Cumulativity and intensional interveners

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FWF Der Wissenschaftsfonds.

Today's talk

1. Present problem for standard view of cumulativity: Intensional interveners

   (1) *The two girls fed two dogs.*
   (2) *The two girls* **believed** *that two monsters were roaming the castle.*

2. Briefly motivate a notion of 'parthood' for propositions
   - Cumulative belief w.r.t. pluralities of propositions
   - To get to their 'parts', we must recur to the embedded plural expression

   (3) { that one vampire was roaming the c.+ that one zombie was roaming the c.,
   that one griffin was roaming the c.+ that one zombie was roaming the c., . . . }

3. Propose an analysis in terms of 'plural projection'

   Schmitt (2017), Haslinger and Schmitt (2018)

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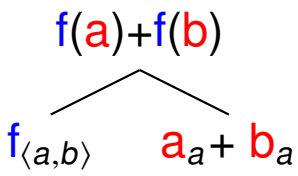

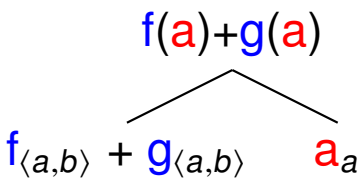

Cumulativity and its standard analysis

Sentences with plural DPs: ‘adding up’ of properties (cumulative inferences):

Link (1983) a.o.

(5) a. *Ada fed Carl. Bea fed Dido.*
b. *Ada and Bea fed Carl and Dido.*
c. *The two girls fed the two dogs.*

(6) Each atomic part of A+B fed at least one atomic part of C+D &
each atomic part of C+D was fed by at least atomic part of A+B

Cumulativity for sentences with two plural DPs derived by operation ** on predicate extensions

Krifka 1986, Sternefeld 1998 a.o.

(7) $^{**}P_{\langle e\langle et\rangle\rangle}$: smallest function f s.th. for all $x, y \in D_e$, if $P(x)(y) = 1$, then $f(x)(y) = 1$ and for all $S, S' \subseteq D_e$, s.th for every $x' \in S$ there is an $y' \in S'$ and $f(x')(y') = 1$ and for every $y' \in S'$ there is an $x' \in S$ and $f(x')(y') = 1$, $f(+(S))(+(S')) = 1$.

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Expansion to non-lexical predicates

(9) *The two girls wanted to feed the two dogs.*

adapted from (Beck and Sauerland 2000)

(10) SCENARIO: Ada wanted to feed Carl. Bea wanted to feed Dido.

(11) $\lambda x_e.\lambda y_e.y$ wanted to feed x required input for **

Syntactic derivation of required predicate (Beck and Sauerland 2000)

(12) [[*the two girls*] [[*the two dogs*] ** $\underbrace{[1[2[t_2 \textit{ wanted to feed } t_1]]]}_{\lambda x_e.\lambda y_e.y \text{ wanted to feed } x}$]]

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- Cumulativity is the result of the operation ** on predicate extensions.
- In sentences with two plural expressions A, B, we will only find cumulativity if we find a binary predicate **P** that takes A, B as its arguments and can form the input to **.

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A problem

Examples here in German – analogous judgments found in English (modulo speaker variation)

(13) SCENARIO + CONTEXT Ada and Bea spent the night at Gene's castle. Ada believes in griffins and Bea in zombies. Around midnight, Ada heard a sound in her bedroom and was certain that it was caused by a single griffin going crazy. A little later, Bea heard a sound in her bedroom, and took it to be caused by a lone zombie, crying for help. In the morning, they each took Gene aside and told him what they believed was going on at his castle. Later, Gene tells me:
Well, I had invited Ada and Bea to spend the night at the castle. Bad idea! I know, of course, that people find it a little spooky here, but guess what...

(14) die haben echt geglaubt, dass da Greife und Zombies unterwegs waren!
DEM have indeed believed that there griffins and zombies roaming were
'They really believed that griffins and zombies were roaming the castle!' (German)
true in scenario (13)

(15) die haben echt geglaubt, dass da zwei Monster unterwegs waren!
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Description of the problem

(16) *Ada und Bea haben echt geglaubt, dass da zwei Monster unterwegs waren!*
'Ada and Bea really believed that four monsters were roaming the castle!'

- This looks like cumulativity: Neither of the following holds

  (17) a. Ada believes that two monsters were roaming the castle.
  b. Bea believes that two monsters were roaming the castle

- The belief is *de dicto* (there are no monsters – and no 'indirect' *res*)
- Deriving cumulativity via standard analysis: Cumulate the relation in (18)

  (18) $\lambda x_e.\lambda y_e.y$ believes that x was roaming the castle

- Hence we need an LF like (19), where zwei Monster outscopes BELIEVE

  (19) [[A und B] [[zwei M] ** $\underbrace{[1\,[\,2\,[\,t_2\ \textit{glauben}\,[\,\textit{dass}\ t_1\ \textit{unterwegs waren}]]]]]}_{\lambda x_e.\lambda y_e.y \text{ believe that } x \text{ was roaming the castle}}$

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Two analytical options

1. direct analysis: cumulative relation between A+B and ⟦two monsters⟧

(20) a. Ada and Bea believed that two monsters were roaming the castle.
b. $\lambda x_e.\lambda y_e.y$ believes that x was roaming the castle

- Q1: How do we dissociate wide-scope indefinite from existential import?
  cf. e.g. *Hob-Nob* problem Geach (1967), Edelberg (1986))
- Q2: How do we generalize this analysis to other kinds of cumulative belief?
  Schmitt (2013, 2017)

(21) *They believed [$_p$ [$_q$ that a griffin was going berserk] and [$_r$ (that) a zombie was crying for help]], but neither of them took into account that there might simply be something wrong with the heating!*

2. indirect analysis: cumulative relation between A+B and embedded proposition p.

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Rationale

- We need a 'part-whole' relation among propositions to apply existing theories of cumulativity

(23) A plurality **a** of individuals cumulatively believes **p** iff there is a set $S_{\mathbf{p}}$ of parts of **p** such that:

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b. Every atomic part of **a** believes some part of **p** and every part of **p** is believed by an atomic part of **a**.

- Not obvious how to define parthood in terms of entailment or logical compatibility
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(26) a. $[\![(26)]\!] = 1$ iff there is a proposition **q** believed by Ada and a proposition **r** believed by Bea and **q** and **r** jointly (contextually) entail **p**.

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This correctly predicts the cumulative inferences discussed above . . . but it is too weak!

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This predicts that Ada and Bea cumulatively believe any nonempty $\mathbf{p}$ that is jointly entailed by Ada's beliefs and Bea's beliefs.

Semantic plurality should be irrelevant. This doesn't seem correct (Lucas Champollion, p.c.):

(29) a. *Ada believes that there are no criminals in Vienna.*
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c. $\not\Rightarrow$ *Ada and Bea believe that there are no archeologists in Vienna.*

Generalization: Cumulative readings of attitude verbs are only available if the complement clause contains a semantically plural expression.

Note: conjunctions of any type (including propositions) count as semantically plural expressions

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Intuition

- The embedded proposition only has non-trivial ‘parts’ in the sense relevant here if the embedded clause contains a semantically plural expression.
- The part structure of the plural expression will ‘project’, in a sense, to the embedded proposition.
- We introduce pluralities of propositions (cf. Schmitt 2013, 2017 for additional motivation). They correspond to nonempty sets of propositions.
- Embedded clause denotes a set of pluralities of propositions. Very informally speaking, for our example, we want something like:

(30) $\{\lambda w$.a monster with property P is roaming the castle in $w + \lambda w$.a monster with property Q is roaming the castle in $w \mid P$ and Q are properties and $P(w') \cap Q(w') = \emptyset$ in all relevant belief-worlds $w'\}$

- (31) must somehow be able to denote a set of this kind (possibly in addition to other readings)

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- The semantic domain D_a of type a contains partial functions from possible worlds to extensions of type a, and pluralities thereof. Pluralities correspond to nonempty sets of atomic domain elements.
  - D_e contains partial individual concepts and pluralities thereof
  - D_t contains partial propositions and pluralities thereof ...
- For any type, an operation + maps a set $S \subseteq D_a$ to its sum, a plurality in D_a.

(32) toy examples

a. $D_e = \{\lambda w.\textbf{Ada}, \lambda w.\textbf{Bea}, \lambda w.\text{the mayor of Vienna in } w, \lambda w.\textbf{Ada}+\lambda w.\textbf{Bea}, \lambda w.\textbf{Ada}+\lambda w.\text{the mayor of Vienna in } w \ldots\}$

b. $D_{\langle e,t\rangle} = \{\lambda w.\lambda x.\textbf{smoke}(w)(x), \lambda w.\lambda x.\textbf{dance}(w)(x), \lambda w.\lambda x.\textbf{smoke}(w)(x) + \lambda w.\lambda x.\textbf{dance}(w)(x) \ldots\}$

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- For every type a, we also have a type a^* of 'plural sets' with elements from D_a – i.e. plural sets contain pluralities of intensions of type a.
- The domain D_{a^*} is disjoint from $\wp(D_a)$, but has the same algebraic structure.

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Cumulative composition (1/3)

Informally, a cover of (P, x) is a relation between atomic parts of P and atomic parts of x in which each atomic part of P and each atomic part of x occurs at least once.

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Intuition: Cumulative truth conditions wrt. pluralities P, x and relation $R \approx$ there is some cover such that for each pair (P', x') in the cover, $R(P', x')$ holds.

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- We define an operation $\mathcal{C}$ that encodes plural projection and cumulation.
- It takes two plural sets P^* and x^* and gives us another plural set.
- Essentially, we take all covers of some plurality from P^* and some plurality from x^*.
- If P^* is of type $\langle a, b\rangle^*$ and x of type a^*, then for each cover R, we take the sum of the set $\{\lambda w.P(w)(x(w)) \mid (P, x) \in R\}$ ($\approx$ extensional functional application)
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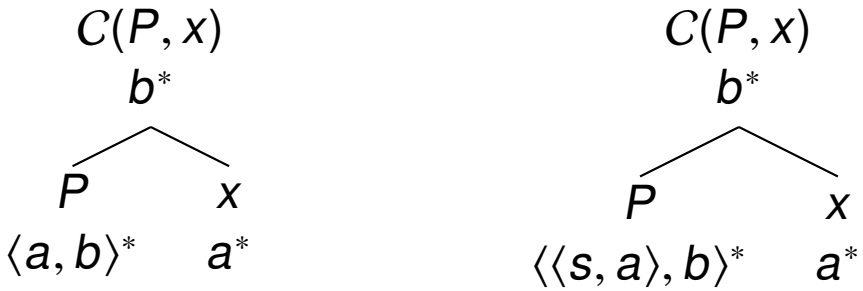
- It takes two plural sets P^* and x^* and gives us another plural set.
- Essentially, we take all covers of some plurality from P^* and some plurality from x^*.
- If P^* is of type $\langle a, b\rangle^*$ and x of type a^*, then for each cover R, we take the sum of the set $\{\lambda w.P(w)(x(w)) \mid (P, x) \in R\}$ ($\approx$ extensional functional application)
- If P^* is of type $\langle\langle s, a\rangle, b\rangle^*$ and x of type a^*, then for each cover R, we take the sum of the set $\{\lambda w.P(w)(x) \mid (P, x) \in R\}$ ($\approx$ intensional functional application)

Cumulative composition (3/3)

The operation C is available as a compositional rule in addition to functional application.

(36)



A plural set S of propositions is true in a world w iff S contains at least one element p, s.th. all atomic parts p' of p are true in w.

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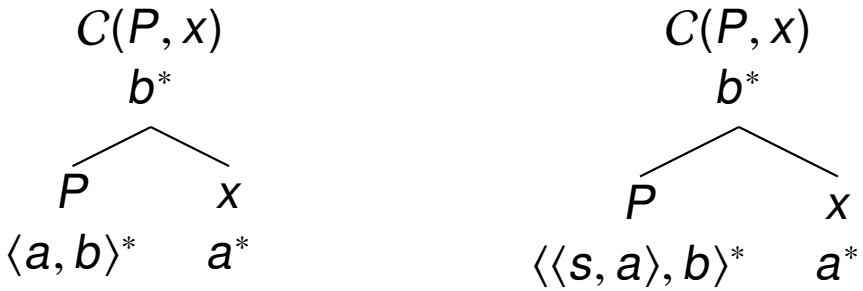

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Example

(37) Ada and Bea fed Carl and Dido.

a. $[\![\text{Carl and Dido}]\!] = [\lambda w.\mathbf{C} + \lambda w.\mathbf{D}]$

b. $[\![\text{Carl and Dido}]\!] = \mathcal{C}([\lambda w.\lambda y.\lambda x.\mathbf{fed}(w)(y)(x)], [\![\text{Carl and Dido}]\!]) =$
$\mathcal{C}([\lambda w.\lambda y.\lambda x.\mathbf{fed}(w)(y)(x)], [\lambda w.\mathbf{C} + \lambda w.\mathbf{D}]) =$
$[\lambda w.\lambda x.\mathbf{fed}(w)(\mathbf{C})(x) + \lambda w.\lambda x.\mathbf{fed}(w)(\mathbf{D})(x)]$

c. $[\![\text{Ada and Bea fed Carl and Dido}]\!] =$
$\mathcal{C}([\![\text{fed Carl and Dido}]\!], [\lambda w.\mathbf{Ada} + \lambda w.\mathbf{Bea}])$
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Cumulative belief

(38) *They believed [$_p$ [$_q$ that a griffin was going berserk] and [$_r$ (that) a zombie was crying for help]], but neither of them took into account that there might simply be something wrong with the heating!*

The basic idea (I ignore the semantics of indefinites here – actually they introduce plural sets):

(39) a. $[\![\text{p}]\!] =$
$[\lambda w.\text{a griffin was going berserk in } w + \lambda w.\text{a zombie was crying for help in } w]$

b. $[\![\text{believe p}]\!] = C([\lambda w.\lambda p_{\langle s,t\rangle}.\lambda x.\mathbf{believe}(w)(p)(x)], [\![\text{p}]\!])$
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Indefinite plurals

- We consider individual concepts f such that for every world w in which f is defined, $f(w)$ is a monster in w.
- *Two monsters* will denote a set of pluralities of such concepts.
- The individual concepts have to be partial since not every world contains monsters.
- $[\![$two monsters$]\!]$ will only contain pluralities $f + g$ such that in every world w in which both f and g are defined, $f(w) \neq g(w)$. So in every world in which they are both defined, two distinct monsters exist.
- (40) a. LF: [two [F monster]]

  b. $[\![\text{F}]\!] = \lambda w.\lambda P_{\langle s,et\rangle}.[f^* \mid f^*$ is a plurality of partial individual concepts
  $\wedge \forall f \leq_{AT} f^*.\forall w(w \in dom(f) \rightarrow P(w)(f(w)))$
  $\wedge \forall f, g \leq_{AT} f^*.\forall w(w \in dom(f) \wedge w \in dom(g) \rightarrow f(w) \neq g(w))]$

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Back to our example (1/3)

(41) *The two girls* **believed** *[p that two monsters were roaming the castle]*

(42) a. $[\![\text{two monsters}]\!] = [f + g \mid f, g$ partial individual concepts
$\wedge \forall w(w \in dom(f) \rightarrow \textbf{monster}(w)(f(w))) \wedge \forall w(w \in dom(g) \rightarrow$
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b. $[\![\text{p}]\!] = [\lambda w.\textbf{roam-the-castle}(w)(f(w)) +$
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Back to our example (2/3)

(43) *The two girls* **believed** *[p that two monsters were roaming the castle]*

(44) SCENARIO: Ada believes a griffin was roaming the castle. Bea believes a zombie was roaming the castle. (Ada and Bea believe one can't be a zombie and a griffin at the same time.)

- We can find an individual concept f that maps each w among Ada's belief-worlds to a griffin roaming the castle in w.
- Similarly there is an individual concept g that maps each w among Bea's belief-worlds to a zombie roaming the castle in w.
- For each w that is both in Ada's and in Bea's belief worlds, $f(w)$ and $g(w)$ are distinct. For the other worlds, we can always choose f and g such that their values are distinct.
- Ada believes $\lambda w.$**roam-the-castle**$(w)(f(w))$. Bea believes $\lambda w.$**roam-the-castle**$(w)(g(w))$

Prediction: (43) true in scenario (44).

Back to our example (3/3)

(45) *The two girls* **believed** *[p that two monsters were roaming the castle]*

(46) SCENARIO: Ada believes a griffin was roaming the castle. Bea believes monsters don't exist.

- We can find an individual concept f that maps each w among Ada's belief-worlds to a griffin roaming the castle in w.
- But we can't find an individual concept g that maps any w among Bea's belief worlds to a monster in w.
- Ada believes $\lambda w.$**roam-the-castle**$(w)(f(w))$. But there is no g satisfying the conditions imposed by ⟦two monsters⟧ for which Bea believes $\lambda w.$**roam-the-castle**$(w)(g(w))$.
- Every partial individual concept g that only yields monsters as values will be undefined in Bea's belief worlds.
- So we can't find a plurality in ⟦p⟧ that Ada and Bea cumulatively believe.

Prediction: (45) not true in scenario (46).

Problems: Undefinedness (1/3)

We will now briefly look at two problems for the proposed analysis.

(47) *The two girls* **believed** *[p that two monsters were roaming the castle]*

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- In this scenario, for any plurality $f + g$ of individual concepts in ⟦two monsters⟧, both f and g will be undefined in Bea's belief worlds.
- So, under standard assumptions about presupposition projection, we predict that for any plurality $p + q$ in ⟦two monsters were roaming the castle⟧, both f and g are neither true nor false in Bea's belief worlds.
- But intuitively (47) is not a presupposition failure in the given scenario. It is simply false.
- Problem: In cases where the domains of the individual concepts introduced by indefinites are 'too small' – i.e. don't include all the relevant belief worlds – we don't want to predict a presupposition failure.
- This shouldn't be solved by stipulating that *x believes p* is always false if *x* has some belief-worlds in which the presupposition of *p* is not met. We want to allow for some cases of presupposition projection.

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Problems: Undefinedness (2/3)

(49) *The two girls* **believed** *[p that two monsters were roaming the castle]*

(50) SCENARIO: Ada believes a griffin was roaming the castle. Bea believes monsters don't exist.

- Potential solution: Instead of pluralities of partial individual concepts, the indefinites introduce pluralities of quantifiers of the following form:

  (51) $\lambda w.\lambda P_{\langle e,t\rangle}.w \in dom(f) \wedge P(f(w))$ for some partial individual concept f

- (51) is defined for every world w and every predicate P unless P introduces a presupposition of its own.
- Assume that the value of f is a monster in every world where f is defined. Then in a world w where monsters don't exist, $w \in dom(f) \wedge$ **roam-the-castle**$(f(w))$ is false.

Problems: Undefinedness (2/3)

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- Assume that the value of f is a monster in every world where f is defined. Then in a world w where monsters don't exist, $w \in dom(f) \wedge$ **roam-the-castle**$(f(w))$ is false.

Problems: Undefinedness (2/3)

(49) *The two girls* **believed** *[p that two monsters were roaming the castle]*

(50) SCENARIO: Ada believes a griffin was roaming the castle. Bea believes monsters don't exist.

- Potential solution: Instead of pluralities of partial individual concepts, the indefinites introduce pluralities of quantifiers of the following form:

  (51) $\lambda w.\lambda P_{\langle e,t\rangle}.w \in dom(f) \wedge P(f(w))$ for some partial individual concept f

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Problems: Undefinedness (3/3)

(52) *The two girls* **believed** *[p that two monsters were roaming the castle]*

(53) SCENARIO: Ada believes a griffin was roaming the castle. Bea believes monsters don't exist.

- Assume that the value of f is a monster in every world where f is defined. Then in a world w where monsters don't exist, $w \in dom(f) \wedge$ **roam-the-castle**$(f(w))$ is false.
- So we can predict that (57) is false in scenario (53), given a definition of the following sort:

(54) a. A plurality p in D_t is true in a world w iff $p'(w) = 1$ for each $p' \leq_{AT} p$.
b. A plurality p in D_t is false in a world w iff $w \in dom(p')$ for each $p' \leq_{AT} p$, and $p'(w) = 0$ for at least one $p' \leq_{AT} p$.
c. A plural set p^* in D_{t^*} is true iff at least one of its elements is true, and false iff all of its elements are false.

- Whether the falsity conditions in (54) are too strong, and how to integrate cumulativity with presupposition projection in general, is a matter for future research.

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Problems: Contradictory beliefs

A second problem, which will be left open here:

(55) *Die zwei Mädchen* ***glauben,*** [p *dass zwei Zombies im Schloss unterwegs waren].*
the two girls believe that two zombies in.the castle on.the.way were
'The two girls believe that two zombies were roaming the castle.' (German)

(56) SCENARIO: Ada and Bea both believe a single zombie is roaming the castle. Ada believes it is raining in Vienna today. Bea believes it is not raining in Vienna today.

Since the set of Ada's belief worlds is disjoint from the set of Bea's belief worlds, it's always possible to find two distinct partial individual concepts $f + g$ such that:

- $f(w)$ is a monster in any world w where f is defined; $g(w)$ is a monster in any world w where g is defined
- The domains of f and g are disjoint, so there is no world in which both f and g yield the same monster

So (55) should be true in this scenario, but intuitively this doesn't seem correct, because nothing in the context suggests that the zombies in Ada's and Bea's respective belief worlds have distinct properties in a relevant sense.

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(58) SCENARIO: Ada and Bea both believe a single zombie is roaming the castle. Ada believes it is raining in Vienna today. Bea believes it is not raining in Vienna today.

Several potential approaches to explore in future work:

- Only consider beliefs that are in some sense 'relevant' (cf. explicit higher order quantifiers – don't range over arbitrary propositions (Zimmermann 2006))

  (59) Ada believes something that Bea (also) believes.

- Stricter conditions on the pluralities of propositions that are considered when interpreting attitude verbs.

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Conclusion

- Semantically plural expressions in the scope of an attitude verb like *believe* allow for cumulativity with plurals in the matrix clause.
- The embedded plural can be read *de dicto*, which poses a problem for existing analyses of cumulativity.
- We suggested that this problem can be solved using a notion of parthood among propositions, which is formalized in terms of pluralities of propositions.
- The plural projection approach can be extended to account for such examples if we assume that plural indefinites can denote pluralities of partial individual concepts.

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Appendix 1: Examples with embedded plurals

More examples

(60) SCENARIO: Prepper Mike has been hiding in a bunker for the past couple of days. Noone knows where he is, but many people have crazy theories: Ada thinks that Mike was eaten by a zombie. Bea is certain that a mermaid fetched him and drowned him in the sea. Gene believes that he shape-shifted into a crocodile and is now secretly running everything. Harry tells the newspaper:
Well, Mike went missing – and my friends Ada, Bea and Gene all have absurd theories about this – but Gene certainly is the worst.

(61) *While Ada and Bea believe [$_p$ [$_q$ that Mike was eaten by a zombie] and [$_r$ (that he was) drowned in the sea by a mermaid]], Gene actually thinks that he turned into a crocodile and became the secret world leader.* true in scenario (60)

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