Asymmetries in cumulative construals of non-referential DPs

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1 In a nutshell

Point of departure
Cumulative readings of sentences with (modified) numerals and plural universals:

(1) Exactly three volunteers (between them) contacted all registered voters.

Observations regarding German

(i) We sometimes find two types of ‘non-maximal’ readings for (modified) numerals and plural universals (see Schmitt [2015] for German, Buccola and Spector [2016] for English)

(2) Upward non-maximality
a. In meinem Bezirk haben gerade mal drei Aktivisten alle Wähler
   in my district have no more than three activists all voters
   kontaktiert.
   contacted
   ‘In my district, no more than three activists contacted all voters.’
   b. SCENARIO: There are 100 voters. Activist Ada contacted 30 voters, activist Bea another 30. Carl contacted the remaining 40. Dean also contacted some voters.
      (2-a) true

(3) Downward non-maximality
a. In meinem Bezirk haben alle Aktivisten gerade mal zehn Wähler
   in my district have all activists no more than ten voters
   kontaktiert
   contacted
   ‘In my district, all the activists contacted no more than 10 voters.’
   b. SCENARIO: There are 100 activists. 10 of them each contacted a different voter. The rest didn’t contact anyone.
      (3-a) true

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1 Landman (2000) notes examples of what we call downward non-maximal readings in English, but describes them differently.
(ii) Non-maximal readings depend on the scope of the DP in question: Non-maximality only for DPs with plurals/degree expressions in their scope

(iii) Non-maximal readings also depend on pragmatic factors: Non-maximality only if pluralities introduced by DP are surprisingly small/big relative to the predicate

<table>
<thead>
<tr>
<th>Claims</th>
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<tr>
<td>(i) Scope asymmetries and pragmatic restrictions in the distribution of non-maximality provide further evidence for an asymmetric approach to cumulativity: Plural projection (Schmitt 2017, Haslinger and Schmitt t.a.)</td>
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<td>(ii) Both non-maximal readings can be derived from maximal readings without lexical ambiguity by assuming that plural expressions introduce two-layered meanings</td>
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<td>(iii) The pragmatic restrictions make reference to the predicate that combines with a potentially non-maximal DP, so plural projection correctly predicts that scope should influence availability of non-maximal readings.</td>
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2 Symmetric account of cumulativity: Predicate analysis


- Cumulative truth conditions in sentences with two or more plural expressions:

  (4) a. Ada and Bea wanted to feed Carl and Dean.
      b. SCENARIO: Ada wanted to feed Carl, Bea wanted to feed Dean.
      c. true iff: A and B each wanted to feed C or D & for each of C and D, A or B wanted to feed him.

- We get to these truth-conditions by cumulating the relation between the pluralities:

  (5) predicate P: \{\langle a, c \rangle, \langle b, d \rangle \}  
      cumulated predicate **P: \{\langle a, c \rangle, \langle b, d \rangle, \langle a \oplus b, c \oplus d \rangle \}

  (6) Ada and Bea wanted to feed Carl and Dean.  
      ⇒ cumulate \( \lambda x.\lambda y.x \text{ wanted to feed } y \)

- Cumulativity between \( n \) different pluralities, requires an \( n \)-ary relation as input for cumulation.

Predicate analysis is symmetric: Scope asymmetries are neutralized.
3 Non-maximality

The distribution of non-maximality exhibits asymmetries. This seems at odds with the symmetric account of cumulativity.

Two variants of non-maximality, upward and downward. In each case, we want to know:

- how do we characterize the phenomenon?
- what are the syntactic/semantic restrictions on when it occurs?
- what are the pragmatic restrictions on when it occurs?

3.1 Upward non-maximality

3.1.1 Characterizing upward non-maximality

- Upward non-maximality: Only observable with DPs that usually introduce an upper bound.

(7) genau vier Buben (‘exactly four boys’), gerade mal vier Buben (‘no more than four boys’), höchstens vier Buben (‘at most four boys’), weniger als vier Buben (‘less than four boys’), ...

- When a DP introducing an upper bound combines with a predicate $P$, the resulting interpretation usually imposes the following two requirements (a.o.):
  
  There is a plurality $x$ in the NP denotation such that $x$ satisfies $P$ and
  
  (i) **basic condition**: $x$ is smaller than or equal to the upper bound introduced by the DP and
  
  (ii) **maximality condition**: the individuals in the NP denotation that are not part of $x$ don’t satisfy any plural alternative of $P$

Some examples of what we mean by ‘plural alternative of $P$’:

- For the predicate read exactly ten books, the plural alternatives are all predicates of the form read $x$, where $x$ is a book.
- For the predicate show no more than three books to every student, the plural alternatives are all predicates of the form show $x$ to $y$, where $x$ is a book and $y$ is a student.

- For exactly four boys, (i) says that some plurality of four or fewer boys satisfies $P$, and (ii) says that no plurality of more than four boys satisfies $P$.

Upward non-maximality essentially means that the maximality condition doesn’t hold.

- Some examples of contexts where upward non-maximality is possible:

(8) a. *In meinem Bezirk haben gerade mal drei Aktivisten alle Wähler kontaktiert.*
    in my district have no more than three activists all voters contacted
    ‘In my district, no more than three activists contacted all voters.’
b. SCENARIO: There are 100 voters. Activist Ada contacted 30 voters, activist Bea another 30. Carl contacted the remaining 40. Dean also contacted some voters. (8-a) true

(9) a. Letztes Jahr haben genau zehn Wiener insgesamt über 500 Strafzettel bekommen.
Last year have exactly ten Viennese people in total over 500 parking tickets got
‘Last year, exactly ten Viennese people got more than 500 parking tickets between them.’

b. SCENARIO: We are looking at the statistics on parking violations in Vienna. Most drivers got one or two parking tickets last year, but there are ten exceptionally careless drivers. Five of them got 40 parking tickets each, four of them got 60 tickets each and one even got 70 tickets. (9-a) true

• Upward non-maximality: Sentence can be judged true if some plurality that is ‘picked out’ by the DP cumulatively fulfills the predicate, even if a bigger plurality also fulfills the predicate.

3.1.2 Conditions on upward non-maximality

• There are a number of contexts where we don’t find upward non-maximality.

• We don’t find it with distributive predicates.

(10) a. Letztes Jahr haben genau zehn Wiener jeweils 50 Strafzettel
Last year have exactly ten Viennese people each 50 parking tickets got
‘Last year, exactly ten Viennese people got 50 parking tickets each.’

b. SCENARIO: We are looking at the statistics on parking violations in Vienna. Most drivers got one or two parking tickets last year, but there are twelve exceptionally careless drivers who got 50 parking tickets each. (10-a) false

• Two conditions on when we find upward non-maximality with cumulative predicates:
  
  (i) Scope asymmetries: Upward non-maximal readings only occur when a plural DP has another plural or degree expression in its scope. E.g. (12) is true in scenario (11).

  (11) SCENARIO: There are 50 people living in a remote village. Three Viennese people have a lot of friends there: Anna is friends with 15 people from the village, Sarah is friends with another 15 people and Paul is friends with the remaining 20. There are also other people from Vienna who have one or two friends in the village.

  (12) Es ist überraschend, dass gerade mal drei Wiener mit allen Leuten
it is surprising that no more than three Viennese people with all people
aus diesem Dorf befreundet sind.
from this village friendly are
‘It is surprising that no more than three Viennese people are friends with all the people in this village.’ true in (11)

This seems to change when the object-DP is scrambled above the subject: Preferred reading of (13) is not true in (11) (Non-maximal reading available, but dispreferred?)
Es ist überraschend, dass mit allen Leuten aus diesem Dorf gerade mal drei Wiener befreundet sind. ‘It is surprising that no more than three Viennese people are friends with all the people in this village.’

This shows that asymmetry cannot be explained via thematic roles (as proposed for English in [Buccola and Spector 2016]).

(ii) **Pragmatic factors**: The non-maximal DP, on its ordinary meaning, introduces an upper bound on the size of pluralities satisfying the predicate.\(^2\)

Non-maximal reading only available in contexts where this upper bound counts as surprisingly low given the predicate \(\text{→ Unexpected that such a small group of individuals managed to achieve so much.}\)

In (14), upward non-maximal reading is available, but in (15) it is hard to get: Not unexpected that ten activists, between them, managed to call ten voters – calling ten voters does not count as a particularly high achievement.

(14) *Heute haben gerade mal 10 Aktivisten genau 500 Wähler angerufen.*

‘Today, no more than 10 activists called exactly 500 voters.’

**Scenario:** There are 500 voters in the district and fifteen activists are participating in the election campaign. Today, ten of the activists worked very hard: They managed to call 50 voters each, so that in total, every voter received a call from one of them. The other five activists were lazy and only called one or two voters each.

(14) **true**

(15) *Heute haben gerade mal 10 Aktivisten genau 10 Wähler angerufen.*

‘Today, no more than 10 activists called exactly 10 voters.’

**Scenario:** There are 500 voters in the district and fifteen activists are participating in the election campaign. Usually, ten of the activists are very hard-working. But today, each of the 15 activists only made one phone call to a voter. Only the first ten voters in the phone book were called by the activists, so in total, there were 15 calls and 10 voters received one or two calls.

(15) **inadequate**

\(^2\)For DPs introducing no upper bound (at least five boys), we cannot diagnose an upward non-maximal reading. So we don’t claim that the semantic operation deriving upward non-maximal readings cannot apply to such DPs.
3.1.3 Interim summary

**Generalizations**: Upward non-maximality …

(i) doesn’t require a separate lexical entry for the determiner/numeral
(ii) only occurs if the non-maximal DP has a plurality in its scope
(iii) doesn’t occur with distributive predicates
(iv) only occurs if the non-maximal DP imposes a ‘surprisingly low’ upper bound

3.2 Downward non-maximality

3.2.1 Characterizing downward non-maximality

Conditions on downward non-maximality are almost analogous and raise analogous questions.

- Downward non-maximality: Only observable with DPs that usually introduce a lower bound:

  (16) a. plural universals: *alle Wiener* (‘all Viennese’)
      
  b. DPs with unmodified and modified numerals: *1,8 Millionen Wiener* (‘1.8 million Viennese’), *genau 1,8 Millionen Wiener* (‘exactly 1.8 million Viennese’)

- Usually, when a lower-bounded DP combines with a predicate $P$, the relevant part of the truth conditions (ignoring a potential upper bound contributed by the DP) can be described as follows:

  There is a plurality $x$ in the NP denotation that satisfies $P$ and

  (i) **basic condition**: $x$ is larger than or equal to the lower bound introduced by the DP

  (ii) **maximality condition**: and the individuals in the NP denotation that are not part of $x$ don’t satisfy any plural alternative of $P$

- For (17), this means: There is a plurality $x$ of students that cumulatively read exactly ten books, s.th. (i) $x$ contains more than five students and (ii) the students who are not in $x$ did not read any books.

  (17) **More than five students read exactly ten books.**

- On a downward non-maximal reading, requirement (i) no longer has to hold.

  **Downward non-maximality essentially means that basic condition doesn’t hold.**

- In some contexts: Maximal plurality satisfying the predicate does not have to meet the lower bound imposed by the DP:

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3Arguably, also with plural definites, but non-maximality in plural definites has a different distribution than what we are considering here (see e.g. [Brisson 1998]), and thus might also have a different explanation.
In my district, all the activists contacted no more than 10 voters.

Scenario: There are 100 activists. 10 of them each contacted a different voter. The rest didn’t contact anyone. \((18\text{-a})\) true

Last year, exactly 1.8 million Viennese people got exactly 5000 parking tickets between them.

Scenario: We are looking at the statistics on parking violations in Vienna. There are 1.8 million people living in Vienna. 5000 people got exactly one parking ticket each. The other people didn’t get any parking tickets. \((19\text{-a})\) true

- **Downward non-maximality:** It is sufficient if *some* plurality \(x\) in the NP denotation satisfies the predicate, as long as no individual outside \(x\) satisfies any plural alternative of the predicate.

3.2.2 Conditions on downward non-maximality

- There are a number of contexts where we don’t find downward non-maximality

- We do not find it with distributive predicates.

Last year, exactly 1.8 million Viennese people got exactly two parking tickets each.

Scenario: We are looking at the statistics on parking violations in Vienna. There are 1.8 million people living in Vienna. 5000 people got exactly two parking ticket each. The other people didn’t get any parking tickets. \((20\text{-a})\) false

- Three conditions on when we find downward non-maximality with cumulative predicates:

  (i) **Scope asymmetries:** At least with some predicates, availability of downward non-maximal interpretation is influenced by relative scope of upper-bounded DP and other plural DPs. 

  (22) is true in scenario (21), which requires a downward non-maximal interpretation.

Scenario: There are 50 people living in a remote village. Three Viennese people have a lot of friends there: Anna is friends with 10 people from the village, Sarah is friends with another 10 people and Paul is friends with yet another 10. Nobody else from Vienna has any friends there.

Es ist überraschend, dass alle Leute aus diesem Dorf mit genau mal freundlich sind.

It is surprising that all people from this village with no more than three Viennese people friendly are
‘It’s surprising that all the people in this village are friends with no more than three Viennese people.’

Like the asymmetry we observed with the upward non-maximal interpretation, facts change with scrambling: (23) is inadequate in the scenario above.

(23)  
\textit{Es ist überraschend, dass mit gerade mal drei Wienern alle Leute aus diesem Dorf befreundet sind.}

\textit{it is surprising that with no more than three Viennese people all people from this village are friends.}

‘It is surprising that all the people in this village are friends with no more than three Viennese people.’

The non-maximal reading is available/preferred only if the DP stands in a cumulative relation with another plural expression (or degree expression) in its scope.

(ii) ‘Maximality’ of DP meaning: Additional condition on downward non-maximality that doesn’t seem to have counterpart for upward non-maximality: Downward non-maximal reading only available if the DP picks out the maximal plurality in the NP denotation.

For DPs with universals and plural definites, this condition is trivially satisfied, but relevant for DPs like genau 1,8 Millionen Wiener ‘exactly 1.8 million Viennese’: Allows for downward non-maximal reading in a context with exactly 1.8 million Viennese people in total (25-a). If DP doesn’t pick out the maximal plurality of Viennese people, non-maximal reading seems unavailable:

(24)  
\textit{Letztes Jahr haben genau 1,8 Millionen Wiener insgesamt genau 5000 Strafgelder bekommen.}

\textit{last year have exactly 1.8 million Viennese people in total exactly 5000 parking tickets got}

‘Last year, exactly 1.8 million Viennese people got exactly 5000 parking tickets between them.’

(25)  
\begin{align*}
  &\text{a. SCENARIO: We are looking at the statistics on parking violations in Vienna. There are 1.8 million people living in Vienna. 5000 people got exactly one parking ticket each. The other people didn’t get any parking tickets.} \\
  &\text{(25) true} \\
  &\text{b. SCENARIO: We are looking at the statistics on parking violations in Vienna. There are 2 million people living in Vienna. 5000 people got exactly one parking ticket each. The other people didn’t get any parking tickets.} \\
  &\text{(25) false}
\end{align*}

\textbf{Unexpected given our paraphrase of downward non-maximal reading:} Lower bound introduced by the DP should not influence the truth conditions of the downward non-maximal reading at all, but contrast between (24) and (25) shows this is not correct.

(iii) Other pragmatic factors: Non-maximal DP, on its ordinary meaning, introduces lower bound on the size of pluralities satisfying the predicate.\footnote{For DPs that don’t introduce a lower bound (at most five boys) we cannot diagnose a downward non-maximal reading.}
Non-maximal reading only available in contexts where this lower bound counts as surprisingly high given the `low bar’ represented by the predicate \( \sim \). Unexpected that so many individuals achieved so little.

In (26), the downward non-maximal reading is available, but in (27) it is hard to get because calling 500 voters does not count as an unexpectedly low achievement for all the activists.

(26) \textit{Heute haben alle Aktivisten genau 10 Wähler angerufen.} today have all activists exactly 10 voters called ‘Today, all the activists called exactly 10 voters.’

\text{SCENARIO:} There are 600 voters in the district and 100 activists are participating in the election campaign. Today, the activists were not working very hard: Ten of them each called a different voter; the other activists did not call any voters. \( (26) \text{ true} \)

(27) \textit{Heute haben alle Aktivisten genau 500 Wähler angerufen.} today have all activists exactly 500 voters called ‘Today, all the activists called exactly 500 voters.’

\text{SCENARIO:} There are 600 voters in the district and 12 activists are participating in the election campaign. Today, ten of the activists were working very hard: They each called 50 voters, so that in total, 500 voters received a call. The other two activists did not call any voters. \( (27) \text{ inadequate} \)

The downward non-maximal reading is available/preferred only if the lower bound introduced by the DP is unexpectedly high given the predicate.

3.2.3 Interim summary

\textbf{Theoretical task:} Model non-maximal readings in such a way that they

(i) don’t require a separate lexical entry for the determiner/numeral

(ii) only occur if the non-maximal DP has a plurality in its scope

(iii) don’t occur with distributive predicates

(iv) only occur if the non-maximal DP imposes ‘surprisingly low’ upper bound (upward non-maximality) or ‘surprisingly high’ lower bound (downward non-maximality)

\textbf{Here} we give a semantics that meets goals (i) and (ii). Our current research goal: Pragmatic account that explains (iii), (iv) and is compatible with a generalized account of cumulativity.
What do non-maximal readings tell us about cumulativity in general?

- **Scope asymmetries** ⇒ We don’t want a ‘symmetric’ theory of cumulativity that neutralizes surface scope relations.

- **Pragmatic condition** (‘unexpectedly low/high given the predicate’) ⇒ Same thing: Sensitive to the meaning of the entire predicate a plural DP combines with, including scopally lower DPs it cumulates with.

- **Effects of scrambling** ⇒ Asymmetries must be derived in syntactic/semantic composition, cannot (only) be result of ‘special’ properties of certain thematic positions (as in Buccola and Spector 2016).

- **In sum**: Cumulative readings aren’t ‘scopeless’ readings (vs. e.g. Landman 2000), semantic scope is not inherently related to distributivity.

What do we do with these observations?

- Predicate (symmetric) analysis could be adapted to yield non-maximal readings: Introduce ‘non-maximal’ versions of ** and stipulate that LF movement deriving cumulative readings cannot change relative scope relations between plurals.

- **Our approach**: plural projection (Schmitt 2017, Haslinger and Schmitt 2018). Asymmetric theory of cumulativity motivated by independent problems with the predicate analysis. Does not predict connection between cumulativity and ‘scopelessness’, so it should be easy to model non-maximal readings in this framework.

4 Plural projection

Next: Introduce asymmetric plural projection mechanism as our theory of cumulativity. This will be the backbone of our account of non-maximality.

- Recall symmetric account: Cumulativity between $n$ different pluralities requires $n$-ary relation as input for cumulation.

  \[
  (28) \quad \text{If } Ada \text{ and } Bea \text{ wanted to feed } Carl \text{ and } Dean \Rightarrow \text{derive binary relation } \\
  \quad \text{If } Ada \text{ and } Bea \left[ \text{Carl and Dean } \left[ 2 \left[ 1 \text{ wanted to feed } t_2 \right] \right] \right] \Rightarrow \text{cumulate } \lambda x.\lambda y.\lambda e. y \text{ wanted to feed } x
  \]

- **Problem** for symmetric account: Schein-sentences (Schein 1993). Asymmetries in ‘mixed’ cumulative-distributive reading, (Schein 1993, Zweig 2008 a.o.).

  \[
  (29) \quad \begin{align*}
  \text{a. Ada and Bea taught every dog two tricks.} & \quad \text{adapted from Schein (1993)} \\
  \text{b. Scenario: There are two dogs, Carl and Dean. Ada taught Carl tricks 1 and 2.} \\
  & \quad \text{Ada taught Dean trick 3 and Bea taught Dean trick 2.}
  \end{align*}
  \]

  - \(29\) does not involve symmetric cumulation of binary or ternary relations: The part structure of
the scopally dependent plural \textit{two tricks} must be preserved, since it doesn’t have to be the case that every dog was taught his two tricks by the same person.

Higher two plural DPs seem to cumulate, but \textit{two tricks} scopally dependent of \textit{every dog}.

- **Existing asymmetric analyses:** Multiple instances of cumulation in the syntax \cite{Champollion2010}, or event-based \cite{Shen1993, Kratzer2003, Landman2000, Ferreira2005, Zweig2008}. Both approaches problematic with more complex examples \cite{HaslingerAndSchmitt2018}.

- **Basic intuition behind our approach to (30-a):** \texttt{Ada+Bea} are in a cumulative relation with one of predicate pluralities in the set in (30-b), \cite{HaslingerAndSchmitt2018}.

\begin{align*}
(30) & \quad a. \text{Ada and Bea taught every dog two tricks.} \\
& \quad b. \{\text{taught}(T_1)(C) + \text{taught}(T_2)(C) + \text{taught}(T_1)(D) + \text{taught}(T_2)(D), \\
& \quad \text{taught}(T_1)(C) + \text{taught}(T_2)(C) + \text{taught}(T_2)(D) + \text{taught}(T_3)(D), \\
& \quad \text{taught}(T_1)(C) + \text{taught}(T_3)(C) + \text{taught}(T_1)(D) + \text{taught}(T_2)(D), \ldots\}\}
\end{align*}

- **How do we derive this set?** **Plural projection** (motivated independently in \cite{Schmitt2017}.

> Plural projection: Main ideas

(i) All semantic domains contain pluralities (i.e. we don’t only have pluralities of individuals, but also pluralities of predicates, etc.)

(ii) Pluralities ‘project’: Every node dominating a plurality also denotes a plurality

(iii) Cumulativity is derived by step-by-step via the projection rule along the lines of the syntactic structure, rather than by enriching a predicate.

- every semantic domain contains pluralities (plurality-formation via +):

\begin{align*}
(31) & \quad a. \quad D_e = \{ \text{Ada, Bea, Ada+Bea,} \ldots \} \\
& \quad b. \quad D_{(et)} = \{ \text{smoke, drink, smoke+drink,} \ldots \} \\
& \quad c. \quad \ldots
\end{align*}

- every semantic domain has an ‘associated’ semantic domain of ‘plural sets’, indicated by \*-types (we use square brackets for these plural sets):

\begin{align*}
(32) & \quad D_{e^*} = \{ [\ ], [\text{Ada}], [\text{Bea}], [\text{Ada+Bea}], [\text{Ada, Bea}], [\text{Ada, Ada+Bea }], [\text{Bea}, \text{Ada+Bea }], [\text{Ada, Bea, Ada+Bea }], \ldots\}
\end{align*}

- Some simplified denotations:

\begin{align*}
(33) & \quad a. \quad [\text{Ada and Bea}] = [\text{Ada+Bea}] \\
& \quad b. \quad [\text{smoke and dance}] = [\text{smoke+dance}] \\
& \quad c. \quad [\text{Ada or Bea}] = [\text{Ada, Bea}] \\
& \quad d. \quad [\text{smoke or dance}] = [\text{smoke, dance}]
\end{align*}

- **Plurals project:** If a plurality enters the derivation, dominating nodes will also denote pluralities. (Plural sets containing pluralities of values.)
"Projection" via a compositional rule **Cumulative Composition**: Encodes **cumulativity**.

(34) \( feed \text{ and brush } Carl \)

\[
\begin{array}{c}
\text{[brush(C)+ feed(C)]} \\
\text{[feed+brush] [Carl]}
\end{array}
\]

(35) \( Ada \text{ and Bea fed } Carl \text{ and Dean.} \)

\[
\begin{array}{c}
\text{[fed(C)(A)+fed(D)(B), fed(C)(B)+fed(D)(A), ...]} \\
\text{Ada+Bea [fed(C)+fed(D)]} \\
\text{fed [Carl and Dean]}
\end{array}
\]

- Plural set of truth-values true iff one of the pluralities in it consists of true atoms only
- Two aspects of this system allow us to model cumulative readings of *every* DPs:

  (i) **asymmetry**: *every* DP combines with the meaning of a (possibly plural) predicate
  (ii) result of this combination is a plural set, which can form input for cumulativity

This system is **inherently asymmetric**: We should be able to extend it to asymmetries with non-maximality by positing additional (derived) meanings for the DPs, but **without changing the basic mechanism that accounts for cumulativity**.
Formal implementation:

(36) The set \( T \) of **semantic types** is the smallest set such that \( e \in T, t \in T \), for any \( a, b \in T \), \( \langle a, b \rangle \in T \), and for any \( T, a^* \in T \).

(37) Let \( A \) be the (nonempty) set of atomic individuals. For each type \( a \), there is an **atomic domain** \( A_a \) and a **full domain** \( D_a \) with the following properties:

a. \( D_a \) is a set such. \( A_a \subseteq D_a \) and there is an operation \( +_a : \mathcal{P}(D_a) \setminus \{\emptyset\} \to D_a \).

b. There is a function \( pl_a : \mathcal{P}(A_a) \setminus \{\emptyset\} \to D_a \) s.t.:
   i. \( pl_a(\{x\}) = x \) for each \( x \in A_a \)
   ii. \( pl_a \) is an isomorphism from \( \mathcal{P}(A_a) \setminus \{\emptyset\} \) to \( (D_a, +_a) \).

c. For any type \( b \neq a \), \( D_a \) and \( D_b \) are disjoint.

(38) a. \( A_e = A \), the set of individuals; \( A_t = \{0,1\}^W \), where \( W \) is the set of possible worlds

b. For any types \( a, b \), \( A_{(a,b)} = D_{(a,b)} \), the set of partial functions from \( D_a \) to \( D_b \).

c. For any type \( a \), \( A_{a^*} \) is a set that is disjoint from \( \mathcal{P}(D_a) \) and on which the operations \( \cup, \cap \) and \( \setminus \) are defined. There is a function \( pl_{a^*} : \mathcal{P}(D_a) \to A_{a^*} \) that is an isomorphism wrt. \( \cup, \cap \) and \( \setminus \).

(39) The operation \( \bigoplus_a : \mathcal{P}(D_a) \setminus \{\emptyset\} \to D_a \) is defined for any type \( a \) as follows:

a. For any type \( a \) that is not of the form \( b^* \), and any nonempty \( S \subseteq D_a \), \( \bigoplus S = + S \).

b. For any type \( b^* \) and any nonempty \( S \subseteq D_{b^*} \), \( \bigoplus S = \bigcup_{f \in S} f = A \exists f : f \) is a function from \( S \) to \( D_{b^*} \setminus \forall X^* \in S : f(X^*) \in p\mathcal{P}^{-1}(X^*) \wedge X = \bigoplus_{b^*}(f(X^*) \mid X^* \in S) \)

(40) Let \( P \in D_a, x \in D_b \). A relation \( R \subseteq A_a \times A_b \) is a **cover** of \( (P, x) \) iff \( +((P^\prime | \exists x^\prime : (P^\prime, x^\prime) \in R)) = P \) and \( +((x^\prime | \exists P^\prime : (P^\prime, x^\prime) \in R)) = x \).

(41) **Cumulative Composition**

a. For any \( P^* \in D_{(a,b)^*} \) and \( x^* \in D_{a^*} \):
   \[
   C(P^*, x^*) = \bigoplus\left(\left\{P^\prime(x^*) \mid (P^\prime, x^*) \in R\right\}\right) | \exists P \in p\mathcal{P}^{-1}(P^*), x \in p\mathcal{P}^{-1}(x^*) : R \text{ is a cover of } (P, x)
   \]

b. For any meaningful expressions \( \phi \) of type \( \langle a, b \rangle^* \) and \( \psi \) of type \( a^* \), \( [\phi \psi] \) is a meaningful expression of type \( b^* \), and \([\phi \psi] = C([\phi], [\psi]) \).
Application to Schein sentences

• Idea for Schein sentences (42): non-classical meaning for every DPs that manipulates plural sets

\[(42)\]
- A da and Bea taught every dog two tricks. 
- scenario: There are two dogs, Carl and Dean. A da taught Carl tricks 1 and 2. A da taught Dean trick 3 and Bea taught Dean trick 2.

• In (42-a), every dog combines with a plural set:

\[(43)\] taught two tricks = \[teach(t1) + teach(t2), teach(t1) + teach(t3), teach(t2) + teach(t3) \ldots \] 

• We consider all the different ways of mapping the individual dogs to elements of this set, e.g.:

\[(44)\]
- Carl $\rightarrow$ teach(t1) + teach(t2), Dean $\rightarrow$ teach(t1) + teach(t3)
- Carl $\rightarrow$ teach(t1) + teach(t3), Dean $\rightarrow$ teach(t1) + teach(t2)
- Carl $\rightarrow$ teach(t1) + teach(t2), Dean $\rightarrow$ teach(t3) + teach(t2) etc.

• The semantic contribution of the every DP: For each mapping of dogs to the predicate-sums in (43), we combine each dog with its respective predicate-sum and sum up the results.

\[(45)\]
- teach(t1)(Carl) + teach(t2)(Carl) + teach(t1)(Dean) + teach(t3)(Dean)
- teach(t1)(Carl) + teach(t3)(Carl) + teach(t1)(Dean) + teach(t2)(Dean)
- teach(t1)(Carl) + teach(t2)(Carl) + teach(t3)(Dean) + teach(t2)(Dean) etc.

• We then collect all the results into a plural set:

\[(46)\] teach(t1)(Carl) + teach(t2)(Carl) + teach(t1)(Dean) + teach(t3)(Dean), teach(t1)(Carl) + teach(t3)(Carl) + teach(t1)(Dean) + teach(t2)(Dean), teach(t1)(Carl) + teach(t2)(Carl) + teach(t3)(Dean) + teach(t2)(Dean) \ldots \] 

• This is exactly the set we wanted to derive.

\[(47-a)\] is true iff Ada and Bea cumulatively satisfy at least one predicate sum in the set (46): We can combine [Ada + Bea] with (46) via our general composition rule for plural sets.

5 Semantic proposal for plural quantifiers

Next: Combine plural projection mechanism with novel treatment of plural quantifiers. This will derive us (some aspects of) non-maximality.

5.1 The theoretical challenge

• Main semantic goal: Model cumulative readings of plural quantifiers – maximal and non-maximal – in the plural projection framework. Accordingly:
Conditions our semantics has to meet

(i) maximal (upper-bounded and lower-bounded) readings should be available for plural quantifiers

(ii) non-maximal readings should be derivable from them

(iii) analysis should be compatible with the plural projection framework, i.e. plural quantifiers introduce plural sets

• Additional motivation for condition (iii): Plural quantifiers (e.g. modified numerals) behave just like unmodified numerals in Schein sentences:

(47) a. Ada and Bea taught every dog exactly two tricks. adapted from Schein (1993)
   b. SCENARIO: There are two dogs, Carl and Dean. Ada taught Carl tricks 1 and 2. Ada taught Dean trick 3 and Bea taught Dean trick 2.

   Our analysis of Schein sentences can only be extended to (47) if taught exactly two tricks can denote a plural set.

• Problem: How do we correctly predict the maximal (upper-bounded) reading of modified numerals like exactly two students? We can’t simply take the set of all pluralities of exactly two students, since that is our meaning for two students, which is not upper-bounded:

(48) a. \[ \text{two students} = [A + B, B + F, A + F \ldots] \]

   b. \[ \text{two students came} = [\text{came}(A) + \text{came}(B), \text{came}(B) + \text{came}(F), \text{came}(A) + \text{came}(F) \ldots] \]

• By our definition, (48-b) is true if it contains at least one plurality all parts of which are true. Also the case if A, B and F all came. In this scenario, (49-a) is true, but (49-b) is false.

(49) a. Two students came.
   b. Exactly two students came.

• For modified numerals, it is not enough to existentially quantify over pluralities that meet the lower and the upper bound introduced by the numeral. The semantics must introduce a maximality condition saying that no plurality above the upper bound satisfies the predicate.

• We now spell out a system that includes such a maximality condition

Core features of our proposal

(i) Expressions introduce two-layered meanings

(ii) In the case of plural DPs, the two layers can be ‘non-trivial’, i.e. distinct

(iii) For each layer, composition proceeds via the plural projection mechanism

(iv) Truth is defined by appealing to a relation between the two meaning levels

(v) The difference between maximal and non-maximal readings ‘plays out’ on one of the two layers of meaning
5.2 Our proposal (informal version), part 1

We start by considering the proposal without the treatment of non-maximality.

- In cumulative sentences, the maximality condition gives rise to a well-known compositionality problem (e.g. [Krifka 1999, Landman 2000, Brasoveanu 2010]). We circumvent this problem by introducing two-layered meanings: Plural quantifiers introduce not one, but two plural sets.

- Basic idea: A semantically plural expression $\alpha$ of some ‘plural type’ $\alpha^*$ actually denotes not just one plural set, but two plural sets $[\alpha]^{\{\}}$ (the ‘upper set’) and $[\alpha]^{\{\}}$ (the ‘lower set’).

What these sets are and will end up doing for us (very informally)

(i) the lower set will essentially present us with candidates that could fulfill the predicate. For our purposes: Contains all pluralities that are at most as big as the upper threshold ‘set’ by determiner.

(ii) the upper set will essentially tell us which candidates actually qualify. For our purposes: Contains all pluralities that are at least as big as the lower threshold ‘set’ by determiner.

(manipulation of this set will derive the maximality/non-maximality distinction)

(50) exactly two students

a. students $= [A, B, F, A + B, A + F, B + F, A + B + F]$

b. $[\text{exactly two students}]^{\{\}} = \{x \in D_e \mid \text{students}(x) \land |x| \leq 2\}$

$c. [\text{exactly two students}]^{\{\}} = \{x \in D_e \mid \text{students}(x) \land |x| \geq 2\}$

$\leadsto$ lower set: all pluralities consisting of at most two students

$\leadsto$ upper set: all pluralities consisting of at least two students

In the case of a non-plural lexical item like came, both sets are singletons:

(51) a. $[\text{came}]^{\{\}} = \{\lambda x. \text{came}(x)\}$

b. $[\text{came}]^{\{\}} = \{\lambda x. \text{came}(x)\}$

$\leadsto$ only plural expressions can have more than one element in each of the sets

$\leadsto$ only plural ‘quantifiers’ (universals, modified numerals) can exhibit interesting differences between the two sets

In semantic composition, we combine the upper sets and the lower sets in parallel, using our Cumulative Composition rule:

(52) a. $[\text{exactly two students came}]^{\{\}}$

$= [\{\{\{\text{came}(y) \mid y \leq a x\} \mid x \in D_e \land \text{students}(x) \land |x| \leq 2\}]$
Nina Haslinger & Viola Schmitt

The maximality condition is built into the definition of truth for plural expressions:

\[(53)\] An expression \(\alpha\) of type \(t^*\) (i.e. an expression denoting a pair of plural sets of propositions) is true iff
\[\text{a. there is a plurality } p \in \alpha^\uparrow \cap \alpha^\downarrow \text{ such that every atomic part of } p \text{ is true}\]
\[\text{b. and there is no } p' \in \alpha^\downarrow \text{ such that every atomic part of } p' \text{ is true.}\]

Truth is defined via an interaction of the two layers of meaning:

(i) there must be a true element that is an element of both sets.
(role of the lower set: Provides us with candidates – at least one of them has to be true,
role of the upper set: tells us which pluralities of the lower set are big enough)

(ii) for any such element: No ‘bigger’ true element in the higher set
(role of the upper set: tells us which pluralities are too big)

For (52) \((\text{exactly two students came})\):

a. \((53-a)\) says: There must be a plurality of two distinct propositions of the form \(\text{came}(x)\),

where \(x\) is a student, that are both true \(\Rightarrow\) Two students came.

b. \((53-b)\) says: There is no larger plurality of distinct propositions of this form that are all true \(\Rightarrow\) It is not the case that three or more students came.

This allows us to model the contributions of other numeral modifiers and of quantifiers like \(\text{all}.\)

\[(54)\] \(\text{students} = [A, B, F, A + B, A + F, B + F, A + B + F]\)

In (55) and (56), there is no upper bound, so the lower set contains all pluralities of students:\[6\]

\[(55)\]
\[\text{a. } \boxed{[\text{at least two students}]^\downarrow = [x \in D_e \mid \text{students}(x)] = [A, B, F, A + B, A + F, B + F, A + B + F]}\]
\[\text{b. } \boxed{[\text{at least two students}]^\downarrow = [x \in D_e \mid \text{students}(x) \wedge |x| \geq 2] = [A + B, A + F, B + F, A + B + F]}\]

\[(56)\]
\[\text{a. } \boxed{[\text{all students}]^\downarrow = [x \in D_e \mid \text{students}(x)] = [A, B, F, A + B, A + F, B + F, A + B + F]}\]

\[\text{In (i), there is no lower bound, so the upper set contains all pluralities of students:}\]

\[(57)\]
\[\text{a. } \boxed{[\text{at most two students}]^\downarrow = [x \in D_e \mid \text{students}(x) \wedge |x| \leq 2] = [\text{came}(A), \text{came}(B), \text{came}(F), \text{came}(A) + \text{came}(B), \text{came}(A) + \text{came}(F), \text{came}(B) + \text{came}(F)]}\]
\[\text{b. } \boxed{[\text{at most two students}]^\downarrow = [x \in D_e \mid \text{students}(x)] = [A, B, F, A + B, A + F, B + F, A + B + F]}\]

In this talk, we ignore the problem that (ii-b) can be true if no student came. This observation can be integrated into the present account, but in a somewhat stipulative way (cf. [Landman 2000, 2004] for relevant discussion).
b. \([\text{all students}]^\downarrow = [\bigoplus \{x \mid \text{students}(x)\}] = [A + B + F]\)

- For examples like (57), the predictions of this system roughly coincide with standard GQ theory.

(57)  
  a. Exactly two students came.
  b. At least two students came.
  c. All students came.

- The ‘upper set’/‘lower set’ mechanism has two advantages, however:
  
  (i) It predicts the right truth conditions for cumulative sentences with quantifiers.
  
  (ii) It allows us to model the two kinds of non-maximal readings in a unified way: Both correspond to operations that modify the upper set.

- Let’s look at (58-a) to illustrate advantage (i). We want to derive the maximal reading (58-b).

(58)  
  a. Exactly two activists called exactly three voters.
  b. ‘Exactly two activists called voters, and exactly three voters received a call from an activist.’

(59)  
  a. \([\text{called exactly three voters}]^\downarrow = [\bigoplus \{\text{call}(y) \mid y \leq x \} \mid x \in D_x \wedge \text{voters}(x) \land |x| \leq 3]\)
  
  \(= [\text{call}(A) + \text{call}(B) + \text{call}(C), \text{call}(A) + \text{call}(B), \text{call}(A) + \text{call}(C), \text{call}(A), \text{call}(B) + \text{call}(C), \ldots]\)

  b. \([\text{called exactly three voters}]^\uparrow = [\bigoplus \{\text{call}(y) \mid y \leq x \} \mid x \in D_x \wedge \text{voters}(x) \land |x| \geq 3]\)
  
  \(= [\text{call}(A) + \text{call}(B) + \text{call}(C), \text{call}(A) + \text{call}(B), \text{call}(A) + \text{call}(B) + \text{call}(D), \text{call}(A) + \text{call}(B) + \text{call}(C) + \text{call}(D), \ldots]\)

(59-a) contains all predicate-sums that, together, amount to the property of calling some plurality of three or fewer voters. (59-b) contains all predicate-sums that, together, amount to the property of calling some plurality of three or more voters.

- We combine (59) with our meaning for exactly two activists via Cumulative Composition.

This rule considers all possible combinations of some predicate plurality and some argument plurality such that every part of the predicate plurality occurs at least once, and every part of the argument plurality occurs at least once:

(60)  
  a. \(\text{voters} = \{A, B, C, D, A + B, \ldots, A + B + C + D\}\)
  b. \(\text{activists} = \{X, Y, Z, X + Y, \ldots, X + Y + Z\}\)
  c. \([\text{exactly two activists called exactly three voters}]^\downarrow = [\text{call}(A)(X), \ldots, \text{call}(D)(Z), \ldots, \text{call}(A)(X) + \text{call}(A)(Y), \text{call}(A)(X) + \text{call}(A)(Z), \ldots, \text{call}(A)(X) + \text{call}(D)(X), \text{call}(A)(X) + \text{call}(D)(Y), \text{call}(B)(Y) + \text{call}(C)(Z), \ldots, \text{call}(A)(X) + \text{call}(B)(Y) + \text{call}(D)(X), \text{call}(A)(X) + \text{call}(B)(Z) + \text{call}(D)(Z), \text{call}(B)(Y) + \text{call}(C)(Y) + \text{call}(D)(Z), \ldots]\)
  
  \(\approx \text{all pluralities of propositions of the form call}(x)(y)\) that ‘cover’ two or fewer activists and three or fewer voters
  
  d. \([\text{exactly two activists called exactly three voters}]^\uparrow = [\text{call}(A)(X) + \text{call}(B)(Y) + \text{call}(D)(X), \text{call}(A)(X) + \text{call}(B)(Z) + \text{call}(D)(Z), \text{call}(B)(Y) + \text{call}(C)(Y) + \text{call}(D)(Z), \ldots, \text{call}(A)(X) + \text{call}(A)(Z) + \text{call}(B)(Y) + \text{call}(D)(X), \text{call}(A)(X) + \text{call}(B)(Z) + \text{call}(B)(Y) + \text{call}(D)(X), \text{call}(A)(X) + \text{call}(B)(Z) + \text{call}(D)(Z), \text{call}(A)(Y) + \text{call}(B)(Y) + \text{call}(C)(Y) + \text{call}(D)(Z), \text{call}(A)(Y) + \text{call}(B)(Y) + \text{call}(C)(Y) + \text{call}(D)(Z), \text{call}(A)(Y) + \text{call}(B)(Y) + \text{call}(C)(Y) + \text{call}(D)(Z), \ldots]\)
call(B)(Y) + call(C)(X) + call(D)(Z), \ldots \]
\approx \text{all pluralities of propositions of the form call}(x)(y) \text{ that ‘cover’ two or more}
activists and three or more voters

• Our truth definition now requires two things:
  – There is some plurality \( p \) in \((60\text{-c}) \cap (60\text{-d})\) all parts of which are true.
    \( \rightsquigarrow \) there is a sum \( x \) of exactly two activists and a sum \( y \) of exactly three voters such that \( x \)
cumulatively called \( y \)
  – There is no \( p' > p \in (60\text{-d})\) all parts of which are true.
    \( \rightsquigarrow \) for any sums \( x' \) of two or more activists and \( y' \) of three or more voters such that \( x' \)
cumulatively called \( y' \), \( x = y \)

So the system can derive the maximality conditions associated with modified numerals like *exactly* \( n \) in a way that resembles the scalar analysis in [Krifka (1999)].

5.3 Back to non-maximal readings

- Preliminary idea for modeling upper and lower bound of a plural expression:
  
  **Upper bound** of an expression \( X \) of some plural type \( a^* \): \( \text{UB}(X) = \max\{n \mid \exists Y \in [X]^\uparrow \cap [X]^\downarrow, |Y| = n\} \)

  **Lower bound** of an expression \( X \) of some plural type \( a^* \): \( \text{LB}(X) = \min\{n \mid \exists Y \in [X]^\uparrow \cap [X]^\downarrow, |Y| = n\} \)


  \[
  \begin{align*}
  (61) & \quad \text{a. UB(exactly two students)} = 2 \text{ if there are two or more students; otherwise undefined} \\
  & \quad \text{b. LB(exactly two students)} = 2 \text{ if there are two or more students; otherwise undefined} \\
  (62) & \quad \text{a. UB(at least two students)} = |\{x \mid \text{student}(x)\}| = \text{the number of all students if there are two or more students; otherwise undefined} \\
  & \quad \text{b. LB(at least two students)} = 2 \text{ if there are two or more students; otherwise undefined} \\
  (63) & \quad \text{a. UB(at most two students)} = \min\{2, |\{x \mid \text{student}(x)\}|\} \text{ if there is at least one} \\
  & \quad \text{student; otherwise undefined}
  \end{align*}
  \]
b. \( \text{LB(at most two students)} = 1 \) if there is at least one student; otherwise undefined

(64) a. \( \text{UB(all students)} = |\{x \mid \text{student}(x)\}| \) = the number of all students if there is at least one student; otherwise undefined

b. \( \text{LB(all students)} = |\{x \mid \text{student}(x)\}| \) = the number of all students if there is at least one student; otherwise undefined

More research needed for treatment of undefinedness; here we concentrate on simple scenarios where all the relevant bounds are defined.

- **Upward non-maximality**: We want to capture cases like (65):

(65) *Es ist überraschend, dass gerade mal drei Wiener mit allen Leuten aus diesem Dorf befreundet sind.*

'It is surprising that no more than three Viennese people are friends with all the people in this village.'

- The semantic shift implementing upward non-maximality removes all but the ‘smallest’ elements from the upper set, and thereby removes the effect of the maximality condition.

**Upward non-maximality (informal version):**

For any plural DP \( A \), \( \text{UPWARD([A])} = ([A]^\#, [A]^\# \cap [A]^\#) \).

The operation \( \text{UPWARD} \) only applies to a plural DP \( A \) if it combines with a predicate \( P \) such that \( \text{UB}(A) \) is surprisingly low relative to \( [P] \).

- Let us first illustrate this manipulation with our example above, *exactly two students*

(66) *exactly two students*

a. \( \text{students} = [A, B, F, A + B, A + F, B + F, A + B + F] \)

b. \( ([\text{exactly two students}])^\# = [x \in D_e \mid \text{students}(x) \land |x| \leq 2] \)
   \( = [A, B, F, A + B, A + F, B + F] \)
   \( \sim \) lower set: all pluralities consisting of at most two students

c. \( ([\text{exactly two students}])^\# = [x \in D_e \mid \text{students}(x) \land |x| \geq 2] \)
   \( = [A + B, A + F, B + F, A + B + F] \)
   \( \sim \) upper set: all pluralities consisting of at least two students

d. \( \text{UPWARD}([\text{exactly two students}])^\# = [x \in D_e \mid \text{students}(x) \land |x| = 2] \)
   \( = [A + B, A + F, B + F] \)
   \( \sim \) upper set: all pluralities consisting of exactly two students

- Note that the effect of this manipulation is that the upper set is less restrictive: It includes no ‘bigger’ pluralities anymore. This means that there will be no maximality condition anymore.

For our example (67), we assume for simplicity that *gerade mal n* means the same thing as *genau n ‘exactly n’*. We first interpret the VP in the usual way:

(67) *Es ist überraschend, dass gerade mal drei Wiener mit allen Leuten aus diesem Dorf befreundet sind.*

'It is surprising that no more than three Viennese people with all people from this village are friends.'
‘It is surprising that no more than three Viennese people are friends with all the people in this village.’

(68) a. \[[\text{mit allen Leuten aus diesem Dorf befreundet sein}]\] = \[
\bigplus\{\lambda x.\text{friends}(y')(x) \mid y' \leq_a y\} \mid \text{people-from-this-village}(y)\]

\[= \{\text{friends}(A), \text{friends}(B), \text{friends}(A)+\text{friends}(B), \text{friends}(A)+\text{friends}(C), \ldots,\]

\[+\{\text{friends}(x) \mid \text{person-from-this-village}(x)\}\}\]

b. \[[\text{mit allen Leuten aus diesem Dorf befreundet sein}]\] = \[
\bigplus\{\lambda x.\text{friends}(y')(x) \mid y' \leq_a y\} \mid \text{people-from-this-village}(y)\]

\[= \{\text{friends}(A), \text{friends}(B), \text{friends}(A)+\text{friends}(B), \text{friends}(A)+\text{friends}(C), \ldots,\]

\[+\{\text{friends}(x) \mid \text{person-from-this-village}(x)\}\}\]

(68-a) contains all sums of predicates of the form \(\text{friends}(x)\) that correspond to the property of being friends with some people from the village.

(68-b) contains a single predicate sum, corresponding to the property of being friends with all the people from the village.

• Now we assume a context where it is unexpected for a sum of three Viennese people to satisfy the predicate in (68). This licenses the shift \textit{upward}. We combine (68) with the shifted meaning of the subject DP, (69):

(69) a. \text{UPWARD}([\text{gerade mal drei Wiener}]) = \[x \mid \text{viennese}(x) \land |x| \leq 3\]

b. \text{UPWARD}([\text{gerade mal drei Wiener}]) = \[x \mid \text{viennese}(x) \land |x| = 3\]

• Informally, the \textbf{resulting lower set} contains all sums of propositions of the form \(\text{friends}(x)(y)\) that cover some plurality of people from the village and some plurality of three or fewer Viennese people.

The \textbf{resulting upper set} contains all sums of propositions of the form \(\text{friends}(x)(y)\) that cover the sum of all people from the village and a plurality of \textbf{exactly three} Viennese people.

• Our truth definition now requires two things:

  – There is some plurality \(p\) in the intersection of upper and lower set, all parts of which are true.
    \(\leadsto\) there is a sum \(x\) of exactly three Viennese people such that \(x\) is cumulatively friends with all the people from the village

  – There is no \(p' > p\) in the upper set such that all parts of \(p'\) are true.
    \(\leadsto\) this condition has no effect since pluralities covering more than three Viennese people won’t be in the upper set

\text{So the effect of the shift \textit{upward} is to remove the maximality condition.}

• \textbf{Downward non-maximality}: We want to capture cases like (70):

(70) \textit{Es ist überraschend, dass alle Leute aus diesem Dorf mit gerade mal drei Viennern befreundet sind.}\

‘It’s surprising that all the people in this village are friends with no more than three Viennese people.’
The semantic shift implementing downward non-maximality enriches the upper set with ‘smaller’ elements, and thereby removes the lower bound on the maximal plurality satisfying the predicate.

**Downward non-maximality (informal version):**
For any plural DP $A$, $\text{DOWNWARD}([A]) = ([A]^{\downarrow}, [A]^{\uparrow} \cup [A]^{\uparrow})$.
The operation $\text{DOWNWARD}$ only applies to a plural DP $A$ if it combines with a predicate $P$ such that $\text{LB}(A)$ is surprisingly high relative to $[P]$.

Let us first illustrate this manipulation with our example above, **all students**

\begin{align*}
\text{a. } \quad [\text{all students}]^{\downarrow} &= \{x \in D_e \mid \text{students}(x)\} = [A, B, F, A + B, A + F, B + F, A + B + F] \\
\text{b. } \quad [\text{all students}]^{\uparrow} &= \{\lambda x.\text{friends}(y')(x) \mid y' \leq_a y\} \end{align*}

\begin{align*}
\text{c. } \quad \text{DOWNWARD}([\text{all students}]^{\downarrow})^{\uparrow} &= \{x \in D_e \mid \text{students}(x)\} = [A, B, F, A + B, A + F, B + F, A + B + F]
\end{align*}

The effect of this manipulation is that the upper set is less restrictive: It now includes ‘smaller’ pluralities.

For our example (71), we first interpret the VP in the usual way:

\begin{align*}
\text{(71) } \quad \text{Es ist überraschend, dass alle Leute aus diesem Dorf mit gerade mal drei} \\
\text{Viennern befreundet sind.}
\end{align*}

It’s surprising that all the people in this village are friends with no more than three Viennese people.

\begin{align*}
\text{(72) a. } \quad [[\text{mit gerade mal drei Wienern befreundet sein}]]^{\downarrow} &= \{\lambda x.\text{friends}(y')(x) \mid y' \leq_a y\} \end{align*}

\begin{align*}
\text{b. } \quad [[\text{mit gerade mal drei Wienern befreundet sein}]]^{\uparrow} &= \{\lambda x.\text{friends}(y')(x) \mid y' \leq_a y\}
\end{align*}

(72-a) contains all sums of predicates of the form $\text{friends}(x)$ that correspond to the property of being friends with some sum of three or fewer Viennese people.

(72-b) contains all sums of predicates of the form $\text{friends}(x)$ that correspond to the property of being friends with some sum of three or fewer Viennese people.

Now we assume a context where the sum of all people from the village is unexpectedly large relative to the predicate in (72). This licenses the shift $\text{DOWNWARD}$. We combine (72) with the shifted meaning of the subject DP, (73):

\begin{align*}
\text{(73) a. } \quad \text{DOWNWARD}([[\text{alle Leute aus diesem Dorf}]]^{\downarrow})^{\uparrow} &= \{x \mid \text{people-from-the-village}(x)\} \\
\text{b. } \quad \text{DOWNWARD}([[\text{alle Leute aus diesem Dorf}]]^{\downarrow})^{\uparrow} &= \{x \mid \text{people-from-the-village}(x)\}
\end{align*}
Informally, the resulting lower set contains all sums of propositions of the form friends(x)(y) that cover some plurality of people from the village and some plurality of three or fewer Viennese people.

The resulting upper set contains all sums of propositions of the form friends(x)(y) that cover some plurality of people from the village and some plurality of three or more Viennese people.

Our truth definition now requires two things:

- There is some plurality \( p \) in the intersection of upper and lower set, all parts of which are true.
  \[ \leadsto \text{there is a sum } x \text{ of exactly three Viennese people such that } x \text{ is cumulatively friends with some plurality of people from the village} \]

- There is no \( p' > p \) in the upper set such that all parts of \( p' \) are true.
  \[ \leadsto \text{there no sum } x' > x \text{ of Viennese people such that } x' \text{ is cumulatively friends with some plurality of people from the village} \]

The effect of the shift DOWNWARD is to remove the condition that all people from the village must be part of the maximal sum satisfying the predicate.

Intuitively, the scope asymmetry is built into the pragmatic conditions licensing the UPWARD and DOWNWARD shifts:

(74) In the upward non-maximal example (11):
   a. The lower argument, mit allen Leuten aus diesem Dorf ‘with all the people from
      this village’, combines with the lexical predicate befreundet ‘friends [with]’.
      It is not unexpected that there is some plurality that all the people of this village
      are cumulatively friends with.
      \[ \leadsto \text{downward non-maximal reading not licensed} \]
   b. The upper argument, gerade mal drei Wiener ‘no more than three Viennese people’,
      combines with the complex predicate mit allen Leuten aus diesem Dorf befreundet
      ‘friends with all the people from this village’.
      We assume that the upper bound, 3, is unexpectedly low relative to the predicate.
      \[ \leadsto \text{upward non-maximal reading licensed} \]

(75) In the downward non-maximal example (71):
   a. The lower argument, mit gerade mal drei Wienern ‘with no more than three Viennese people’,
      combines with the lexical predicate befreundet ‘friends [with]’.
      It is not unexpected that there is some plurality that no more than three Viennese
      people are cumulatively friends with.
      \[ \leadsto \text{upward non-maximal reading of this argument not licensed} \]
   b. The upper argument, alle Leute aus diesem Dorf ‘all the people from this village’,
      combines with the complex predicate mit gerade mal drei Wienern befreundet
      ‘friends with no more than three Viennese people’.
      We assume that the total number of people in the village – the lower bound – is
      unexpectedly high relative to this predicate.
      \[ \leadsto \text{downward non-maximal reading licensed} \]

---

7Here, we assume for simplicity that additional argument positions are closed off existentially. This allows us to evaluate the condition locally for predicates with arbitrarily many semantic arguments. The actual solution to this problem might be more complex.
• In sum, whether the upper/lower bound of the DP is ‘unexpectedly low’ or ‘unexpectedly high’ relative to the predicate depends (partly) on which plural expressions occur in the predicate.

• **Goal for future research**: precise implementation of the notion ‘surprisingly low/high relative to a predicate’, possibly based on an ordering relation similar to the one employed by *even*.

6 Conclusion and open questions

<table>
<thead>
<tr>
<th>Take-home messages</th>
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<tbody>
<tr>
<td>• cumulative readings of plural quantifiers can be ‘weakened’ under certain pragmatic conditions: <strong>upward and downward non-maximality</strong> (cf. Landman 2000, Buccola and Spector 2016 for English)</td>
</tr>
<tr>
<td>• <strong>cumulativity ≠ scopelessness</strong></td>
</tr>
<tr>
<td>– <strong>new challenge for scopeless analyses</strong> (Schmitt 2015): availability of upward and downward non-maximal readings depends (partly) on position relative to other plural expressions</td>
</tr>
<tr>
<td>• ‘plural projection’ semantics built to model asymmetrical aspects of cumulativity</td>
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<tr>
<td>– any expression containing a semantically plural subexpression denotes a <strong>plural set</strong></td>
</tr>
<tr>
<td>– cumulativity built into composition rules for plural sets</td>
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<tr>
<td>– no special LF syntax for cumulativity – predicates combine with their arguments in the order determined by syntax, just like in distributive sentences</td>
</tr>
<tr>
<td>• <strong>possible solution for upward/downward maximality</strong>: Combine plural projection with <strong>two-layered meaning</strong>, where certain pragmatic conditions correlate with manipulation of one of these two layers</td>
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</tbody>
</table>

Open questions

(i) With some predicates, scrambling doesn’t seem to block non-maximal readings. For instance, in our judgment, (76-a) and (76-b) (modeled after an example from Krifka 1999) both allow for a non-maximal reading of *gerade mal drei Leuten*.

(76) a. *Bezeichnenderweise gehören in diesem Ort gerade mal drei Leuten* significantly belong.to in this town no.more.than three people.DAT mehr als 1000 Grundstücke.
more than 1000 plots.of.land.NOM ‘Significantly, no more than three people own more than 1000 plots of land in this town.’

b. *Bezeichnenderweise gehören in diesem Ort mehr als 1000* significantly belong.to in this town more than 1000

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Our approach makes two predictions about such predicates:

- Scope inversion should be easier to get with such predicates than with predicates we used above like kontaktieren ‘contact’ or befreundet sein ‘be friends with’.
- With such predicates, other phenomena in plural semantics that seem to be scope-dependent – such as cumulative readings of every (Champollion 2010, Haslinger and Schmitt 2018) – should occur in both surface configurations.

We don’t know if these predictions are borne out (question for experimental work!) A similar issue arises in the case of syntactic movements that easily allow for scope inversion in German (e.g. topicalization).

(ii) So far, we don’t have story for distributive readings of modified numerals in Schein sentences.

(78) Ada and Bea taught exactly two tricks to exactly two dogs.

References


