjunctions with conjuncts of various categories display the behavior of pluralities in that they give rise to cumulativity when co-occurring with another plural expression, I already gave an example involving VP-predicate conjunction in section 3.2 above, which is repeated in (40). We saw that the sentence in (40-b) is true in the scenario in (40-a). In this scenario, it is not the case that each girl made Gene brush Harry (or that each girl made Gene feed Ivo) as a distributive interpretation of predicate conjunction (e.g. von Stechow 1974, Gazdar 1980, Partee & Rooth 1983 a.o.) would require. Analyses of cumulative predicate conjunction like Link 1984, Krifka 1990, Heycock & Zamparelli 2005 won’t help, either. They assume that P and Q denotes a property of pluralities that have a P-part and a Q-part, so their scope is restricted to cases where the predicate conjunction directly combines with a semantically plural argument - which is not the case in (40). (41) shows that we can construct similar cases where the predicate conjunction occurs inside the complement of an attitude predicate and cumulates with the matrix subject – (41-b) is true in the scenario in (41-a) – which rules out a treatment in terms of an event-based analysis of conjunction and cumulation. Rather, these data suggest that predicate conjunctions of the form P and Q denote pluralities that have P and Q as their atoms.

   b. The two girls made Gene [feed Ivo]p [and brush Harry]q when all he wanted to do was take care of his hamster.

   b. The Georgian ambassador called this morning, the Russian one at noon. They think that Trump should [p talk to Putin] and [q build a hotel in Tbilisi], but neither addressed the Caucasus conflict! adapted from Schmitt 2018

Similar facts obtain for propositional conjunctions: The sentence (42-b) is true in the scenario in (42-a), where Bea believes p but is agnostic wrt. q and Carl believes q but is agnostic wrt. p. The plurality consisting of Bea and Carl thus seems to be in a cumulative (belief-) relation w.r.t. the plurality of propositions consisting of p and q. (The phenomenon is not limited to clausal conjunctions embedded under believe, see Schmitt 2018.)

(42) a. Scenario: Ada, Bea and Gene will have a joint birthday party, organized by Roy, and they each have their private theories about what will happen at the party. Ada is certain that Roy hired Criss Angel. Bea believes that he arranged a performance by Kiss. Gene thinks that The Smiths will play at the party. Roy tells me:
   b. Well, Ada and Bea have high hopes for the party – they believe [s I hired Criss
Angel] and [\text{that} \text{Kiss will be performing}]...but neither of them is as crazy as Abe: He believes that I talked The Smiths into reuniting!

Hence, it seems that conjunctions of the form \textit{X and Y} generally denote pluralities with atoms \textit{X} and \textit{Y}. As \textit{X}, \textit{Y} can denote ‘functional’ elements, like predicates or propositions, we must expand our notion of plurality to domains other than the ‘primitive’ domains of individuals or events – i.e. we must admit ‘higher-order’ pluralities to our system.

The second assumption of the plural projection analysis is that pluralities ‘project’ in the sense sketched above. It is motivated, amongst other things, by the flattening cases from section 3.2 above. There, we observed that (43-b) is true in the scenario in (43-a) and that these truth-conditions do not fall out from an analysis that cumulates syntactically derived relations. But we can easily state those truth-conditions of (43-b) if we appeal to cumulative relations involving higher-order pluralities: The two girls must cumulatively satisfy the predicate plurality in (44)– i.e. each girl must satisfy at least one predicate in the sum in (44), and each predicate must be satisfied by at least one girls

   b. The two girls made Gene [[feed the two cats] \textit{P} and [brush Harry] \textit{Q}] when all he wanted to do was take care of his hamster. (Schmitt 2018)

(44) [[feed the two cats and brush Harry]] = \textit{feed(I)}+\textit{feed(J)}+\textit{brush(H)}

We therefore require a system that derives (44) as the denotation of the VP conjunction in (43). This system must guarantee that if one plural expression is contained in another, the resulting expression denotes a single ‘flat’ plurality that preserves the part structure of the embedded plural expression (\textit{the two cats} in (43)). Plural projection is a mechanism that will do just that.

4.2 Plural projection: The basic system

I now spell out the basic mechanism, which derives the data discussed in section 4.1. I start with an extensional version in section 4.2.1, which follows Schmitt (2018) and in particular Haslinger & Schmitt (2018b) (with some slight modifications), and then expand it to intensions in section 4.2.2 (I keep the discussion rather short; a more elaborate and intuitive discussion of the technical aspects can be found in Schmitt (2018) and Haslinger & Schmitt (2019a).) In section 4.3 I extend this system in a way so that it will cover DDC-readings.

4.2.1 Plural projection: Extensional version

\textbf{Ontology} As sketched above, the semantic composition rule will operate on plural sets. These plural sets should not interact with the composition rules in the same way as unary predicates (i.e. the ‘ordinary’ sets), so their status is reflected in the type system: For any semantic type \textit{a}, there is also a type \textit{a*} for plural sets with elements of type \textit{a}.

(45) The set \textit{T} of \textbf{semantic types} is the smallest set such that \textit{e} \in \textit{T}, \textit{t} \in \textit{T}, for any \textit{a, b} \in \textit{T}, \langle\textit{a, b}\rangle \in \textit{T}, and for any \textit{a} \in \textit{T}, \textit{a*} \in \textit{T}.

The elements of plural sets are pluralities. As mentioned above, they are available for all semantic types. I thus employ a cross-categorial notion of sum: Sums of any semantic type stand in a one-to-one correspondence to nonempty sets of atomic meanings of that type. In other words, each semantic domain is enriched by a ‘flat’ plural semantics. This idea is spelled out in (46): The cross-categorial operation \textbf{+} maps any nonempty set of denotations (of the same type) to its sum. For any type \textit{a}, the set \textit{Aa} of atomic elements of the domain is extended to a set \textit{Da} that also includes sums. Clause (46-a) says that the sum operation on \textit{Da} is isomorphic to the union of nonempty sets of atomic meanings
from \(A_a\). Clause (46-c) bans pluralities from being identified with such sets, as they are distinguished by the composition rules.

(46) Let \(A\) be the (nonempty) set of atomic individuals. For each type \(a\), there is an **atomic domain** \(A_a\) and a **full domain** \(D_a\) with the following properties:
   a. \(D_a\) is a set s.th. \(A_a \subseteq D_a\) and there is an operation \(\vdash a : \mathcal{P}(D_a) \setminus \{\emptyset\} \rightarrow D_a\).
   b. There is a function \(p^a_{\vdash a} : \mathcal{P}(A_a) \setminus \{\emptyset\} \rightarrow D_a\) s.th.:
      (i) \(p^a_{\vdash a}([x]) = x\) for each \(x \in A_a\)
      (ii) and \(p^a_{\vdash a}\) is an isomorphism from \((\mathcal{P}(A_a) \setminus \{\emptyset\}, \cup)\) to \((D_a, \vdash)\).
   c. For any type \(b \neq a\), \(D_a\) and \(D_b\) are disjoint.\(^{12}\)

The atomic domains are defined recursively in the usual way, (47). For types of the form \(a^*\), the domain is isomorphic to, but disjoint from the power set of \(D_a\). The disjointness condition allows for the definition of operations that are sensitive to whether their arguments are plural sets (type \(a^*\)) or ‘regular’ sets (type \((a, t)\)).

(47) a. \(A_e = A\), the set of individuals; \(A_t = \{0, 1\}^W\), where \(W\) is the set of possible worlds
   b. For any types \(a, b\): \(A_{(a, b)} = D_b^{D_a}\), the set of partial functions from \(D_a\) to \(D_b\). 
   c. For any type \(a\), \(A_{\vdash a}\) is a set that is disjoint from \(\mathcal{P}(D_a)\) and on which the operations \(\cup, \cap\) and \(\setminus\) are defined. Further, there is a function \(p^a_{\vdash a} : \mathcal{P}(D_a) \rightarrow A_a\) that is an isomorphism wrt. \(\cup, \cap\) and \(\setminus\).

In the following, I’ll use the notational conventions employed by [Haslinger & Schmitt (2018b)]:

(48) a. I use ‘starred’ variables like \(x^*, P^*\) etc. for types of the form \(a^*\).
   b. I sometimes omit type subscripts on cross-categorial operations.
   c. For variables \(x, x_1, \ldots, x_n\) of any type, I write \([x_1, \ldots, x_n]\) for the plural set \(p^x_{\vdash x}([x_1, \ldots, x_n])\), and \([x | \phi]p^x_{\vdash x}\) for \(p^x_{\vdash x}(\lambda x.\phi)\).
   d. For any type \(b\) and \(x, y \in D_b\):
      (i) \(x + y =_{\text{def}} \vdash b([x, y])\)
      (ii) \(x \leq y \iff_{\text{def}} x + b y = y\)
      (iii) \(x \leq a y \iff_{\text{def}} x \leq y \land y \in A_b\)

(49) illustrates the ontological assumptions for the domain predicates of individuals: \(D_{(e,t)}\) will now contains sums of predicates in addition to ‘atomic’ predicates, (49-b). The domain of \(A_{(e,t)}\) of plural sets of predicates is isomorphic to the power set of \(D_{(e,t)}\), \(D_{(e,t)}^{(e,t)}\), in turn, would contain sums of such plural sets, e.g. \([\lambda x.\text{smoke}(x)] + [\lambda x.\text{smoke}(x) + \lambda x.\text{dance}(x)]\), plus the elements of \(A_{(e,t)}^{(e,t)}\).

(49) a. \(A_{(e,t)} = [\lambda x.\text{smoke}(x), \lambda x.\text{dance}(x), (\lambda x.\text{smoke}(x) \lor \text{dance}(x)), \ldots]\)
   b. \(D_{(e,t)} = [\lambda x.\text{smoke}(x), \lambda x.\text{dance}(x), (\lambda x.\text{smoke}(x) \lor \text{dance}(x)), \lambda x.\text{smoke}(x) + \lambda x.\text{dance}(x), \lambda x.\text{smoke}(x) + (\lambda x.\text{smoke}(x) \lor \text{dance}(x)), \lambda x.\text{dance}(x) + (\lambda x.\text{smoke}(x) \lor \text{dance}(x)), \lambda x.\text{smoke}(x) + (\lambda x.\text{smoke}(x) \lor \text{dance}(x)), \ldots]\)
   c. \(A_{(e,t)}^{(e,t)} = \{[], [\lambda x.\text{smoke}(x)], [\lambda x.\text{dance}(x)], [\lambda x.\text{smoke}(x) \lor \text{dance}(x)], [\lambda x.\text{smoke}(x) + \lambda x.\text{dance}(x)], [\lambda x.\text{smoke}(x) + (\lambda x.\text{smoke}(x) \lor \text{dance}(x))], [\lambda x.\text{dance}(x) + (\lambda x.\text{smoke}(x) \lor \text{dance}(x))], \ldots\}\)

‘Typeshift’ \(\uparrow\) Before I consider the denotations of plural expressions, I add the assumption that all lexical elements with a denotation in \(D_a\), such that \(a\) does not contain any ‘starred’ types – i.e. elements that don’t make reference to plural sets – are ‘taken’ to plural sets right after they enter the derivation. This can be done via a morpheme denoting the operation \(\uparrow\), which maps any element

\(^{12}\)The empty partial function should be exempt from the disjointness conditions.
X from $D_a$ to $[X]$ in $A_{a'}$. (For the sake of simplicity, I will also employ $\uparrow$ as its object language representation.) The motivation of this move is that plural sets will be treated in a ‘special’ way in the semantic composition, namely, they will be combined with their sisters via the ‘projection’ rule Cumulative Composition defined below. For technical reasons that will become clear below, I want this rule to apply to all elements except those that operate on plural sets. (In effect, the projection rule will only give us interesting results for those structure that contain a plurality-denoting expression, but using it in ‘simple’ structure won’t hurt.)

Denotations of plural expressions I now provide denotations for the plural expressions encountered in the preceding sections: Plural definites, indefinites and conjunctions. (In all cases, the discussion here is still limited to extensions.)

Both plural definites and indefinites denote plural sets of type $e^*_S$, as in (50). The denotations of indefinites can have more than one element, e.g. two pets denotes the set of all sums of two pets. I omit the compositional treatment of these expressions here (see Haslinger & Schmitt 2018b), as I will discuss indefinites in intensional contexts in some detail below.

(50) a. $\llbracket \text{the girls} \rrbracket = \llbracket \text{the [m. girl]} \rrbracket = [\text{Ada + Bea}]
\quad \text{b.} \quad \llbracket \text{two pets} \rrbracket = \llbracket \text{two [m. pet]} \rrbracket = [\text{Ivo + Joe, Ivo + Harry, Joe + Harry}]

The denotation of conjunction appeals to a cross-categorial operation $\oplus$ – an operation that will also be relevant for the projection rule stated below, and which is defined in (51) for an arbitrary number of elements. Informally, (51-b) says that we consider all the different ways of choosing an element from each of the argument sets (represented by the function variable $f$) and, for each such choice, we sum up the selected elements.

(51) The operation $\bigoplus_a : \mathcal{P}(D_a) \setminus \{\emptyset\} \to D_a$ is defined for any type $a$ as follows:
\quad a. For any type $a$ that is not of the form $b^*$, and any nonempty $S \subseteq D_a$, $\bigoplus S = +S$.
\quad b. For any type $b^*$ and any nonempty $S \subseteq D_{b^*}$, $\bigoplus b^* S = [X] \exists f : f$ is a function from $S$ to $D_b \land X X^* \in S : f(X^*) \in pl^{-1}(X^*) \land X = \bigoplus b^* (f(X^*) \mid X^* \in S)]$

Based on (51), the meaning of and as is given in (52)– it denotes a function that takes two plural sets as its arguments and returns us the ‘sum’ in the sense specified in (51).

(52) $\llbracket \text{and } a_{a'}, a'_{a'} \rrbracket = \lambda x a_{a'}, \lambda y a'_{a'} . x \oplus a' y \text{ for any type } a$

This rule has a ‘distributive’ effect, which becomes relevant once the plural sets denoted by the conjuncts contain more than one element: It produces the set of all pluralities that can be obtained by selecting one element from each argument set and summing up all the selected elements. $\llbracket \text{a girl and two pets} \rrbracket$ in (53) ends up denoting a plural set containing all sums of a girl and two pets.

(53) a. $\llbracket \text{smoke and drink} \rrbracket = [\lambda x . \text{smoke}(x)] \oplus [\lambda x . \text{drink}(x)] = [\lambda x . \text{smoke}(x) + \lambda x . \text{drink}(x)]$
\quad b. $\llbracket \text{a girl and two pets} \rrbracket = \llbracket \text{a girl} \rrbracket \oplus \llbracket \text{two pets} \rrbracket = [\text{A, B}] \oplus [\text{I + J, J + H, I + H}]
\quad \quad = [\text{A + I + J, A + J + H, A + I + H, B + I + J, B + J + H, B + I + H}]

Projection rule The final step is the compositional rule, i.e. the projection rule: It combines plural sets with one another and encodes cumulativity. As sketched above, it considers all the different ways of matching parts of the functor plurality with parts of the argument plurality such that each atomic part is covered, and constructs a plurality of values for each such matching. This is formalized by

\[ \text{projection rule} \]

This approach does not extend to non-upward-monotonic indefinites like exactly less than ten pets. See Haslinger & Schmitt 2018a for an expansion of the current system to such expressions.
defining the notion of a **cover** of \((P, x)\) in (54): A cover is a relation between atomic parts of \(P\) and atomic parts of \(x\) in which each atomic part of \(P\) and each atomic part of \(x\) occurs at least once.

(54) Let \(P \in D_a, x \in D_b\). A relation \(R \subseteq A_a \times A_b\) is a **cover** of \((P, x)\) iff \(\exists (P', x') \in R\) = \(P\) and \(\exists ((x' | P') : (P', x') \in R) = x\).

Based on this notion, (55) defines the composition rule for plural sets: For each cover of some plurality in the functor set and some plurality in the argument set, we use regular functional application for all functor-argument pairs related by the cover. The values are ‘summed up’ via \(\oplus\). (In this paper, \(\oplus\) usually coincides with \(\odot\)).

(55) **Cumulative Composition**

For any \(P^* \in D_{(a, b)^*}\) and \(x^* \in D_a^*\): \(C(P^*, x^*) = \bigoplus_{(P'(x') | (P', x') \in R)}(\exists P \in pt^{\ominus-1}(P'), x \in pt^{\ominus-1}(x') : R \text{ is a cover of } (P, x))\)

Syntactic structures will thus be interpreted as in (56): Two plural sets will be combined via the rule CC in (56-a), i.e. via Cumulative Composition. Functional application (FA) in (56-b) on the other hand, will be used whenever we combine a function-denoting element (which is not itself a plural set) with an element that denotes an appropriate argument for that function (which can, but does not have to be a plural set). Given my assumptions from above, this means that FA applies when we are dealing with expressions denoting functions to or from plural sets (like and or \(\ominus\)) – because all other functions (elements of \(D_{(a,b)}\)) are automatically mapped to plural sets once they enter the derivation.

(56) a. For any meaningful expressions \(\phi\) of type \(\langle a, b \rangle^*\) and \(\psi\) of type \(a^*\), \([\phi \psi]\) is a meaningful expression of type \(b^*\), and \([\phi \psi] = C([\phi], [\psi])\). **CC**

b. For any meaningful expressions \(\phi\) of type \(\langle c, d \rangle\), and \(\psi\) of type \(c\), where \(c = a \text{ or } c = a^*\), and \(d = b \text{ or } d = b^*\), if \([\phi]\) is in the domain of \([\phi]\), \([\phi \psi]\) is a meaningful expression of type \(d\), and \([\phi \psi] = C([\phi], [\psi])\). **FA**

(57) illustrates the effect of the CC-rule for some examples. (57-a) is similar to examples discussed in section \[3\] above where we witnessed cumulative construals of predicate conjunction: For each way of matching the two girls with the two predicates such that both predicates and both girls are covered, we obtain a plurality of truth-values (represented here by sentences in bold-face) in the output set. (57-b), on the other hand, is analogous to examples involving ‘standard’ cumulativity, which I discussed at the very beginning of this paper: We start off with the indefinite, which denotes a plural set of pluralities of two individuals, [57-b-i] This plural set projects to an analogous set of predicate pluralities, such that each predicate plurality ‘preserves’ the part-structure of one of the individual pluralities, [57-b-ii] Finally, this set of predicate pluralities combines with the (singleton) plural set denoted by the subject: For each plurality of two predicates, and each way of matching these predicates with the two girls such that both predicates and both girls are covered, we obtain a plurality of truth-values in the output set. [57-b-iii]

(57) a. **The two girls are smoking and dancing**

\(C([\text{smoke} + \text{dance}])([\text{Ada} + \text{Bea}]) = [\text{smoke(A)} + \text{dance(B)} + \text{dance(A)} + \text{smoke(B)} + \text{smoke(B)} + \ldots]\).

b. **Ada and Bea fed two pets**.

(i) \([\text{two pets}] = [I + J, I + H, J + H]\)

(ii) \([\text{fed two pets}] = C([\text{fed}])([I + J, I + H, J + H]) = [\text{fed(I)} + \text{fed(J)}, \text{fed(I)} + \text{fed(H)}, \text{fed(J)} + \text{fed(H)}]

(iii) \([\text{Ada and Bea fed two pets}] = C([A + B])([\text{fed(I)} + \text{fed(J)}, \text{fed(I)} + \text{fed(H)}, \text{fed(J)} + \text{fed(H)}]) =

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The plural sets of truth-values are related to truth conditions via (58):

\[ \{ \text{fed}(I)(A) + \text{fed}(J)(B), \text{fed}(I)(B) + \text{fed}(J)(A), \text{fed}(I)(A) + \text{fed}(H)(B), \text{fed}(J)(B) + \text{fed}(H)(A), \ldots \} \]

The plural sets of truth-values are related to truth conditions via (58):

\[ a. S_2 \text{ The two girls } [VP_2 \text{ made } S_1 \text{ Gene } [VP_1 \{ \text{feed the two cats} \}_P \text{ and } [\text{brush Harry}_Q]\}]] \]

\[ b. (i) \{[F] = C(\text{[feed]})([I + J]) = [\text{feed}(I) + \text{feed}(J)] \]

\[ (ii) \{[VP_1] = \oplus([\text{feed}(I) + \text{feed}(J)])([\text{brush}(H)]) = [\text{f}(I) + \text{f}(J) + b(H)] \]

\[ (iii) \{[S_1] = C([G])([\text{f}(I) + \text{f}(J) + b(H)]) = [\text{f}(I)(G) + \text{f}(J)(G) + b(H)(G)] \]

\[ (iv) \{[VP_2] = C([\text{made}])([\text{f}(I)(G) + \text{f}(J)(G) + b(H)(G)]) = \]

\[ \text{made}(f(I)(G))(A) + \text{made}(f(J)(G))(A) + \text{made}(b(H)(G))(B), \text{made}(f(I)(G))(A) + \text{made}(f(J)(G))(B) + \text{made}(b(H)(G))(B) \ldots ] \]

\[ \text{(59)} \]

Deriving the Flattening effect Here is how the proposal derives the flattening effect discussed in sections 3.2 and 4.1. The example from (30) above is repeated in (59-a) below, whereas (59-b) gives the relevant steps of the derivation. For the first VP-conjunct, we obtain a singleton plural set via CC, sections 3.2 and 4.1. The example from (30) above is repeated in (59-a) below, whereas (59-b) gives one way of choosing an element from each set, \( \oplus \) – made with the meaning of the matrix predicate yielding the by now familiar projection behaviour, (59-b-iii). The result is combined, again via CC, resulting in the set of pluralities of truth-values in this complication here for the sake of simplicity. Finally, the predicate sum obtained in (59-b-iv) is combined with the sum of the two girls via CC, resulting in the set of pluralities of truth-values in (59-b-v). Via the definition of truth in (58), we obtain the correct truth-conditions.

\[ a. S_2 \text{ The two girls } [VP_2 \text{ made } S_1 \text{ Gene } [VP_1 \{ \text{feed the two cats} \}_P \text{ and } [\text{brush Harry}_Q]\}]] \]

\[ b. (i) \{[F] = C(\text{[feed]})([I + J]) = [\text{feed}(I) + \text{feed}(J)] \]

\[ (ii) \{[VP_1] = \oplus([\text{feed}(I) + \text{feed}(J)])([\text{brush}(H)]) = [\text{f}(I) + \text{f}(J) + b(H)] \]

\[ (iii) \{[S_1] = C([G])([\text{f}(I) + \text{f}(J) + b(H)]) = [\text{f}(I)(G) + \text{f}(J)(G) + b(H)(G)] \]

\[ (iv) \{[VP_2] = C([\text{made}])([\text{f}(I)(G) + \text{f}(J)(G) + b(H)(G)]) = \]

\[ \text{made}(f(I)(G))(A) + \text{made}(f(J)(G))(A) + \text{made}(b(H)(G))(B), \text{made}(f(I)(G))(A) + \text{made}(f(J)(G))(B) + \text{made}(b(H)(G))(B) \ldots ] \]

4.2.2 Expanding the system to intensions

The derivation in (59) already suggests that we need an expansion to the intensional part of the language – which is also required for some of the examples discussed in Section 4.1 (60) (repeated from (41-a), for instance, involves a cumulative relation between the subject plurality and the predicate conjunction inside the embedded clause, which itself is the argument of the intensional verb think.

\[ \text{(60) } \text{The agents think } S_1 \text{ that Trump will } [P \text{ talk to Putin}] \text{ and } [Q \text{ invest in Tbilisi}]. \]

I thus modify the basic mechanism from the previous section to include world parametrization: We add type \( s \) to our set of types and the set \( W \) of all possible worlds to our semantic domains. All lexical elements that don’t make reference to plural sets – i.e. elements like Ada, smoke etc. – are assigned functions from worlds to extensions in that respective world, as illustrated for smoke in (61).

\[ \{ \text{smoke} \} = \lambda w. \lambda x. \text{smoke}(x)(w) \]

Cumulative Composition is relativized to extensional and intensional functional application as in (62): If a plural set of function-pluralities that are extensional wrt. their argument combines with
a plural set of arguments, the relevant parts of the functor plurality and of the argument plurality combine via extensional functional application in (62-a) (‘ECC’). If a plural set of function-pluralities that are intensional wrt. their argument combines with a plural set of arguments, the relevant parts of the functor plurality and of the argument plurality combine via intensional functional application in (62-b) (‘ICC’). In both cases, the result will again be a plural set.

(62) Cumulative Composition

a. For any $P \in D_{(a,b)}$ and $x \in D_{(a,b)}$:

$$C(P^*, x) = \{ (EFA[P^*, x] | (P^*, x) \in R) \} \mid \exists P \in pl^{\text{EFA}}(P^*, x) \in pl^{\text{EFA}}(x^*), R \}$$

where for any for any $Q \in D_{(a,b)}, y \in D_{(a,b)}$, $EFA[Q, y] = \lambda w. Q(w)(y(w))$  

b. For any $P^* \in D_{(a,b)}, x^* \in D_{(a,b)}$:

$$C(P^*, x^*) = \{ (IFA[P^*, x^*] | (P^*, x^*) \in R) \} \mid \exists P \in pl^{\text{IFA}}(P^*, x^*) \in pl^{\text{IFA}}(x^*), R \}$$

where for any $Q \in D_{(a,b)}, y \in D_{(a,b)}$, $IFA[Q, y] = \lambda w. Q(w)(y)$

While our CC-rule is now ‘sensitive’ to the difference between extensional and intensional functions, the other compositional rule from above – Functional Application – remains unchanged, i.e. it is not relativized to extensions and intensions. This won’t be necessary, because I assume that lexical elements that appeal to plural sets in their denotations – that take plural sets as their arguments or return them as their values (elements like and) are not assigned functions from worlds to extensions. Thus, while the denotation of an expression like smoke in (61) is relativized to worlds as in (61), the denotation of and is not and will remain as in (62), above, i.e. (63).

(63) $[[\text{and}]_{[a', (a', a)]]] = \lambda x_{a'} \lambda y_{a'.} x \oplus_{a'} y$ for any type $a$

For illustration, consider the derivation of smoke and dance: The shift $\uparrow$ takes the intensions of smoke and dance to the respective singleton plural set. $[[\text{and}]]$ applies to these two plural sets via FA, so we end up with the plural set containing the sum of the intensions of smoke and dance.

(64) $[[\uparrow \text{smoke} \land [\uparrow \text{dance}]] = [[\text{and}]] (\uparrow \text{FA} \lambda x. \text{smoke}(x)(w))(\uparrow \text{FA} \lambda x. \text{dance}(x)(w)) =
[[\text{and}]] ((\lambda w. \lambda x. \text{smoke}(x)(w))(\lambda w. \lambda x. \text{dance}(x)(w))) =
[\lambda w. \lambda x. \text{smoke}(x)(w) + \lambda w. \lambda x. \text{dance}(x)(w)]$

Using this richer system will give us plural sets of propositions – functions from worlds to truth-values – at the matrix level. Thus, our truth-definition from above must be reformulated as in (65).

(65) A plural set $p^* \in A_{(a,b)}$ of propositions is true in a world $w$ iff there is a plurality $P \in pl^{\text{EFA}}(p^*)$ such that for all $q \leq a$, $q(w) = 1$, and false iff for all pluralities $p \in pl^{\text{EFA}}(p^*)$, there is a $q \leq a$ such that $q(w) = 0$.

Let us now consider the relevant steps of the derivation of a sentence containing an intensional operator, i.e. (60) above. The denotation of the predicate conjunction is analogous to what we saw in (64). Ignoring tense, we employ ECC to derive the denotation of the embedded clause – a plural set containing a single propositional plurality, (66-b). The next step is to combine the denotation of believe – for which I essentially assume the standard denotation, (Hintikka [1969], (66-c) – with this plural set of propositional pluralities. This step is shown in (66-d): Because believe is intensional on its first argument, we use ICC. This gives us a plural set of predicates that contains a single predicate plurality, which consists of the predicate of believing $p$ and the predicate of believing $q$. The final step is to combine this plural set with the subject. This being an extensional context, we again resort to ECC, (66-e). As a result, we obtain a plural set of propositions, which is mapped to true in a world $w$ (via (65)) just in case one of the propositional pluralities in it is such that all of its atoms are true in
w. These are the correct truth-conditions.

(66) a. \([\text{talk to Putin and invest in Tbililsi}] = [\lambda w.\text{talk to P. in w} \oplus \lambda w.\text{invest in T. in w}]\]

b. \([S1] = [\lambda w.\text{Trump talks to P. in w} \oplus \lambda w.\text{Trump invests in T. in w}]\]

c. \([\lambda \text{believe}] = [\lambda w.\lambda x. (\lambda p.\lambda x.\text{BEL}_x w \subseteq p)]\]

d. \(C([\lambda \text{believe}])([S]) = C([\lambda w.\lambda x.\lambda x.\text{BEL}_x w \subseteq p])([\lambda w.\text{Trump talks to P. in w} \oplus \lambda w.\text{Trump invests in T. in w}]) = [\lambda w.\lambda x.\text{BEL}_x w \subseteq \lambda w'].\text{Trump talks to P. in w'} \oplus \lambda w.\lambda x.\text{BEL}_x w \subseteq \lambda w'.\text{Trump invests in T. in w'}\]

e. \([S2] = C([\lambda \text{the agents}])([66-c]) = C([\lambda w.\lambda x.\lambda x.\text{BEL}_x w \subseteq p])([\lambda w.\lambda x.\lambda x.\text{BEL}_x w \subseteq p])([\lambda w.\text{BEL}_{a1,w} \subseteq \lambda w'.\text{Trump talks to P. in w'} \oplus \lambda w.\text{BEL}_{a2,w} \subseteq \lambda w'.\text{Trump invests in T. in w'}])\]

4.3 Expanding the system to DDC-readings

The final step is to expand the system to DDC-sentences like (67).\(^{14}\)

(67) Bea and Carl believed that two monsters are roaming the castle.

As indicated above, the basic idea will be that two monsters denotes a plural set of pluralities of partial individual concepts. These pluralities will each consist of two ‘distinct’ atomic individual concepts, so the plural set intuitively corresponds to the one in (68) (where individual concepts are represented informally by definites):

(68) [the griffin, the zombie, the werewolf, the ork, ...]

This plurality will then project via the CC-rule from above to a plural set of propositions, which roughly corresponds to (69). Crucially, the propositional pluralities in the set reflect the part structure of the pluralities of individual concepts.

(69) [that the griffin is roaming the castle, that the zombie is roaming the castle, that the werewolf is roaming the castle, that the ork is roaming the castle, ...]

This plural set combines with the matrix predicate by means of the CC-rule, which will give us a plural set of predicates that intuitively corresponds to (70):

(70) [believe that the griffin is roaming the castle, believe that the zombie is roaming the castle, believe that the werewolf is roaming the castle, believe that the ork is roaming the castle, ...]

Finally, the plural subject combines with this plural set of predicates (again via the CC-rule), resulting in a plural set of propositions that will correspond to something like (71). Quer(65) the sentence will be true in a world w iff at least one of the propositional pluralities in this set is true.

(71) [that A believes that the griffin is roaming the castle, that B believes that the zombie is roaming the castle, that A believes that the griffin is roaming the castle, that B believes that the griffin is roaming the castle, that A believes that the zombie is roaming the castle, that B believes that the werewolf is roaming the castle, that A believes that the werewolf is roaming the castle, ...]

I will give two implementations of this idea. The first one is more ‘direct’ in terms of how it relates to the intuitions just sketched, as it will involve pluralities of actual individual concepts. But as it

\(^{14}\) I here cover the basic examples. Analogous assumptions will have to be made regarding other instances of DDC-sentences, e.g. the cases listed in (20) in section 2 above.
runs into a problem having to do with the partiality of individual concepts, I replace it by a slightly more ‘abstract’ version, which uses pluralities of quantifiers over individual concepts.

### 4.3.1 Pluralities of individual concepts

Let us first consider the idea that *two monsters* denotes a plural set of individual concepts. We start with the denotation of *monster*, which I assume to be standard, i.e. a function from worlds to sets of individuals. (72-a). It is immediately taken to a plural set via $\llbracket$, (72-b).

\begin{equation}
\llbracket\text{monster}\rrbracket = \lambda w. \lambda x_c. \text{monster}(x)(w)
\end{equation}

\begin{equation}
\llbracket\llbracket \text{monster} \rrbracket \rrbracket = [\lambda b. \lambda x_c. \text{monster}(x)(w)]
\end{equation}

The plural morpheme $\text{pl}$ in (73) takes plural sets of NP-intensions (i.e. objects like (72-b)) and maps them to plural sets of individual concepts. (Analogously for $\text{sg}$, which I leave out as it does not matter for my purposes.) Crucially, it also encode ‘distinctness’: It imposes the requirement of ‘cross-world non-overlap’. Condoravdi et al. (2001) make a very similar proposal regarding the counting of individual concepts – the only difference lies in the technical implemention and in the fact that I consider partial functions. What (73) says, essentially, is that when we apply the denotation of $\text{pl}$ to a property $P$, it will yield us all those pluralities of individual concepts that are such that (i) each of their atoms yields a $P$-individual in every world where it is defined and (ii) all of their atoms are ‘distinct’ in the sense that there is no world where they return the same individual. (In the more complex derivations below, I will sometimes simplify the notation, writing ‘when defined, each $f$-atom is a P’ and ‘all $f$-atoms are distinct’ for the first and the second conjunct in (73), respectively.)

\begin{equation}
\llbracket \text{pl} \rrbracket = \lambda P_{\langle s,e \rangle}. \{ f \in D_{\langle s,e \rangle} : \forall g \leq \text{AT} \exists f (\forall w' \in \text{dom}(g)(P(w')(g(w'))))) \wedge \forall g, g' \leq \text{AT} f(g \neq g' \rightarrow (\forall w' \in \text{dom}(g) \cap \text{dom}(g')(g(w') \neq g'(w')))) \}
\end{equation}

We combine the denotation of $\text{pl}$ with (72-b) via the FA-rule, which gives us (74):

\begin{equation}
\llbracket \text{pl} \llbracket \text{monster} \rrbracket \rrbracket = [f \in D_{\langle s,e \rangle} : \forall g \leq \text{AT} \exists f (\forall w' \in \text{dom}(g)(\text{monster}(w')(g(w'))))) \\
\wedge \forall g, g' \leq \text{AT} f(g \neq g' \rightarrow (\forall w' \in \text{dom}(g) \cap \text{dom}(g')(g(w') \neq g'(w'))))]
\end{equation}

For illustration, take a tiny model with worlds $w_1, w_2, w_3, w_4$. Let the extensions of *monster* be the ones in (75-a). We consider the four individual concepts in (75-b). Applying $\llbracket \text{pl} \rrbracket$ to the intension of *monster* in this model gives us the plural set in (76): $f_5$ doesn’t appear in any of its elements, because there are worlds $f_5$’s domain where its value is not a monster. As for $f_1 - f_4$, we only obtain those pluralities that meet the distinctness requirement (which includes the ‘atomic’ functions $f_1, f_2, f_3, f_4$):

\begin{equation}
\llbracket \text{pl} \llbracket \llbracket \text{monster} \rrbracket \rrbracket \rrbracket = [f_1, f_2, f_3, f_4, f_1 + f_2, f_1 + f_4, f_3 + f_4]
\end{equation}

Finally, let us consider the contribution of *two* (and analogous upward-monotone numerals) in (77). Numerals are assigned a more or less standard denotation (modulo the particulars of the system

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There is no specific reason to distribute the workload in this particular way. One certainly has the intuition that (some) number-morphemes and numerals require the objects in their argument set to be distinct (see in particular Wagener 2013), but this intuition could be encoded in a number of ways.

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presented here): They take a plural set of individual concepts, and return us that subset thereof, that only contains pluralities ‘made up’ of the relevant number of atoms.

\[(77) \quad \text{two} = \lambda x^{(s,x)} . \{ y | \lambda y \cdot x^x \land |y| = 2 \}\]

Combining (77) with (74) gives us (78): The plural set of all pluralities of individual concepts, such that they consist of two ‘distinct’ atoms.

\[(78) \quad \text{two}(\text{PL} \{ \text{monster} \}) = \{ f | \lambda f : \text{two} = 2 \land \forall g \subseteq \text{AT} \cdot (\forall w' \in \text{dom}(g)(\text{monster}(w')(g(w')))) \land \forall g, g' \subseteq \text{AT} \cdot (f \neq g' \rightarrow (\forall w' \in \text{dom}(g) \cap \text{dom}(g')(g(w') \neq g'(w'))))\]

Returning to our tiny model in (75) above, this gives us (79):

\[(79) \quad \text{two}(\text{PL} \{ \text{monster} \}) = \{ f_1 + f_2, f_1 + f_4, f_3 + f_4 \}\]

Let us use this tiny model to show how the projection mechanism works in our original example, repeated in (80-a). For the denotation of the embedded clause, we combine the denotation of two monsters with the denotation of are roaming the castle (abbreviated as ‘roam’) via ECC, which yields a plural set of proposition, (80-b). The pluralities of propositions in this set reflect the part-structure of the pluralities of individual concepts in (79). Next, we combine the denotation of believe, (80-c), with this plural set of propositions via ICC, (80-d). This gives us a plural set of predicate pluralities, that still reflect the part-structure of our original pluralities of individual concepts in (79). Finally, we combine this plural set of predicates with the subject plurality via ECC, which results in a plural set of propositions, (80-e). According to the truth-definition from (65) the sentence is thus true if there is a plurality f\_g consisting of two distinct individual concepts that yield monsters in every world where they are defined, and for all of Ada’s belief worlds w where f is defined, f(w) is roaming the castle in w and in all of Bea’s belief worlds w where g is defined, g(w) is roaming the castle in w. This seems like the correct result.

\[(80) \quad \begin{align*}
\text{a.} & \quad \text{S}_2 \text{ Ada and Bea } \{ \text{VP believe } \{ \text{S}_1 \text{ that two monsters are roaming the castle} \})\} \\
\text{b.} & \quad \text{S}_2 \text{ Ada and Bea } \{ \text{VP believe } \{ \text{S}_1 \text{ that two monsters are roaming the castle} \})\} \\
\text{c.} & \quad \text{S}_2 \text{ Ada and Bea } \{ \text{VP believe } \{ \text{S}_1 \text{ that two monsters are roaming the castle} \})\} \\
\text{d.} & \quad \text{S}_2 \text{ Ada and Bea } \{ \text{VP believe } \{ \text{S}_1 \text{ that two monsters are roaming the castle} \})\} \\
\text{e.} & \quad \text{S}_2 \text{ Ada and Bea } \{ \text{VP believe } \{ \text{S}_1 \text{ that two monsters are roaming the castle} \})\} \\
\end{align*}\]

4.3.2 Pluralities of quantifiers

There is, however, one problem with this proposal: Since the individual concepts are partial functions (simply because there will be worlds without monsters), there will, of course, be worlds for which they are not defined. Now consider again our tiny model from above and assume that the sets of Ada’s and Bea’s belief-worlds are \{w1, w2\} and \{w4\}, respectively: Bea doesn’t believe in monsters, therefore, none of the individual concepts summed up in (79) yields a value. Assuming that
presupposition projects through the belief-operator, we thus get presupposition failure for at least one atom of all the propositional pluralities in (80-e). Accordingly, the sentence in (80-a) should not be considered false in the scenario in (81), but rather should be judged as a presupposition failure.

(81) Ada believes that a zombie is roaming the castle. Bea doesn’t believe in monsters.

This result doesn’t seem right. The same problem occurs, of course, if we consider cases without embedding, e.g. (82-a), which will denote the plural set of propositions in (82-b). In a world without monsters like \( w_4 \) above, the sentence will be judged false, but the current system again predicts an instance of presupposition failure.

(82) a. Two monsters are roaming the castle.
   b. \([Aw.\text{roam}(w)(f_1(w)) + Aw.\text{roam}(w)(f_2(w)), Aw.\text{roam}(w)(f_1(w)) + Aw.\text{roam}(w)(f_4(w)),
       Aw.\text{roam}(w)(f_3(w)) + Aw.\text{roam}(w)(f_4(w))]\)

So what we want, in effect, is that an atomic part of plural proposition counts as false relative to a world \( w \) of evaluation if the individual concept ‘included’ in this part is not defined for \( w \). One way to achieve this goal – suggested to me by Nina Haslinger (pc) – is to map pluralities of individual concepts to quantifiers, which themselves will be total functions. To achieve this, I assume a (silent) morpheme \( F \) with the denotation in (83): If takes an individual concept \( f \) and predicate extension \( P \) and maps them to true in a world \( w \) if \( \text{f}(w) \) is in \( P \) in \( w \), and to false otherwise.

(83) \[ \llbracket F \rrbracket = \lambda w.\lambda (f,e)\cdot \lambda P(e).w \in \text{dom}(f) \land P(f(w)) \]

I assume that \( F \) attaches in the structure right above the numeral, as in (84). (Because the lexical entry of \( F \) doesn’t make reference to plural sets, it will immediately shifted to a plural set by means of \( \llbracket f \rrbracket \).) The denotation we get for this structure, again based on our tiny model and (79) above is given in (85): The singleton plural set containing the operator’s denotation combines with the plural set of individual concepts. The resulting plural set contains quantifier pluralities that directly reflect the part structure of the pluralities of individual concepts.

(84) \[ \llbracket \llbracket \llbracket F \rrbracket \rrbracket \rrbracket = (C)(\llbracket \llbracket \llbracket F \rrbracket \rrbracket \rrbracket)(f_1 + f_2, f_1 + f_4, f_3 + f_4) =
    \{Aw.\lambda P(e).w \in \text{dom}(f_1) \land P(f_1(w)) + Aw.\lambda P(e).w \in \text{dom}(f_2) \land P(f_2(w)),
    Aw.\lambda P(e).w \in \text{dom}(f_1) \land P(f_1(w)) + Aw.\lambda P(e).w \in \text{dom}(f_3) \land P(f_3(w)),
    Aw.\lambda P(e).w \in \text{dom}(f_1) \land P(f_1(w)) + Aw.\lambda P(e).w \in \text{dom}(f_4) \land P(f_4(w))\} \]

Qua this assumption, we are now able to derive the correct truth-conditions: (82-a) from above will now have the denotation in (86), accordingly, it will come out as false in all worlds where it is not the case that two monsters came – including worlds, crucially, where less than two monsters exist. Likewise, if (86) occurs as the complement of believe in our original sentence in (80-a) the sentence is correctly predicted false also in cases where Ada or Bea doesn’t believe in monsters.

(86) \[ \llbracket \text{Two monsters are roaming the castle} \rrbracket = C(\llbracket \llbracket \llbracket F \rrbracket \rrbracket \rrbracket)(\text{roam})
    = \{Aw.w \in \text{dom}(f_1) \land \text{roam}(w)(f_1(w)) + Aw.w \in \text{dom}(f_2) \land \text{roam}(w)(f_2(w)),
    Aw.w \in \text{dom}(f_1) \land \text{roam}(w)(f_1(w)) + Aw.w \in \text{dom}(f_3) \land \text{roam}(w)(f_3(w)),
    Aw.w \in \text{dom}(f_1) \land \text{roam}(w)(f_1(w)) + Aw.w \in \text{dom}(f_4) \land \text{roam}(w)(f_4(w))\} \]

4.4 Interim summary

In the previous paragraphs, I motivated and outlined the plural projection mechanism by Schmitt (2018), Haslinger & Schmitt (2018b, 2019a), which assumes that pluralities ‘project’ their part struc-
ture to the denotations of dominating nodes and takes cumulativity to be encoded by a compositional rule. I then expanded this mechanism to intensional contexts and provided an analysis for DDC-sentences, repeated again in (87).

(87) Ada and Bea believed that two monsters were roaming the castle!

I assumed that expressions like two monsters denote plural sets containing pluralities of individual concepts, which are subsequently mapped to plural sets containing quantifier pluralities which preserve their part-structure. These plural sets ‘project’ to plural set containing pluralities of propositions – which roughly correspond to (88). At least one of the elements of this set must be cumulatively believed by the plural subject in order for the sentence to be true.

(88) [that the griffin is roaming the castle + that the zombie is roaming the castle, that the werewolf is roaming the castle + that the ork is roaming the castle, . . . ]

This analysis gave us the correct truth-conditions. Since it doesn’t assume that cumulativity w.r.t. \( n \)-many pluralities is the result of enriching an \( n \)-ary relation, and thus doesn’t have to syntactically derive the required relation, the account is superior to the alternatives discussed in sections 2 and 3 in two respects, because it avoids all the problems those analyses ran into: We can interpret the indefinite in situ and thus have a ‘natural’ way of deriving the de dicto reading, without having to assume ambiguity of indefinites (i.e. a second meaning for indefinites without existential import). Further, we have no problem dealing with flattening cases.

5 Consequences of the proposal

So far, my motivation for plural projection was more or less technical - as opposed to the alternative proposals discussed before, it is a generalized mechanism for cumulativity, which derives the correct truth-conditions for DDC-sentences. In the following, I present two pieces of ‘positive’ evidence for the proposal, which set it apart from the account in (Pasternak 2018a,b), which analyzes DDC-sentence via a notion collective belief. I then briefly address a general problem for all types of analyses discussed here.

5.1 Arbitrary sums of propositions and cumulative belief

One of the reasons why ‘traditional’ theories of plurality like (Link 1983) assume that the domain of individuals contains all possible sums of individuals is because cumulative inferences show that we are able to ‘sum up’ any two or more individuals. For instance, for any individual-denoting expressions \( a, b, c, d \), the inference from (89-a) and (89-b) to (89-c) will be valid.

(89) a. \( a \text{ fed } c \).
   b. \( b \text{ fed } d \).
   c. \( a \text{ and } b \text{ fed } c \text{ and } d \).

Above I assumed (and motivated) that natural language ontology contains pluralities of objects of any semantic type, including pluralities of propositions. These pluralities are ‘arbitrary’ in the sense that we can ‘sum up’ any two or more objects of the relevant domain, just as we can ‘sum up’ any two or more individuals (or events) in standard plural semantics. This notion also extended to pluralities of propositions, which I treated as ‘arbitrary’ sums of propositions, i.e. objects that are not constrained by logical compatibility. Furthermore, the mechanism I used predicts that a plurality of individuals should be able to cumulatively believe any such plurality of propositions: I didn’t
formulate a special notion of ‘collective’ belief (or analogous attitudes° but rather assumed that if a plurality of individuals believes a plurality of propositions, we are in effect simply dealing with the ‘sum’ of atomic individuals believing atomic propositions (just as in (89), where we can get from atomic individuals feeding atomic individuals to pluralities of individuals feeding pluralities of individuals). Therefore, I predict that (90-c) follows from (90-a) and (90-b) just as (89-c) follows from (89-a) and (89-b) – irrespective of the logical relation between \( p \) and \( q \) (we allow for ‘arbitrary’ sums of propositions), and irrespective of the other beliefs of \( a \) and \( b \) (no special notion of ‘collective’ belief that would impose special restrictions on these inferences).°

\( \text{(90)} \)
\begin{enumerate}
  \item \( a \) believes \( p \).
  \item \( b \) believes \( q \).
  \item \( a \) and \( b \) believe \( p \) and \( q \).
\end{enumerate}

Let’s consider the second point first. (91) shows that a plurality of individuals \( a+b \) can be said to believe \( p \) and \( q \) even if their beliefs are incompatible (while \( p \) and \( q \) are not): In the scenario in (91-a), Adas’s beliefs (that AC/DC will be the only act at the party) are incompatible with Bea’s beliefs (that Kiss will be the only act at the party), but the sentence in (91-b) is true (see Schmitt 2013, 2018, but see also Pasternak 2018b for related observations for slightly different examples).

\( \text{(91)} \)
\begin{enumerate}
  \item \textbf{scenario:} Roy is organizing a joint birthday party for Ada, Bea and Gene. He hasn’t told them anything, except that there will be exactly one performance by a well-known band. Ada, Bea and Gene each have their private theories about which band it will be. Ada believes that AC/DC will the one band playing at the party. Bea believes that Kiss will perform, and no other band. Abe thinks that The Smiths will play. Roy tells me:
  \item Well, Ada and Bea certainly have high hopes for the party – they believe that AC/DC will play at the party and (that) I got Kiss to perform – but neither of them is as crazy as Gene... he believes that I got The Smiths to reunite !
\end{enumerate}

This point carries over to our original DDC-sentences, which is to be expected if, as I argued above, they also involve what essentially boils down to cumulative belief w.r.t. pluralities of propositions: (92-b) is true in the scenario in (92-a), despite Ada and Bea having incompatible beliefs (namely, that lions live in Europe vs. that lions are extinct in Europe).

\( \text{(92)} \)
\begin{enumerate}
  \item Roy told Ada and Bea, who don’t know each other, that he would have a ‘surprise’ when returning from his hunting trip in Austria. Ada and Bea each believe (based on different clues they think they picked up) that Roy will shoot something for them – the most dangerous animal available! Ada doesn’t know that lions are extinct in Europe, so she believes that Roy will kill a lion for her. Bea knows that lions are extinct and

°The possibility of collective belief (as distinct from simple cumulative belief) pointed out to me by Ede Zimmermann and Irene Heim (both pc). It also used by Pasternak [2018b], whose proposal I return to below.

°°It might appear that cumulativity with attitude predicates like believe is ‘harder to get’ than cumulativity with predicates of individuals such as like in sentences like (i). In particular, it seems that we require a lot of context to get a cumulative reading for sentences like (i) in scenarios where Ada believes \( p \) but considers \( q \) impossible and Bea believes \( q \) but considers \( p \) impossible (see [91] and [93] for the actual examples).

\( \text{(i)} \) \quad Ada and Bea believe \( p \) and \( q \).

However, I believe that there is no difference between such cases and examples like (ii) and that the restrictions we observe on cumulative readings with attitude predicates are identical to those we find with predicates of individuals: If it is very salient that Ada likes Carl and Bea likes Dido, but also salient that Ada hates Dido and Bea hates Carl, it seems very hard to utter (ii).

\( \text{(ii)} \) \quad Ada and Bea like Ivo and Joe.
believes that Roy will shoot a European bear for her. Roy tells me: You won’t believe it... I just wanted to surprise them with the news about my engagement to Gene, but...

b. die glauben tatsächlich, dass ich zwei riesige Tiere schießen werde!

‘They actually believe that I will shoot two gigantic animals!’

Accordingly, these data show that subjects can cumulatively believe the content of a declarative clause, irrespective of how their individual beliefs ‘match up’.

Now consider the claim that the content of the declarative in cases of cumulative belief is indeed a plurality of propositions, i.e. that we can form ‘arbitrary’ sums of propositions $p+q$ that can then be cumulatively believed by individuals. There is indeed no restriction on the propositions being compatible: While in (91-b) $p$ and $q$ (the denotations of the individual conjuncts) are compatible, they are incompatible in (93-b) (that Roy lives in a trailer in Alabama vs. that Roy lives in a cottage in Alaska), yet the sentence is true in the scenario in (93-a). (See Schmitt (2013, 2018) for analogous examples.)

(93) a. scenario: Roy hasn’t been seen for a while. His friends have different theories about what happened, based on their individual past encounters with Roy. Ada believes that he lives in a trailer in Alabama, trying to become a country singer. Bea believes that he built a cottage in Alaska and started making chainsaw art. Gene believes that he lives on a yacht and works for a drug cartel. Sue hears about this and tells me: Well, Ada and Bea certainly have weird ideas...

b. They believe that Roy lives in a trailer in Alabama and (that) he is holed up in a cottage in Alaska... but neither is as crazy as Gene: He believes that Roy resides on a yacht!

In sum, the data suggest that we indeed find the inference schema in (90): We form arbitrary ‘sums’ of propositions and they can be cumulatively believed by pluralities of individuals.

Crucially, this also means that the cumulation system runs ‘blindly’ in the sense that no particular notion of collective belief is required. The theory I propose thus runs contrary to the proposal by Pasternak (2018b), who does assume collective belief is responsible for DDC-sentences: Simplifying greatly, he assumes that $a$ and $b$ believe $p$ is true if the intersection of $a$’s beliefs and $b$’s beliefs entails $p$. This is of course incompatible with examples like (91) and Pasternak is aware of this problem. He thus proposes (again, this is my simplified rendering) that certain ‘irrelevant’ beliefs can be excluded (so that we end up with a set of compatible beliefs). Now, at this stage, it looks like the two proposals (his and mine) basically start off at different points of the scale: While Pasternak’s proposal is too restrictive and must be ‘loosened’, so to speak, mine is excessively liberal (I discuss the data that motivate Pasternak’s approach in the next section). However, I think that there is another crucial point that sets our proposals apart: The fact that I assume that cumulative attitudes relate individuals to pluralities. (93) shows that such pluralities are motivated independently – we need and to be some kind of sum-operation, because a classical meaning of and wouldn’t get us anywhere, as the embedded conjunction should simply be contradictory. Furthermore, I predict that cumulative attitudes only occur whenever the complement clause actually denotes a plurality of propositions, which means that I link the availability of this type of cumulativity to certain grammatical phenomena – an issue that I address in the next section. This does not mean that we should discard Pasternak’s proposal altogether – I think his point about having to distinguish ‘relevant’ vs. ‘irrelevant’ beliefs touches upon an important problem, which I address in section 5.3 below.
5.2 Propositional pluralities and plural expressions

Let us now consider the role of grammatical plurality and how it can help us distinguish between theories. In my analysis above, I view DDC-sentences as instances of cumulativity (albeit cumulativity is of course coded in a ‘non-traditional’ way in my analysis), i.e. they boil down to cumulative relations between pluralities of individuals and pluralities of propositions. As cumulativity is a trait of plurality, my proposal thus predicts that cumulative belief (or other attitudes) is limited to those cases where the complement clause denotes such a plurality of propositions. In the plural projection system, pluralities are the result of ‘plurality-forming’ operations – the conjunction and, the plural morpheme and certain determiners (see Haslinger & Schmitt (2018b) for such a claim about every). Accordingly, cumulative belief should be restricted to those types of complement clauses that either involve a clausal conjunction or a plural expression the denotation of which then ‘projects’ to a plurality of propositions (i.e. subsentential conjunction, a plural definite/indefinite etc.). I already showed that such examples indeed give rise to cumulative readings in the preceding sections (see e.g. section 4.1), so I won’t repeat them here. Rather, I want to discuss the prediction that such constructions are the only type of configuration that gives rise to cumulativity. This means that I predict (94) to be impossible: As long as \( p \) contains no plural expressions, we should not be able to truthfully utter (94-b) in the type of scenario in (94-a), where neither \( a \) nor \( b \) believe \( p \), but the logical conjunction of their beliefs entails \( p \).

\[
(94) \quad \begin{align*}
\text{a. scenario: } & a \text{ believes } q, \ b \text{ believes } r, \ q \not\subseteq p, \ r \not\subseteq p, \ q \cap r \subseteq p \\
\text{b. } & a \text{ and } b \text{ believe } p, \text{ where } p \text{ contains no plural expression.} \quad \text{true in (94)}
\end{align*}
\]

My proposal deviates from Pasternak’s 2018b in this respect – because under his analysis, as soon as \( p \) follows from the conjoined beliefs of \( a \) and \( b \), we should have a valid instance of collective belief. The following example seems to corroborate this: According to Pasternak (2018b), (95-b) is true in the scenario in (95-a). As (95-b) contains no plural expression, it shouldn’t denote a plurality of propositions in my system and would thus be predicted to be false in the scenario.

\[
(95) \quad \begin{align*}
\text{a. scenario: } & \text{Paul just got married and his cousins Arnie and Beatrice, who have never met, just caught wind of it. Arnie supsects that Paul’s husband is rich, and has no other relevant opinions. Beatrice thinks he’s a New Yorker, and has no other relevant opinions.} \\
\text{b. } & \text{Paul’s cousins think he married a rich New Yorker.} \quad \text{(Pasternak 2018b:548 (6))}
\end{align*}
\]

In other words, (95) is an example of the type in (94), which my own proposal predicts to be impossible. However, I still think that my account is on the right track for the simple reason instances of the pattern in (94) are extremely restricted, in two respects. First, the only examples that seem to work all involve predicate modification as in (95): The sentence in (96-b) for instance, cannot be true (for any speaker I consulted) in the scenario in (96-a). An account that only requires the embedded proposition to follow from the conjunction of beliefs of the subject plurality would predict it to be true: After all, the logical conjunction of Ada’s belief and Bea’s belief entails that Roy will fall in love with Ada.

\[
(96) \quad \begin{align*}
\text{a. scenario: } & \text{Ada is looking forward to Sue’s party: She is certain that every man at the party will fall in love with her. Bea is also looking forward to the party: She hates men and is certain that only one man will attend: Roy. Sue tells me: } \text{Ada and Bea are really} \\
\text{b. } & \text{that Roy will fall in love with Ada.}
\end{align*}
\]

\[\text{18This list of ‘plurality-inducing’ and thus plurality-denoting expressions is not exhaustive: Mass nouns (see Link 1983 a.o.), singular nouns, (Wagiel 2018), certain degree expressions, (Dotlacil & Nouwen 2016), embedded questions, (Beck & Sharvit 2002) and the antecendents of conditionals, (Schlenker 2004, Gajewski 2005) have all been argued to display certain traits of pluralities.}\]
looking forward to the party:...

b. Sie glauben, dass Roy sich in Ada verlieben wird! Die spinnen!

They believe that Roy will fall in love with Ada. They are crazy!

Second, even examples with predicate modification appear to be much more constrained than examples containing plural expressions (in my sense above). Pasternak himself notes that contradictory belief that concerns the ‘components’ of the modified structure (i.e. rich and New Yorker) above changes the judgements – according to him, (95-b) is considered false if the scenario in (95-a) is modified as in (97).

(97) Arnie thinks that Paul married a rich Marylander, while Beatrice thinks he married a poor New Yorker. (Pasternak 2018b:549 (7))

This is in contrast with e.g. predicate conjunctions (plural expressions in my system): (98-b) from German is true in the scenario in (98-a), despite Arnie and Beatrice having contradictory beliefs about Paul’s husband. (98-b) thus clearly differs from (95-b) in the scenario in (97) (the judgements for the German correlate of (95-b) are parallel to those reported for English by Pasternak). (98-b) and Pasternak’s cases are not completely parallel in terms of the question under discussion they address (wealth and origin vs. origin only), however, so I added (98-c), which, like (95-b), doesn’t contain a plural expression, but which is analogous to (98-b) in that both the noun and the modifier provide geographical information. (98-c) is false in the scenario in (98-a) and thus – just like (95-b) above – crucially differs from (98-b).

(98) a. scenario: Paul just got married and his cousins Arnie, Beatrice and Carl, who have never met, caught wind of it. Arniesuspects that Paul’s husband is from New York. Beatrice thinks he is from Maryland. Carl thinks he is from France. Paul tells me...

b. Arnie und Beatrice glauben, dass mein Mann ein New Yorker und aus Maryland ist, und der irre Carl denkt, ich hätte einen Franzosen geheiratet!

‘Arnie and Beatrice believe that my husband is from New York and from Maryland, and crazy Carl thinks I married someone from France!’

Nina Haslinger (pc)

But even cases with predicate modification and with compatible beliefs seem severely restricted. I.e. in the scenario in (99-a), which differs from (95-b) only in that Arnie is ascribe a slightly more specific belief and Beatrice’s ignorance is w.r.t. certain properties is made more explicit in the
scenario, the sentence in (99-b) is not judged true.

(99)  a. **scenario**: Paul just got married and his cousins Arnie and Beatrice, who have never met, just caught wind of it. Arnie is certain about one thing: That Paul married a man from France. Beatrice doesn’t know whether he married a man or a woman or where that person might be from, but is certain that this person will be poor. Paul tells me,,

    *Well, Arnie and Beatrice, they are kind of crazy...*

    Die **glauben, dass ich einen armen Mann geheiratet habe.**

    They believe that I a poor man married have

    ‘They believe I married a poor man.’

So even though (95-b) seems to run against my prediction that cases like (94) do not exist, the severe restrictions on such examples suggest that (94) should not straightforwardly follow from any theory of DDC-sentences. Although I do not know to explain the apparent exception in (95-b) I believe that the general pattern of the data supports my assumption that the availability of cumulative belief should be tied to the existence of plural expression inside the embedded clause.

### 5.3 The notion of distinctness

Having motivated the use of pluralities in my proposal, I want to conclude with an open problem that concerns the relation of the ‘atoms’ in these pluralities to one another – or, more specifically, the idea that these atoms must be ‘distinct’.

Recall that my analysis above assumes that in order for (100) to be true, the monsters that Ada and Bea believe to roam the castle must be somehow distinct.

(100)  **Ada and Bea believe that two monsters are roaming the castle.**

This assumption is motivated by the following contrast: Assume again a scenario where Ada and Bea spent the night at Roy’s castle, no monsters exists etc. If we chose the variant of the scenario in (101-a), Roy can truthfully utter (100) (this variant corresponds to our other scenarios). But, if we chose (101-b), Roy cannot truthfully utter (100). Intuitively, what goes wrong here is that we cannot be sure that we are actually dealing with different monsters, i.e. with two monsters rather than one.

(101)  **scenario**: Roy knows there are no monsters in his castle...

    a.  ... Ada tells Roy: There was zombie in my room. Bea tells Roy: There was griffin in my room.

    b.  .... Ada tells Roy: There was monster in my room. Bea tells Roy: There was monster in my room.

While this intuition seems robust, it is unclear how to best model it. In my treatment above, I followed [Condoravdi et al. (2001)](condoravdi2001) in using cross-world overlap: Informally, I assumed that two

20Such examples become even worse if the DP is headed by a quantifier: Although the logical conjunction of Ada’s and Bea’s belief in (i-a) entails the embedded proposition in (i-b), the sentence is not judged true in this scenario.

(i)  a. **scenario**: Ada and Bea, who don’t know each other, are invited to a party. Ada doesn’t feel like going – she is certain no tall woman will attend and she’s trying to hook up with a tall woman. Bea is undecided about going: She believes there will be rich people at the party (she doesn’t have any opinion about their gender), which she likes. But she believes that all rich people are tall, which makes her uneasy about being so short.

    b.  **Ada und Bea glauben, dass keine reiche Frau zur Party kommen wird.**

    Ada and Bea believe that no rich woman will to-the party come will.

    ‘Ada and Bea believe that no rich woman will come to the party.’

21The content of this section has profited greatly from joint work with Nina Haslinger.
monsters denotes a set of pluralities \( f + g \) of individual concepts of monsters that are such that their atoms don’t overlap. This mean that for every world where both \( f \) and \( g \) are defined, their values will differ. The problem is, that this notion of distinctness is actually too weak.

More specifically, the issue is the following: We just saw in section 5.1 that the beliefs of members of the subject plurality (i.e Ada and Bea in (100)) can be incompatible - which means that their sets of belief words don’t overlap. Take for instance the variations of our ‘usual’ scenario in (102): In (102), Ada’s and Bea’s beliefs are incompatible. We obtain the same judgements as in (101): We can truthfully utter (100) in the scenario in (102-a), but not in the scenario in (102-b). However, I actually don’t predict that (100) is out in the scenario in (102-b). As I work with individual concepts as partial functions, and Ada’s and Bea’s belief worlds are distinct, I can always find an two individual concepts that are ‘monster’-concepts and that are distinct in the sense above – for instance, a concept \( f \) that is defined only in Ada’s belief worlds, and another concept \( g \) that is defined only in Bea’s belief worlds.

(102) \textsc{scenario}: Ada and Bea are spending the night at Roy’s castle. Ada believes in zombies, but she believes that griffins don’t exist. Bea also believes in griffins, but she believes that zombies don’t exist...

a. Ada thinks that zombie is roaming the castle. Bea thinks that a griffin is roaming the castle.

b. Ada thinks that a monster is roaming the castle. Bea thinks that a monster is roaming the castle.

I don’t have a solution for this problem at this point (but see Haslinger & Schmitt [2019b] for an idea of the general direction any solution should take). Clearly, we cannot give up the assumption that the individual concepts are partial functions – there simply aren’t monsters in every possible world. But could we make use of Pasternak’s [2018b] approach and ‘remove’ those propositions from Ada’s and Bea’s beliefs that make their beliefs incompatible? While this might help us with (102) (provided we can find mechanism that tells us which propositions we may remove), it is not a general solution: As we show in Haslinger & Schmitt [2019b], whether or not two individual concepts overlap in their domains (i.e. whether or not there is a world where they are both defined) doesn’t seem to matter for our intuitive notion of ‘distinctness’. In other words, we find cases where two individual concepts that apparently have non-overlapping domains, count as distinct. This is exemplified in (103): (103-b) can be true in the scenario in (103-a) even though we seem to be dealing with two individual concepts that will never be defined in the same world: the king of France and the last remaining monarch and the king of Austria and the last remaining monarch (the context makes sure that we are not dealing with kings of both France and Austria).

(103) a. \textsc{scenario}: Ada and Bea, who don’t know each other, are monarchists with crude world views. Ada believes that there is a king of France and that France is the only monarchy in the world. Bea believes that there is a king of Austria and that he is the last existing king in Europe. Both Ada and Bea are invited to Roy’s wedding. Roy is a German aristocrat, so Ada and Bea are very excited: Ada believes she will see the French king at the wedding. Bea is certain that the Austria king will be present. Roy tells me: \textit{Well, Ada and Bea, they have such high hopes ...}

\begin{itemize}
  \item \textit{Die glauben tatsächlich, dass zwei Könige zu meiner Hochzeit kommen werden – They actually believe that two kings to my wedding come will – dabei kommen doch nur Proletarier!}
  \item but come \textit{je} only proletarians!
  \item ‘They actually believe that two kings will come to my wedding – but in fact there will only be proletarians coming!’
\end{itemize}

adapted from Haslinger & Schmitt (2019b)
Even though I cannot resolve this issue here, I don’t think it shows we must discard the proposal I gave above – because I don’t think it is an idiosyncratic property of my account. As we argue in (Haslinger & Schmitt 2019b), it seems to be a very general problem, both in terms of potential analyses of DDC-sentences, as well as in terms of the empirical phenomena that might give rise to it in their analyses.

Concerning the first point, we would have to answer the very same questions if we employed the variant of the predicate analysis that I sketched in section 3.1. Here, the treatment of (100) would mean cumulation of a relation between the denotations of Ada and Bea and two monsters. Since two monsters would have to involve reference to individual concepts (or objects of thought etc.) in such a proposal, we would face the same question: When do two individual concepts (objects of thought etc.) count as ‘sufficiently distinct’? And even though Pasternak’s (2018b) proposal does not appeal to individual concepts, but rather to collective belief (see section 5.1 above), the problem would enter through the back door in this type of analysis as well: Which propositions (about which kind of monster) do Ada and Bea each have to believe so that the logical conjunction of their beliefs entails that two monsters are roaming the castle?

Regarding the second point – the observation that the problem is not limited to the analysis of DDC-sentences – the cases brought up by Condoravdi et al. (2001), i.e. the specific readings of sentences like (104), repeated from (27) above, will also face the same obstacle, if indeed they involve counting of individual concepts (see section 3.1).

22 Condoravdi et al. (2001) don’t address this problem, as they treat individual concepts as total functions.

(104) Negotiations prevented three strikes. (Condoravdi et al. 2001(2))

Furthermore, the problem of ‘distinctness’ is actually quite well-known one when considered from a reverse angle: The question of when two objects that are ‘hidden’ within the beliefs of different subjects count as identical has been thoroughly discussed in the literature on Hob-Nob cases (see Cumming 2014 for an overview): We saw above that the sentence in (105-b) can be true in the scenario in (105-a) (the example is repeated from (26) in section 3.1 above). Of course, we have the intuition that Hob’s belief and Nob’s belief are somehow about the same object, but how exactly do we define this similarity? As far as I can see, no satisfactory solution to this problem has been proposed, or at least none that could grant us further insight into what is going on in DDC-sentences.

(105) a. scenario The Gotham City newspaper reported that a witch, referred to as ‘Samantha’, has been on a rampage. According to the article, she has been blighting farm animals. In reality, there is no such person: the animals all died of natural causes. The reporters simply assumed that a witch was responsible for the mishaps, and dubbed her ‘Samantha’. Hob and Nob both read the Gotham Star and believe the stories about the witch. Hob thinks Samantha must have blighted Bob’s mare, which took ill yesterday. Nob thinks Samantha killed his friend Cob’s sow. adapted from (Edelberg 1986:2)

b. Hob thinks a witch blighted Bob’s mare, and Nob thinks she killed Cob’s sow. (Edelberg 1986:1(1)), adapted from (Geach 1967:628(3))

5.4 Interim Summary

In this section, I tried to motivate the idea that the analysis of DDC-sentences must appeal to pluralities of propositions and to cumulative belief – two ideas that set it apart from the only other existing treatment of DDC-sentences, Pasternak’s (2018b) account in terms of collective belief. I first argued that we need pluralities of propositions as ‘objects’ of belief independently of DDC-sentences and that we seem to get simple cumulative inferences based on these pluralities with attitude verbs. I then argued that the prediction of my proposal – namely, that cumulative belief is only possible if the
embedded clauses denotes a plurality of propositions (or rather, a plural set thereof) and thus if the embedded clause contains a ‘plurality-inducing’ expression – is for the most part correct. As a final point, I addressed the problem of distinctness, which I claimed to be of a very general nature: When do ‘objects’ that individuals have beliefs about count as sufficiently distinct that we can count or pluralize them? I have no answer to this question here, so I must leave this matter to future research.

6 Conclusion and open questions

In this paper, I investigate cumulative readings of sentences like (106) in which some, but not all of the plural expressions have a *de dicto* reading.

(106) *Ada and Bea believe that two monsters are roaming the castle.*

My first point was to show that such examples are problematic for standard treatments of cumulativity, i.e. different versions of analyses, where we either cumulate relations between pluralities of individuals (i.e. the denotations of *Ada and Bea* and *two monsters* in (106)) or thematic relations: Analyses that cumulate relations between individuals will have to interpret *two monsters* outside of the scope of *believe*, in order to derive an adequate input relation for cumulation. They therefore must assume a ‘non-standard’ meaning for the indefinite in order to derive the *de dicto*-reading, but even with this assumption they are not sufficiently general: They fail to derive the correct truth-conditions for DDC-sentences involving ‘flattening’-configurations (Schmitt 2018, Haslinger & Schmitt 2018b), where one plurality-denoting expression is embedded in another one. Analyses that cumulate thematic relations – relations between events and individuals – do not fare better, because they cannot deal with cases where the ‘chain of events’, so to speak, is interrupted by an attitude predicate. I then provided an alternative analysis in terms of the plural projection framework formulated by (Schmitt 2018, Haslinger & Schmitt 2018b), where cumulativity is encoded in the semantic composition: In this system, all semantic domains contain pluralities and embedded pluralities ‘project’ to the denotations of nodes dominating them via a rule encoding cumulativity which appeals to ‘plural sets’ – sets of pluralities associated with each type. I extended this system to cover DDC-sentences by assuming that *two monsters* denotes a plural set of pluralities of individual concepts (or actually, a plural set of pluralities of quantifiers, based on such pluralities of individual concepts), i.e. something roughly equivalent to (107):

(107) \[ f + g : \text{both } f \text{ and } g \text{ yield a monster in every world where they are defined & there is no world } w \text{ where both are defined where } f(w) = g(w) \]

This plurality then projects to a plural set of pluralities of propositions, schematized in (108). The sentence will end up being true if Ada and Bea cumulatively believe at least one plurality in this set.

(108) \[ \text{that a griffin is roaming the castle} + \text{that a zombie is roaming the castle, that a werewolf is roaming the castle} + \text{that a zombie is roaming the castle, . . .} \]

I provided independent motivaton for this proposal by showing that we require cumulative belief of pluralities of propositions independently of DDC-sentences and that the proposal correctly predicts – with one exception – that cumulative belief should only be licensed if the embedded clause denotes a plurality, which means it must contain a plural expression. I argued that these points set the proposal apart from the account in (Pasternak 2018a,b), who essentially assumes collective belief of a proposition \( p \) can be ascribed to a plurality \( a + b \) as long as \( p \) follows from the logical conjunction of \( a \)’s and \( b \)’s beliefs.

There are, of course, a number of open question, most of which comprise both empirical and theoretical aspects. I already addressed the one I find most perplexing – the question of when ‘objects’ of
belief count as distinct – above, so I will use this opportunity to raise a number of other issues (but I
leave technical matters aside).
The first one relates to a point already discussed in section 5.2 above: How do we account for
Pasternak’s case involving predicate modification – and why this case should be so different from
the other ones I discussed in section 5.2? I actually have no answer to this question at this point, but
the way to go is probably to start by considering more examples (including more types of syntactic
configurations and more contexts).
The second question relates to my use of the plural projection mechanism: In (Haslinger & Schmitt
2018b, 2019a, 2018a) we argue that cumulative readings of ‘asymmetrically distributive universals’
like English every or German sowohl A als auch B (‘A as well as B’), but also of modified numerals
are best modelled via this particular system. At this point, I don’t know how such elements interact
with the phenomenon considered here (e.g. can we also get cumulative readings for every-DP in
DDC-configurations and what exactly is the meaning of the resulting sentence), but investigating this
interaction could help us work out the mechanism in greater detail.
This second question is closely linked to a third one, which concerns the scope of the phenomenon:
In section 2, I showed that DDC-sentences are not limited to complement clauses containing (unmod-
ified) numeral DPs. Do plural expressions other than the ones I discussed in this paper (unmodified
numeral DPs, definite plurals, conjunctions) give rise to the same phenomenon? In the same vein, we
could of course also wonder about which predicates allow for DDC-readings. I showed in section 2
that at least in German the phenomenon is wide-spread with attitude verbs, but I have not consistently
investigated whether all classes of attitude verbs (e.g. factive vs non-factive) behave alike or whether
the same phenomenon can also be found with embedded interrogatives (which is where Beck &
Sharvit’s 2002 work on the ‘plurality-like’ behavior of embedded questions could prove relevant).

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