Asymmetrically distributive items and plural projection

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Abstract

English every DPs, their German counterparts and distributive conjunctions in several languages give rise to cumulativity asymmetries: Such asymmetrically distributive universals (ADUs) allow for cumulative readings only if they occur in the scope of another semantically plural expression. We present a novel compositional analysis of these elements which we argue to be empirically superior to existing accounts of cumulativity asymmetries with every DPs (Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008, Champollion 2010) due to two properties: It is surface-compositional and it does not assume an inherent connection between cumulativity and events. More concretely, our analysis expands the ‘plural projection’ framework (Schmitt 2018), which derives cumulativity in a step-by-step process, along the lines of the syntactic structure. It does so by generalizing the notion of plurality to all semantic domains and by defining a new compositional rule that encodes cumulativity. Due to this rule, any constituent containing a semantically plural subexpression denotes a set of (possibly higher-type) pluralities, which reflect the part structure of embedded pluralities. We propose that ADUs operate on such sets of pluralities, which derives the fact that they have to be distributive w.r.t. material in their scope, and return another set of pluralities once they have combined with their scope argument, which correctly predicts that we should observe cumulative readings w.r.t. expressions outscoping them. Since the plural-projection mechanism allows us to preserve the part structure of material scopally dependent on ADUs, we also derive the correct readings for the more complex examples discussed by Schein (1993), where an ADU is ‘sandwiched’ between two plural expressions.

1 The problem: Asymmetrically distributive universals

In English, DPs with the determiner every exhibit a well-known semantic asymmetry when co-occurring with plural expressions like (the) two dogs (Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008, Champollion 2010). In some configurations, every DPs are restricted to a distributive reading: \((1-b)\) is true in scenario \((1-a-i)\) where the predicate fed (the) two dogs applies to each girl individually, but false in scenario \((1-a-ii)\). It thus lacks a cumulative reading, where the feeding relation holds cumulatively of the girls and the dogs – i.e. where each girl fed at least one dog, and each dog was fed by at least one girl.

\[
(1) \quad \text{a. CONTEXT: There are two girls, Ada and Bea, and two dogs, Carl and Dean.}
\]

\[
(i) \quad \text{‘DISTRIBUTIVE’ scenario: Ada fed Carl and Dean. Bea fed Carl and Dean.}
\]

\[
(ii) \quad \text{‘CUMULATIVE’ scenario: Ada fed Carl. Bea fed Dean.}
\]
b. *Every girl* in this town fed *(the) two dogs.* 

But cumulative readings of *every* DPs are available if there is a semantically plural expression in a higher syntactic position: (2) is true in the ‘cumulative’ scenario (1-a-ii) and differs from (1-b) only in that the *every* DP occurs in object position, while *(the) two girls* is the subject.

(2) *(The) two girls fed every dog* in this town. 

This asymmetrical behavior of *every* DPs is not shared by plural definites or indefinites: The sentence in (3), where *(the) two girls* is the subject and thus in the same surface-syntactic position as the *every* DP in (1-b) above, is true in scenario (1-a-ii) and hence has a cumulative reading.

(3) *The two girls in this town fed *(the) two dogs.*

We now sketch the compositionality problem that such asymmetries give rise to and our solution to it.

1.1 The class of asymmetrically distributive universals

The contrast between (1-b) and (2) illustrates the basic property of what we call asymmetrically distributive universals (ADUs): They are limited to distributive readings relative to syntactically ‘lower’ plural expressions, but allow for cumulative readings relative to syntactically ‘higher’ plural expressions.

The class of ADUs includes DPs with universal determiners, like *every* DPs in English or their German correlates with *jed- ‘every’,* which behaves analogously for many speakers. But it is a hitherto unnoticed fact that ‘distributive conjunctions’ (henceforth D-conjunctions) in German, Hungarian and Polish seem to follow the same pattern, which suggests that it reflects a more general trait of ‘distributive’ elements.

(4) exemplifies the asymmetry for the German D-conjunction pattern *sowohl A als auch B* ‘A as well as B’ (see Section 4 for the other languages). As with *every* DPs, the position of the ADU relative to the other plural expression affects the availability of a cumulative reading: (4-b), where the ADU is the subject and thus syntactically higher than the definite plural, is false in scenario (4-a). (4-c), on the other hand, is true in the scenario and involves an ADU in object position, below the definite plural subject.

(4) German

a. ‘*Cumulative* scenario: Two skiing races took place today. Ada and Bea were the only German participants. Ada competed in the downhill and won. Bea competed in the slalom and won.’

b. *Heute haben sowohl die Ada als auch die Bea die zwei Rennen gewonnen!*

‘Today have the Ada and the Bea the two races won’ 

false in (4-a)

c. *Heute haben die zwei Deutschen sowohl die Abfahrt als auch den Slalom gewonnen!*

‘Today have the two Germans the downhill and the slalom won’

true in (4-a)

1.2 Basic asymmetries

The observed interpretative differences correlating with subject-object asymmetries could reflect a structural asymmetry, as we have implicitly assumed above, or a thematic asymmetry, as [Kratzer (2003)] suggests. In the latter case, the availability of cumulative readings would depend on the thematic roles
assigned to the ADU and the other plural expressions.

Here, we take the structural view, following [Champollion (2010)] – we assume that the asymmetry reflects scope, viewed as c-command at LF. (But see Section 5 for more discussion.)

One argument for this comes from German sentences like (5-b), which permit the cumulative reading. In (5-b), the ADU is an agent, just like in (1-b). However, the syntactic position of the ADU is different: In (5-b), but not in (1-b), it occurs below another plural expression, in the subject position of the infinitival clause. A restriction in terms of thematic roles would not explain this contrast.

(5) a. scenario: Detectives Ada and Bea were observing two suspects, Carl and Dean. Ada saw Carl smoke a cigar. Bea saw Dean smoke a cigar.
    b. Ada und Bea haben jeden Verdächtigen eine Zigarre rauchen gesehen. Ada and Bea have every suspect a cigar smoke seen
       ‘Ada and Bea saw every suspect smoke a cigar.’

1.3 Schein sentences

The simple structural asymmetry illustrated above is not the only property of ADUs that any theory of cumulativity has to account for. We also observe a particular interaction between cumulativity and distributivity when ADUs are ‘sandwiched’ between two plural expressions, as in (6-b). Since these cases were first discussed in detail by [Schein (1993)], we call them Schein sentences.

(6) a. Scenario: There are two dogs, Carl and Dean. Ada taught Carl tricks 1 and 2. Ada taught Dean trick 3 and Bea taught Dean trick 2.
   b. Ada and Bea taught every dog two new tricks.

(6-b) is true in scenario (6-a). On the relevant reading, every dog seems to cumulate with Ada and Bea, as it is not the case that each of the girls taught every dog two tricks. But every dog is distributive wrt. two tricks, since each dog is taught two (potentially different) tricks.

This reading cannot be straightforwardly captured in terms of a single cumulative relation holding between individuals. We cannot say that the relation \( \lambda x. y. \lambda y. z. \) holds cumulatively of the two girls and the two dogs: This would predict that for each girl \( x \), some dog was taught two tricks by \( x \), which is false in scenario (6-a) – Bea taught only one trick to one dog. Nor is it the case that the relation \( \lambda x. y. \lambda y. z. \) holds cumulatively of the two girls and some plurality of two tricks: This would predict that there must be two tricks that each of the dogs was taught, but in scenario (6-a), the tricks are different for each of the two dogs. Finally, we could not assume that the three-place relation \( \lambda x. y. \lambda y. z. \) holds cumulatively: This does not capture the distributive interpretation of every dog relative to two new tricks and thus predicts that the sentence can be true even if neither dog was taught two tricks.

In other words, all three semantically plural expressions seem to ‘participate’ in the cumulative interpretation, but this cannot be described in terms of a single cumulative relation between individuals since every dog has scope over two tricks.
1.4 Sketch of our proposal

We saw that English and German have expressions – asymmetrically distributive universals (ADUs) – that pattern as follows (properties (7-a) and (7-b), at least, can also be replicated for Hungarian and Polish D-conjunctions):

\[(7)\]
\[
\begin{align*}
\text{a. } & \text{They allow for cumulative readings wrt. syntactically higher plural expressions.} \\
\text{b. } & \text{They prohibit cumulative readings wrt. syntactically lower plural expressions.} \\
\text{c. } & \text{When they occur in Schein sentences, the resulting mixed cumulative/distributive reading cannot be analyzed via a single cumulative relation between individuals.}
\end{align*}
\]

In this paper, we present a new account of ADUs which covers both singular universals like English every DPs and distributive conjunctions. It is based on a novel view of cumulativity proposed by Schmitt (2018), which derives cumulativity in a step-by-step process by means of a special composition mechanism that is sensitive to syntactic structure.

The basic idea is that model-theoretic objects of higher types like predicates or propositions can also form pluralities that participate in cumulative relations. If so, the truth conditions of (6-b) can be paraphrased as follows: We consider all unary predicates of the form \[\lambda z.\text{taught}(x)(y)(z),\] where \(x\) is a trick and \(y\) is a dog.\(^1\) We then form a set containing all pluralities of such predicates that (i) ‘cover’ both of the dogs and (ii) relate each dog to two tricks. This set is sketched in (8), where + symbolizes a cross-categorial sum operation, to be defined in Section 2 below.

\[(8)\]
\[
\begin{align*}
\text{taught}(T_1)(C) + \text{taught}(T_2)(C) + \text{taught}(T_1)(D) + \text{taught}(T_2)(D), \\
\text{taught}(T_1)(C) + \text{taught}(T_2)(C) + \text{taught}(T_2)(D) + \text{taught}(T_3)(D), \\
\text{taught}(T_1)(C) + \text{taught}(T_3)(C) + \text{taught}(T_1)(D) + \text{taught}(T_2)(D), \ldots
\end{align*}
\]

If \(\text{taught every dog two new tricks}\) denotes the set in (8), we can characterize the cumulative reading of (6-b) as follows: (6-b) is true iff Ada and Bea cumulatively satisfy a predicate sum from this set. In other words, there must be some predicate sum in (8) such that each girl satisfies at least one predicate in this sum, and each predicate in this sum is satisfied by at least one girl.

To turn this into a convincing account of Schein sentences, a principled way of deriving denotations like (8) is needed. Adapting Schmitt’s 2018 proposal, we assume that whenever a constituent that would ‘normally’ be assigned semantic type \(a\) contains a plural, it will actually denote a set of pluralities of denotations of type \(a\) – a plural set. The property of denoting a plural set ‘projects’ from a constituent to its mother via special composition rules. Any node dominating a semantically plural expression will itself be semantically plural unless an intervening operator blocks this process. Cumulativity falls out from the rules implementing this ‘projection’ mechanism and is derived step by step, following the syntactic structure. We will argue that this proposal is superior to the existing accounts of ADUs (Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008 on the one hand, Champollion 2010 on the other) since the mechanisms underlying these accounts do not extend to all observable instances of cumulativity.

1.5 Structure of the paper

Section 2 introduces and motivates the plural projection mechanism, which underlies our analysis of every DPs and D-conjunctions in Sections 3 and 4 respectively. In Section 5 we revisit our underlying assumption that cumulativity asymmetries correlate with scope: We present German data which suggest

\(^1\)We will represent the denotations of lexical predicates and proper names by boldfaced versions of (abbreviations of) the respective object-language expressions.
that scope is among the relevant factors, but other constraints also play a role. We compare our proposal with two competing accounts in Section 6 and conclude in Section 7.

2 Plural projection: Basic traits and motivation

We now outline the basic theory of plurals and cumulativity that forms the backbone of our analysis of ADUs. This system – based on Schmitt (2018) – has two main features. First, all semantic domains contain pluralities: In addition to pluralities of ‘primitives’ like individuals, we also have pluralities of properties, relations, propositions etc. For each semantic type, we assume that pluralities stand in a one-to-one correspondence with non-empty sets of the atomic domain elements, but are not identified with such sets (Link 1983). Second, pluralities ‘project’ in the sense that, if a node $\alpha$ dominates an expression denoting a plurality, the denotation of $\alpha$ will reflect the part structure of that plurality. Our formal implementation of this idea is analogous to Alternative Semantics, as applied to questions (Hamblin 1973), focus (Rooth 1985) and indefinites (Krater & Shimoyama 2002). Cumulativity will be built into the composition rule that implements this projection behavior.

Here is a brief preview of the projection mechanism. Whenever we combine a function and an argument and at least one of the two constituents denotes a plurality, we end up with a plurality of values. When a non-plural function combines with a plural argument, the result will be the sum of all values obtained by applying the function to atomic parts of the argument. Likewise, if a function plurality combines with an atomic argument, we get the sum of all values obtained by applying atomic parts of the function to the argument. This is sketched in (9), where $+$ indicates plurality formation.

$$(9) \quad f(a) + f(b) \quad f(a) + g(a) \quad f + g \quad a + b$$

However, in cases where functor pluralities combine with argument pluralities, a single plurality of values would be insufficient: Cumulative truth conditions are compatible with several possible ways of assigning function parts to argument parts. Hence, in our system, a plural expression will denote a set of pluralities – a plural set – rather than a single plurality. Combining a set of function pluralities and a set of argument pluralities will yield a set of pluralities of the values obtained by applying atomic function parts to atomic argument parts. This set will include exactly those pluralities that ‘cover’ all the parts of some function plurality and all the parts of some argument plurality, as schematized in (10). As in (9), however, the part structure of the embedded plural expressions ‘projects up’ the tree.

$$(10) \quad \{f(a) + g(b), f(b) + g(a), f(a) + g(a) + g(b), f(b) + g(a) + g(b), f(a) + f(b) + g(a), f(a) + f(b) + g(b), f(a) + f(b) + g(a) + g(b)\} \quad \{f + g\} \quad \{a + b\}$$

This operation is repeated at any syntactic node that dominates at least one plural expression. In particular, in cumulative sentences, the rule applies at each node intervening between the plural expressions that participate in cumulativity. Sentences will thus end up denoting plural sets of propositions, which count as true if at least one plurality in the set consists exclusively of true propositions.

In Sections 2.1 and 2.2 we introduce this mechanism in greater detail. In Section 2.3 we discuss the examples that originally motivated it. As we will see in Section 3, these data are closely related to Schein sentences.
2.1 Higher-order pluralities

The assumption that all semantic domains contain pluralities is motivated by the behavior of English and-conjunctions and their German counterparts (henceforth ‘conjunctions’). Schmitt (2013, 2018) argues that conjunctions with conjuncts of various types – including predicates or propositions – behave like definite plurals with respect to cumulativity, and that therefore they must be analyzed as denoting pluralities built from the denotations of the individual conjuncts (pluralities of predicates, propositions etc.).

To see the point, consider a ‘standard’ plural sentence with two plural expressions like (3) from above, repeated in (11). We already saw that this sentence has cumulative truth conditions: It is true iff each of the girls fed at least one dog and each of the dogs was fed by at least one girl.

(11) The two girls fed (the) two dogs.

Predicate conjunctions behave analogously. Scenario (12-a) illustrates the cumulative truth conditions of (12-b): For each of the girls it must be the case that she made Gene do \( P \) or \( Q \) and for each of \( P \) and \( Q \), there must be at least one girl who made Gene do it.

   Both girls are going on a trip. Ada asked Gene to feed Fay. Bea asked Gene to brush Dean.
   b. Poor Gene. The two girls made him [\( P \) feed Fay] and [\( Q \) brush Dean]!
   adapted from Schmitt (2018)

Similar observations can be made for propositional conjunctions: (13-b) is true in scenario (13-a), where each agency claimed at least one of \( p \) and \( q \) to be true, and for each of the propositions \( p \) and \( q \), at least one agency claimed it to be true.

(13) a. ‘Cumulative’ scenario: The Paris agency called and claimed that Macron was considering resignation. Later, the Berlin agency called and stated that Merkel had hired new bodyguards.
   b. The agencies claimed [\( p \) that Macron was considering his resignation] and [\( q \) (that) Merkel hired new bodyguards], but neither of them said anything about the Brexit negotiations.
   adapted from Schmitt (2018)

In sum, predicate conjunctions and propositional conjunctions pattern with expressions denoting plural individuals, like the (two) boys or Ada and Bea, when it comes to cumulativity. Schmitt (2013) therefore proposes that conjunctions with conjuncts of any semantic type actually denote pluralities, whose atomic parts are the denotations of the individual conjuncts. More concretely, she adds pluralities to any semantic domain \( D_a \), in analogy to Link’s (1983) treatment of the domain of individuals. Pluralities are assumed to stand in a one-to-one correspondence with non-empty subsets of the domain: Just as \( D_e \) contains atomic individuals and pluralities thereof, \( D_{(e,p)} \) contains predicates and pluralities of predicates etc. Hence, feed Fay and brush Dean denotes a plurality of predicates with atomic parts \([\text{feed Fay}]\) and \([\text{brush Dean}]\), and the clausal conjunction in (13-b) denotes the sum of the two propositions \([\text{that Macron was considering his resignation}]\) and \([\text{that Merkel hired 10 new bodyguards}]\).

Admitting such pluralities gives us a new way to deal with cumulativity (Schmitt 2018). Consider

\footnote{Analyses of cumulative predicate conjunction like Link 1984, Krifka 1990, Heycock & Zamparelli 2005 don’t extend to such cases since they require the predicate conjunction to combine directly with a plurality. The semantic argument of the predicate conjunction in (12) is \( him \), which is not a plural expression.}
first a ‘standard’ treatment of cumulativity in which cumulative truth conditions are derived by adding cumulation operators to the predicate (see Link 1983, Krifka 1986, Sternefeld 1998, Beck & Sauerland 2000). We call this type of account the **predicate analysis**. On the predicate analysis, the cumulative truth conditions of sentence (11) are derived by enriching the binary relation \([feed]\) via the **-operator, which forms ‘pointwise sums’ of the pairs in the basic extension of *feed*, as shown in (14).

\[
\begin{align*}
\text{(14) a.} & \quad \mathcal{L}_{\text{feed}} = \{ \langle a, c \rangle, \langle b, d \rangle \} \\
\text{b.} & \quad **\mathcal{L}_{\text{feed}} = \{ \langle a, c \rangle, \langle b, d \rangle, \langle a + b, c + d \rangle \}
\end{align*}
\]

On this approach, cumulativity is always the result of combining a cumulation operator (** for binary predicates, *** for ternary predicates, etc., see Sternefeld 1998) with a relation-denoting object language expression. This raises the question how to analyze non-lexical cumulative relations, as in (15-a). (15-a) is true in the ‘cumulative’ scenario (15-b), but does not contain any surface constituent that denotes the relation we need, namely \([\lambda x_1. \lambda y \cdot \text{wanted to feed } x \cdot \lambda x_2. \lambda y \cdot \text{wanted to feed } x_2]\). Beck & Sauerland (2000) suggest that in such cases, the two plurals move covertly to create a relation-denoting LF constituent, \(C\) in (15-c), that forms the input to **\[3\]

\[
\begin{align*}
\text{(15) a.} & \quad \text{The two girls wanted to feed the two dogs.} \quad \text{adapted from Beck & Sauerland (2000)} \\
\text{b. scenario:} & \quad \text{Ada wanted to feed Carl. Bea wanted to feed Dean.} \\
\text{c.} & \quad \text{[[the two girls] [[the two dogs] [** [C ] ]]]}
\end{align*}
\]

Plurality of functions, however, give us another way to derive cumulative truth conditions. Consider first the simple case in (16-a). Like (12) above, it has cumulative truth conditions, as illustrated by the ‘cumulative’ scenario in (16-b). In Schmidt’s 2018 terms, this example involves a plurality of individuals (\([\text{the two girls}]\)) combining with a predicate plurality (\([\text{fed } F \text{ and brushed } D]\)). This local configuration of the two pluralities allows us to formulate a rule that encodes cumulativity without the ** operator. Informally speaking, for (16-a), this rule should yield all the propositional pluralities in (16-c), which ‘cover’ all atomic parts of the individual plurality and all atomic parts of the predicate plurality. Then (16-a) is true iff there is at least one such plurality such that all its atomic parts are true.

\[
\begin{align*}
\text{(16) a.} & \quad \text{The two girls fed Fay and brushed Dean.} \\
\text{b.} & \quad \text{‘Cumulative’ scenario: Ada fed Fay. Bea brushed Dean.} \quad \text{(16-a) true} \\
\text{c.} & \quad \text{fed(f)(a)+brushed(d)(b), fed(f)(b)+brushed(d)(a), fed(f)(a)+brushed(d)(b)+fed(f)(b),} \\
& \quad \text{brushed(d)(a)+brushed(d)(b)+fed(f)(b), fed(f)(b)+brushed(d)(a)+fed(f)(a),} \\
& \quad \text{brushed(d)(b)+brushed(d)(a)+fed(f)(a), fed(f)(b)+brushed(d)(b)+brushed(d)(a)+fed(f)(a)}
\end{align*}
\]

Let’s assume that this cumulation rule can be extended to function-argument configurations of arbitrary types. We then have a new way of tackling less local cases of cumulativity such as (17) (= (11)), where the verb intervenes between the two plural expressions: (17) reduces to two instances of the configuration in (16), i.e. a plurality of functions combining with a plurality of arguments. We first apply the composition rule just sketched to combine the functor *fed* with the object plurality \([\text{the two dogs}]\), which gives us the predicate sum \(\text{fed(f)}+\text{fed(d)}\) as the denotation of the VP in (17). This sum combines with the subject plurality introduced by \([\text{the two girls}]\) in the way sketched for (16). The derivation of (17) will therefore involve two applications of the cumulation rule, one at the VP node which yields a set of pluralities of one-place predicates, and one for the IP which yields a set of pluralities of propositions.

\[
\begin{align*}
\text{(17) \quad [IP The two girls [VP fed (the) two dogs]]}.
\end{align*}
\]

\[3\text{We interpret indices as in Heim & Kratzer 1998.}\]
We will now spell out such a system in more detail. In effect, we will define a version of the cumulation operator that generalizes to arbitrary function and argument types, and then apply this operation at any node dominating a plurality. The possibility of iterating the cumulation operation will remove the need for cumulative relations derived in covert syntax.

2.2 Plural projection: Details

We will start by making our ontological assumptions explicit and then combine them with our new compositional rule, which lets embedded pluralities ‘project’ their part-structure up the tree and encodes cumulativity. (The discussion is based on [Schmitt 2018] but introduces some new concepts and generalizations.)

Ontology In order to deal with cumulative readings of conjunctions, we assumed that conjunctions of any semantic category denote pluralities, i.e. sums of atomic domain elements. We therefore extend the notion of sum to all semantic types: Sums of any type stand in a one-to-one correspondence to nonempty sets of atomic meanings of that type. Hence, the set \([\text{smoke}_{(e,t)}, \text{drink}_{(e,t)}, \text{dance}_{(e,t)}]\) corresponds to the sum \(\text{smoke}_{(e,t)} + \text{drink}_{(e,t)} + \text{dance}_{(e,t)}\), which is a possible denotation of type \((e,t)\) since it consists of three atomic parts of type \((e,t)\). For simplicity, we assume a ‘flat’ plural semantics that is insensitive to subgroupings within a plurality (cf. [Schwarzchild 1996] for discussion).

As mentioned above, however, our approach to cumulativity requires a richer ontology in which semantically plural expressions denote sets of pluralities, so-called plural sets. Since ordinary unary predicates do not interact with the composition rules in the same way as plural expressions, our type system will distinguish them from plural sets: For any semantic type \(a\) there is a corresponding type \(a^\ast\) for plural sets with elements of type \(a\), (18).

\[(18) \quad \text{The set } T \text{ of semantic types is the smallest set such that } e \in T, t \in T, \langle a, b \rangle \in T, \text{ for any } a, b \in T, \langle a, b \rangle \in T, \text{ and for any } a \in T, a^\ast \in T.\]

The domains of types \(a^\ast\) and \(\langle a, t \rangle\) are disjoint, although they will have the same algebraic structure. For instance, the plural set with the elements \(a_e + b_e\) and \(a_e + c_e\) will be of type \(e^\ast\) and distinct from its counterpart in \(D_{(e,t)}\), the predicate \(\lambda x. x = a_e + b_e \lor x = a_e + c_e\).

We formalize these ideas by introducing a distinction between the atomic domain \(A_d\) and the full domain \(D_a\) for each type \(a\): \(A_d\) only contains the atomic domain elements, while \(D_a\) also contains their sums. For instance, \(\text{smoke}_{(e,t)} + \text{drink}_{(e,t)} + \text{dance}_{(e,t)}\) will be an element of \(D_{(e,t)}\), but not of \(A_{(e,t)}\). The atomic domains for basic types are stipulated in the usual way (19-a). For higher types – both regular functional types and the starred types corresponding to plural sets – the atomic domain is defined recursively in terms of the full domains of lower types (19-b,c). Thus, a function of type \((e,t)\) may have plural individuals in its domain and return plural values\(^4\). Finally, we assume that the domain of type \(a^\ast\) is disjoint from, but isomorphic to the power set of \(D_a\) (19-c). This condition will allow for operations that are sensitive to whether their arguments are plural sets (type \(a^\ast\)) or ‘regular’ sets (type \(\langle a, t \rangle\)).

\[(19) \quad \begin{align*}
\text{a. } & A_e = A, \text{ the set of individuals; } A_t = \{0, 1\}^W, \text{ where } W \text{ is the set of possible worlds} \\
\text{b. } & \text{For any types } a, b: A_{\langle a, b \rangle} = D_{a}^{D_{a}}, \text{ the set of partial functions from } D_{a} \text{ to } D_{b}. \\
\text{c. } & \text{For any type } a, A_{a^\ast} \text{ is a set that is disjoint from } \mathcal{P}(D_a) \text{ and on which the operations } \cup, \cap \text{ and } \setminus \text{ are defined. Further, there is a function } p_{a^\ast} : \mathcal{P}(D_a) \rightarrow A_{a^\ast} \text{ that is an isomorphism.}
\end{align*}\]

\(^4\)In this paper, we will not need atomic functions with plural values, but they allow us to integrate predicate-level distributivity markers like each into the present framework. Allowing plural arguments for atomic functions might help with the semantics of collective predicates.
We still need to specify how the full domain $D_a$ is derived from the atomic domain $A_a$. We formalize this relation in (20) using a cross-categorial operation $+$ that maps any nonempty set of denotations (of the same type) to its sum. Clause (20-a-ii) expresses the one-to-one correspondence between the sums in $D_a$ and nonempty subsets of $A_a$: The isomorphism $p_{l_a}$ maps any nonempty set of atomic meanings to its unique sum. Clause (20-b) prevents us from directly identifying pluralities with sets of atomic meanings, which is necessary because the composition rules will treat them differently — i.e. $a_e + b_e$ should be among the possible denotations of type $e$, not $(e, l)$ or $e^*$.

(20)  
\begin{enumerate}
\item For each type $a$, there is an atomic domain $A_a$ and a full domain $D_a$ with the following properties:
  \begin{enumerate}
  \item $D_a$ is a set such that $A_a \subseteq D_a$ and there is an operation $+_a : \mathcal{P}(D_a) \setminus \{\emptyset\} \rightarrow D_a$.
  \item There is a function $p_{l_a} : \mathcal{P}(A_a) \setminus \{\emptyset\} \rightarrow D_a$ such that: $p_{l_a}(\{x\}) = x$ for each $x \in A_a$ and $p_{l_a}$ is an isomorphism from $\mathcal{P}(A_a) \setminus \{\emptyset\}$ to $(D_a, +)$.
  \end{enumerate}
\end{enumerate}

b. For any type $b \neq a$, $D_a$ and $D_b$ are disjoint.

To improve readability, we introduce some notational conventions:

(21)  
\begin{enumerate}
\item We use ‘starred’ variables like $x^*$, $P^*$ etc. for types of the form $a^*$.
\item We sometimes omit type subscripts on cross-categorial operations like $+_a$ or $p_{l_a}$.
\item For variables $x, x_1, \ldots, x_n$ of any type, we write $[x_1, \ldots, x_n]$ for the plural set $p_l([x_1, \ldots, x_n])$ with elements $x_1, \ldots, x_n$, and $[x | \phi]$ for the plural set $p_l([x | \phi])$. Informally, square brackets replace the usual set brackets whenever we are dealing with plural sets.
\item For any type $b$ and $x, y \in D_b$:
  \begin{enumerate}
  \item $x +_b y = \text{def} \bigvee_\mathcal{P}([x, y])$ (binary sum operation)
  \item $x \leq y \iff x +_b y = y$ (parthood)
  \item $x \leq_a y \iff x \leq y \land x \in A_b$ (atomic parthood)
  \end{enumerate}
\end{enumerate}

Here are some toy examples for illustration. (22) shows that $D_e$ contains both atoms and pluralities of type $e$, and that the domain $A_{e^*}$ of plural sets of individuals is isomorphic to the power set of $D_e$. ($D_{e^*}$ would then contain sums of such plural sets, like $[A] + [A + B]$, plus the elements of $A_{e^*}$.) (23) illustrates that $D_{(e, l)}$ contains sums of predicates in addition to the familiar ‘atomic’ predicates.

(22)  
\begin{enumerate}
\item $A_e = \{A, B\}$, $D_e = \{A, B, A + B\}$
\item $A_{e^*} = \{[\ldots], [A], [B], [A + B], [A, B], [A, A + B], [B, A + B], [A, B, A + B]\}$
\end{enumerate}

(23)  
\begin{enumerate}
\item $A_{(e, l)} = \{\text{smoke}_{(e, l)}, \text{dance}_{(e, l)}, (\lambda x. \text{smoke}_{(e, l)}(x) \lor \text{dance}_{(e, l)}(x)), \ldots\}$
\item $D_{(e, l)} = \{\text{smoke}_{(e, l)}, \text{dance}_{(e, l)}, (\lambda x. \text{smoke}_{(e, l)}(x) \lor \text{dance}_{(e, l)}(x)), \text{smoke}_{(e, l)} + \text{dance}_{(e, l)}, \text{smoke}_{(e, l)} + (\lambda x. \text{smoke}_{(e, l)}(x) \lor \text{dance}_{(e, l)}(x)), \text{dance}_{(e, l)} + (\lambda x. \text{smoke}_{(e, l)}(x) \lor \text{dance}_{(e, l)}(x)), \ldots\}$
\end{enumerate}

**Semantics of plurals and conjunction**  This rich ontology allows us to assign a uniform type to plural definites and plural indefinites: Both denote plural sets of type $e^*$, as in (24). While *the girls* ends up denoting a singleton set containing the sum of all the girls, a plural indefinite like *two pets* denotes the set of all sums of two pets.\(^6\)

\(^5\)The empty partial function should be exempt from the disjointness conditions.

\(^6\)This is basically a generalization of Kratzer & Shimoyama’s 2002 alternative semantics for indefinites.
The determiner meanings in (25) manipulate plural sets directly. We first define an operation $\mathcal{A}$ that extracts the atomic parts of all the pluralities in a plural set $[\text{pl}]$. Based on this, we define the pluralization operator $\mathbf{pl}$. (25-b) which applies to NPs and forms all the sums of atomic individuals satisfying the corresponding predicate. (25-c) implements the familiar idea that the definite determiner selects the maximal plurality from the NP denotation. Numerals also combine with the output of $\mathbf{pl}$ and filter out the sums of a certain cardinality (25-d).

Apart from the composition rule that implements cumulativity, to be specified below, conjunction is the only binary operation in our system that directly combines two plural sets. The core of both these operations is a cross-categorical operation $\oplus$ that combines multiple plural sets into one. This operation has a ‘distributive’ effect: It yields the set of all pluralities obtained by selecting one element from each argument set and summing up all the selected elements. Thus, $[\text{a girl and two pets}]$ in (26-a) denotes a plural set containing all sums of a girl and two pets. If the arguments are singleton sets as in (26-b) $\oplus$ produces a singleton set, which reflects the semantic parallelism between conjunctions (of any semantic category) and plural definites.

(26) a. $[\text{a girl and two pets}] = [\text{a girl}] \oplus [\text{two pets}]$
   = $[A, B] \oplus [C + D, C + E, D + E]$
   = $[A + C + D, A + C + E, A + D + E, B + C + D, B + C + E, B + D + E]$

b. $[\text{smoke and drink}] = [\text{smoke}_{(e,t)}] \oplus [\text{drink}_{(e,t)}] = [\text{smoke}_{(e,t)} + \text{drink}_{(e,t)}]$

(27) gives a definition of this operation which is general enough to permit arbitrarily many arguments. For arguments of a non-plural type, $\oplus$ coincides with the ordinary sum operation $\oplus$. The ‘distributive’ effect emerges when the arguments are plural sets: (27-b) says that we consider all the different ways of choosing an element from each of the sets. Each such choice corresponds to a possible value for the function variable $f$. The output is the set of all pluralities obtained by summing up the selected elements for some choice of $f$. (28) illustrates this for two functions that apply to the meanings of the two conjuncts in (26-a). If the selected elements are themselves plural sets, ‘summing them up’ means applying $\oplus$ recursively.\footnote{Since the restrictor predicate may itself be a plural set, (25-b) allows us to give a plausible semantics for examples with cumulativity in the restrictor, like the dogs and cats of the linguists.}\footnote{Strictly speaking, $\oplus S$ is undefined if any element of $S$ is a sum of two or more plural sets, since such sums are not in the domain of the shift $pl^{-1}$. As this situation will not occur in our linguistic applications of (27), it is irrelevant how the definition is extended to these cases.}

(27) For any type $a$, the operation $\bigoplus_a : \mathcal{P}(D_a) \backslash \{\emptyset\} \to D_a$ is defined as follows:
For any nonempty $S \subseteq D_a$:

a. If $a$ is a non-plural type (i.e. $a$ is not of the form $b^*$): $\bigoplus_a S = \bigoplus_a S$

b. If $a = b^*$ for some type $b$: $\bigoplus_a S = [\bigoplus_b \{f(X^*) | X^* \in S\}]$ if $f$ is a function from $S$ to $D_b$ and...
∀X* ∈ S : f(X*) ∈ p^l−1(X*)]

(28)  \[ a \text{ girl} \] = [A, B]; \[ \text{two pets} \] = [C + D, C + E, D + E]

a. \[ f : [A, B] \mapsto A, [C + D, C + E, D + E] \mapsto C + D \]
\[ \bigoplus_e (\{f(X^*) | X^* ∈ \{[(a \text{ girl}], \{\text{two pets}])\}) = A + C + D \]

b. \[ f' : [A, B] \mapsto B, [C + D, C + E, D + E] \mapsto D + E \]
\[ \bigoplus_e (\{f'(X^*) | X^* ∈ \{[(a \text{ girl}], \{\text{two pets}])\}) = B + D + E \]

\[ \bigoplus \] lets us formulate an analysis of conjunction that integrates well with our meanings for plural definites and indefinites (29), (30). According to (30), the two arguments of and must be plural sets, so any non-plural conjuncts need to be shifted to singleton plural sets before combining with and. Given (30) and our semantics for definites and indefinites, the results in (26) can be derived.

(29)  Notational convention: For any type \( a \) and any \( x, y ∈ D_a : x ⊕_a y = \bigoplus_a (x, y) \).

(30)  \[ [(\text{and} \langle a, \langle a, a' \rangle \rangle)] = \lambda x_{a'} . \lambda y_{a'} . x ⊕_{a'} y \text{ for any type } a \]

We have now analyzed several classes of expressions that participate in cumulative readings. All of them denote plural sets. The only missing piece is an explicit formulation of the composition rule combining plural sets which will replace cumulation operators like ** or ***.

**Adding plural projection to the compositional system**  
Our goal is to generalize the notion of cumulativity so that it can apply at any compositional step. For instance, to analyze the VP in (31), we want to combine a conjunction of two transitive verbs, which has type \( \langle e, \langle e, t \rangle \rangle^* \), with a plural object of type \( e^* \) and obtain a plural set of intransitive predicates (type \( \langle e, t \rangle^* \)).

(31)  Ada [VP fed and brushed the three pets].

In such cases, we want functional application to ‘apply cumulatively’ to a plural set of functions and a plural set of their arguments. But within existing theories of cumulativity, this does not make sense: cumulativity is only defined for operations that map their arguments to a truth value, while functional application within a transitive VP yields a result of type \( \langle e, t \rangle \). We therefore must define cumulativity for operations with an output type other than \( t \). To do so, we first reformulate the traditional notion of a binary cumulative relation in terms of plural sets. The resulting definition will generalize to other binary operations in an obvious way.

According to the standard view, a relation \( R \) holds cumulatively of two pluralities \( P \) and \( x \) iff each atomic part of \( P \) stands in relation \( R \) to some atomic part of \( x \), and for each atomic part of \( x \), there is an atomic part of \( P \) that stands in relation \( R \) to it. Viewing \( R \) as a set of ordered pairs, we can paraphrase this condition as follows: There must be a subset \( C \) of \( R \) that ‘covers’ all the atomic parts of \( P \) and \( x \), in the sense that 1) the atomic parts of \( P \) are exactly the first components of the pairs in \( C \), and 2) the atomic parts of \( x \) are exactly the second components of the pairs in \( C \). The formal definition of a cover is given in (32-a) (32-b): gives some examples of covers for the verb conjunction and the plural object in (31).

(32)  a. Let \( P ∈ D_a, x ∈ D_b \). A relation \( R ⊆ A_a × A_b \) is a cover of \( \langle P, x \rangle \) iff \( \{P' | \forall x' : (P', x') ∈ R\} = P \) and \( \{x' | \exists P' : (P', x') ∈ R\} = x \).

b. \( P = \text{fed + brushed} \), \( x = C + D + E \)
Covers: \( \langle \{\text{fed}, C\}, \{\text{fed}, D\}, \{\text{brushed}, E\} \rangle, \langle \{\text{fed}, C\}, \{\text{fed}, D\}, \{\text{brushed}, D\}, \{\text{brushed}, E\} \rangle, \langle \{\text{fed}, C\}, \{\text{fed}, D\}, \{\text{brushed}, D\}, \{\text{brushed}, E\} \rangle, \ldots \)
The truth conditions of cumulative sentences like (31) can then be expressed in terms of existential quantification over covers: $R$ holds cumulatively of $P$ and $x$ iff there is a cover $C$ of $(P, x)$ such that $R$ is true of each pair in $C$. Now recall our idea from Section 2.1 that sentences containing plurals denote a plural set of propositions and their truth conditions can be derived by existential quantification over this set. The generalization about covers brings us closer to an implementation of this idea if we can find some way of mapping the covers to sums of propositions. Intuitively, we want a plural set whose elements correspond to the different covers of $(P, x)$ – for instance, the plural set assigned to (31) will contain elements for all the covers in (32-b), as well as all other covers. If we have access to the relation $R$ that applies cumulatively to the two pluralities, there is an obvious way of mapping a cover to a sum of propositions: For each pair $(P', x')$ in the cover, the proposition $R(P', x')$ will be an atomic part of the sum. This generalization is stated in (33-a). Under the assumption that the cumulative relation between the predicate sum $\text{fed} + \text{brushed}$ and the individual sum $C + D + E$ is the one given in (33-b), the covers in (32-b) correspond to the propositional pluralities in (33-c).

\begin{align*}
\text{(33)} & \quad \text{a. A sentence in which a relation } R \text{ applies cumulatively to two pluralities } P \text{ and } x \text{ will denote} \\
& \quad \text{the plural set } \{ \bigoplus ((R(P', x') \mid (P', x') \in C) \mid C \text{ is a cover of } (P, x)) \}.
\end{align*}

b. $R = \lambda P(a, (e, f)) \cdot \lambda x. P(x)(\text{Ada})$

c. $[\text{fed}(C)(\text{Ada}) + \text{fed}(D)(\text{Ada}) + \text{brushed}(E)(\text{Ada}) + \text{fed}(C)(\text{Ada}) + \text{brushed}(D)(\text{Ada}) + \text{brushed}(E)(\text{Ada}) + \text{fed}(D)(\text{Ada}) + \text{brushed}(D)(\text{Ada}) + \text{brushed}(E)(\text{Ada}) \ldots]$

So we found a way of mapping simple cumulative sentences to plural sets of propositions, such that a cumulative sentence is true if and only if there is at least one sum of true propositions in the corresponding set. A formal definition of this notion of truth is given in (34). Accordingly, the plural set in (33-a) counts as true iff the relation $R$ applies cumulatively to $P$ and $x$.

\begin{align*}
\text{(34)} & \quad \text{A plural set } P^* \text{ of propositions is true in a world } w \text{ iff there is a plurality } p \in pl^{r-1}(P^*) \text{ such that for all } q \preceq_a p, q(w) = 1, \text{ and false in a world } w \text{ iff for all pluralities } p \in pl^{r-1}(P^*), \text{ there is a } q \preceq_a p \text{ such that } q(w) = 0.
\end{align*}

How does this help us with interpreting cumulative sentences compositionally? (33-a) is not a compositional rule, but merely a generalization about sentences that somehow involve a cumulative relation: The sentence (31) does not contain any constituent denoting the required cumulative relation in (33-b). We cannot access this relation unless we posit a corresponding LF constituent. However, our reformulation of the predicate analysis in terms of an operation that outputs a plural set will now enable us to leave the predicate analysis behind.

There are two important aspects to (33-a). First, (33-a), unlike our earlier characterization of relations applying cumulatively, is not a direct statement of truth conditions. Truth conditions are now assigned by (34), while (33-a) simply provides a way of computing a plural set. Second, $R$ in (33-a) is assumed to be a binary relation, i.e. a function that maps its two arguments to a truth value, but nothing prevents us from dropping this restriction and defining cumulative versions of binary operations with arbitrary output types. This generalization of (33-a) is formalized in (35-a). It can be further generalized to cover cases where $R$ combines with two plural sets, rather than with two single pluralities (35-b): We simply consider all relations that are covers of some element of $P^*$ and some element of $x^*$.

9The question how cumulativity interacts with presupposition projection, homogeneity and other potential cases of trivalence is left open here.
(35) For arbitrary types $a, b, c$ and any binary operation $R_{(a, b, c)}$:
   a. When $R$ applies cumulatively to two pluralities $P_a$ and $x_b$, the result is the plural set
      \[ \bigoplus ((R'(P', x') | (P', x') \in C)) | C \text{ is a cover of } (P, x) \] of type $c^\ast$.
   b. When $R_{(a, (b, c))}$ applies cumulatively to two plural sets $P_a^\ast$ and $x_b^\ast$, the result is the plural set
      \[ \bigoplus ((R'(P', x') | (P', x') \in C)) | \exists P \in pl^{l-1}(P^\ast), x \in pl^{l-1}(x^\ast) : C \text{ is a cover of } (P, x) \] of type $c^\ast$.

Definition (35-b) provides a general way of lifting any binary operation of an arbitrary type $\langle a, (b, c) \rangle$ to an operation on plural sets, which has type $\langle a^\ast, (b^\ast, c^\ast) \rangle$. If the binary operation maps its two arguments to a truth value, the semantic effect of (35-b) resembles that of the predicate analysis. For instance, applying the relation $R$ in (33-b) cumulatively (in the sense of (35-b)) to the two plural sets $[\text{fed + brushed}]$ and $[C + D + E]$ yields the plural set of propositions in (33-c). Given definition (34), this predicts the same truth conditions as the predicate analysis. But since (35-b) applies to operations of arbitrary type, we can now approach the problem differently: Our original goal was to directly combine the two plural sets $[\text{fed + brushed}]$ and $[C + D + E]$ to obtain a plural set of type $\langle e, t \rangle^\ast$. In plural-less sentences, transitive verbs combine with their type $e$ objects via functional application. Since functional application is a binary operation, we can derive a cumulative version of it from definition (35-b). This cumulative functional application rule takes two plural sets of types $\langle a, b \rangle^\ast$ and $a^\ast$ as arguments, yielding a plural set of type $b^\ast$ (36-a). We add this operation to our semantic system as the composition rule ‘Cumulative Composition (CC)’ (36-b).

(36) Cumulative Composition (CC)
   a. For any $P^\ast \in D_{(a, b)^\ast}$ and $x^\ast \in D_{a^\ast}$:
      \[ C(P^\ast, x^\ast) = \bigoplus ((P'(x') | (P', x') \in R)) | \exists P \in pl^{l-1}(P^\ast), x \in pl^{l-1}(x^\ast) : R \text{ is a cover of } (P, x) \]
   b. For any meaningful expressions $\phi$ of type $\langle a, b \rangle^\ast$ and $\psi$ of type $a^\ast$, $[\phi \psi]$ is a meaningful expression of type $b^\ast$, and $[\phi \psi] = C([\phi], [\psi])$.

The effect of (36-a) is that for any cover of some plurality in the functor set and some plurality in the argument set, we perform functional application for all pairs related by the cover and sum up the results. We then collect all the value pluralities corresponding to different covers into one plural set.

Let’s apply this rule to the VP in (31), repeated in (37). We construct all the relations that are covers of some element of $[\text{fed + brushed}]$ and some element of $[C + D + E]$ – i.e., all the covers of $[\text{fed + brushed}, C + D + E]$. Some examples of such covers are repeated in (37). The output of CC is the plural set of unary predicates indicated in (37-c) (we only give the pluralities corresponding to the three covers in (37-b) – of course every cover of the verb conjunction and the plural object yields an element of the set).

(37) a. $\text{Ada} [\text{VP fed and brushed the three pets}].$
   b. $[\langle \text{fed}, C \rangle, \langle \text{fed}, D \rangle, \langle \text{brushed}, E \rangle], [\langle \text{fed}, C \rangle, \langle \text{brushed}, D \rangle, \langle \text{brushed}, E \rangle], [\langle \text{fed}, C \rangle, \langle \text{fed}, D \rangle, \langle \text{brushed}, D \rangle, \langle \text{brushed}, E \rangle], \ldots$
   c. $[\text{fed}(C) + \text{fed}(D) + \text{brushed}(E), \text{fed}(C) + \text{brushed}(D) + \text{brushed}(E), \text{fed}(C) + \text{fed}(D) + \text{brushed}(D) + \text{brushed}(E), \ldots]$

How does the plural set in (37-c) combine with a semantically singular, type $e$ subject? Since (37-c) has type $\langle e, t \rangle^\ast$, ordinary functional application cannot apply. We invoke our earlier assumption that non-plural denotations can be shifted to singleton plural sets: We shift the subject to the plural set $[\text{Ada}]$, which combines with (37-c) via yet another application of CC. $\text{Ada}$ is not a plurality, so every predicate sum in (37-c) corresponds to a unique cover, which relates each atomic part of the sum to the individual
Ada. We thus end up with a set of propositional pluralities that has the same structure as (37-c), shown in (38). Given \([34]\) this set captures the cumulative truth conditions of (37-a).

\[
(38) \quad [\text{fed}(C)(A) + \text{fed}(D)(A) + \text{fed} \text{brushed}(E)(A), \text{fed}(C)(A) + \text{fed} \text{brushed}(D)(A) + \text{fed} \text{brushed}(E)(A), \\
\quad \text{fed}(C)(A) + \text{fed}(D)(A) + \text{fed} \text{brushed}(D)(A) + \text{fed} \text{brushed}(E)(A) \ldots]
\]

CC thus lets us derive the truth conditions of (37-a) without recourse to syntactically derived cumulative relations. The locus of cumulativity is neither the lexicon nor an operator attaching to relation-denoting expressions, but a composition rule that is essentially a cumulative version of functional application. In order to see how this mechanism differs conceptually from the predicate analysis and why we call it ‘plural projection’, it is instructive to consider an example where the two plurals are not syntactic sisters, like (39-a). The simplest version of the predicate analysis would associate (39-a) with the LF in (39-b), where the cumulative truth conditions arise from the cumulation operator ** modifying the relational expression \(\text{fed}\), and each step involves regular functional application. On the plural projection approach, however, cumulativity is not attributed to a single LF constituent: The lexical meaning of \(\text{fed}\) is a relation between atomic individuals. This is mapped to a singleton plural set by a type-shift we call \(\uparrow\) in (39-c). This shift, crucially, does not turn \(\llbracket \text{fed} \rrbracket\) into a cumulative relation. Rather, cumulativity is the result of two applications of CC, which occur at the nodes marked \(C\) in (39-c).

\[
(39) \quad \begin{align*}
&\text{a. The two girls fed two pets.} \\
&\text{b. \llbracket \text{the two girls} \rrbracket \llbracket \text{fed} \rrbracket \llbracket \text{two pets} \rrbracket]} \\
&\text{c. \llbracket C \text{the two girls} \rrbracket \llbracket C \llbracket \text{fed} \rrbracket \llbracket \text{two pets} \rrbracket]}
\end{align*}
\]

Given the DP meanings in (40-a), composition proceeds as follows: Since there are no proper pluralities in \(\llbracket \text{fed} \rrbracket\), each plurality in the object plural set corresponds to exactly one cover, and CC reduces to applying \(\text{fed}\) to each atomic part of the pluralities of pets, (40-b). Thus, \(\llbracket \text{fed two pets} \rrbracket\) ‘preserves’ the plural structure introduced by \(\llbracket \text{two pets} \rrbracket\): The parts of the predicate sums in (40-b) correspond to the parts of the pluralities of pets in (40-a). This is what we called the ‘projection’ behavior of semantic plurality. The next step is to apply CC to this predicate set and the set containing the sum of the two girls, resulting in a plural set of propositions indicated in (40-c). Importantly, there is a one-to-one correspondence between the covers we consider at this step, which relate girls to atomic parts of the predicate sums in (40-b), and the covers we would construct on the predicate analysis, which relate girls to pets.

\[
(40) \quad \begin{align*}
&\text{a. } \llbracket \text{the two girls} \rrbracket = [A + B]; \llbracket \text{two pets} \rrbracket = [C + D, C + E, D + E] \\
&\text{b. } \llbracket \text{fed two pets} \rrbracket = C(\llbracket \text{fed} \rrbracket, \llbracket \text{two pets} \rrbracket) \\
&\quad = C(\llbracket \text{fed} \rrbracket, [C + D, C + E, D + E]) \\
&\quad = [\text{fed}(C) + \text{fed}(D), \text{fed}(C) + \text{fed}(E), \text{fed}(D) + \text{fed}(E)] \\
&\text{c. } \llbracket \text{(40-a)} \rrbracket = C(\llbracket \text{fed two pets} \rrbracket, \llbracket \text{the two girls} \rrbracket) \\
&\quad = C([\text{fed}(C) + \text{fed}(D), \text{fed}(C) + \text{fed}(E), \text{fed}(D) + \text{fed}(E)], [A + B]) \\
&\quad = [\text{fed}(C)(A) + \text{fed}(D)(B), \text{fed}(C)(B) + \text{fed}(D)(A), \text{fed}(C)(A) + \text{fed}(D)(A) + \text{fed}(D)(B), \ldots, \\
&\quad \text{fed}(C)(A) + \text{fed}(E)(B), \text{fed}(C)(B) + \text{fed}(E)(A) \ldots]
\end{align*}
\]

In sum, the possibility of iterating CC allows us to interpret cumulative sentences surface-compositionally without having to derive ‘pluralized’ versions of predicates. Configurations in which the two plurals are not in a strictly local relation, like (39-a), can be reduced to a series of local steps, because the meaning of an expression containing a plural will preserve certain aspects of the part structure of that plural.
2.3 Independent motivation for plural projection

The predictions of this new system, with its much richer ontology, differ from those of the predicate analysis in at least two relevant respects.

Recall that in the predicate analysis operators like ** attach to relation-denoting constituents in the syntax. If the cumulative relation does not correspond to a surface constituent, as in (15) above, covert movement of the two plurals is invoked to derive a suitable LF constituent. This gives rise to two problems (see Schmitt 2018) that our system avoids.

**Issue 1: Locality** First, since covert movement is often thought to be constrained by syntactic locality, cumulative readings should only be available in configurations known to permit covert movement (see Beck & Sauerland 2000). However, this prediction is problematic: (41-a), for instance, is true in the ‘cumulative’ scenario in (41-b). The predicate analysis thus would have to derive the relation \[AP.λy.y\] thinks that Trump should \(P\) by covert movement – but this movement should arguably be blocked, as it would have to cross a clause boundary.

(41) a. The Georgian ambassador called this morning, the Russian one at noon. They think that Trump should talk to Putin and build a hotel in Tbilisi, but neither addressed the Caucasus conflict! true in (41-b)


While this argument is based on debatable assumptions about the locality of covert movement, the general point is that the plural projection mechanism derives these cases naturally, because it never appeals to syntactically derived predicates as the input for cumulation operators. This will become relevant in Section 6, where we compare our analysis of ADUs to existing proposals.

**Issue 2: The flattening effect** The second problem with the predicate analysis has more immediate relevance for our analysis of ADUs. The core assumption behind the predicate analysis is that cumulative relations always correspond to object-language constituents. This assumption is challenged by cumulative readings of sentences in which one plural expression syntactically contains another. For instance, (42-b) is true in scenario (42-a).


b. The two girls made Gene [[P feed the two dogs] and [Q brush Eric]] when all he wanted to do was take care of his hamster. true in (42-a) (Schmitt 2018)

The predicate conjunction \(P\) and \(Q\) has a cumulative reading relative to the two girls: In scenario (42-a) it is not the case that each girl made Gene brush Eric, as a distributive interpretation of predicate conjunction (e.g. von Stechow 1974, Gazdar 1980, Partee & Rooth 1983 a.o.) would require. Rather, the relation \[AP.λx.x\] made Gene do \(P\) intuitively applies cumulatively to the two girls and the two predicates \(P\) and \(Q\). As there is no surface constituent denoting this relation, the obvious solution within the predicate analysis would be to assume an LF like (43).

(43) [[the two girls] [[feed the two dogs and brush Eric] [[* [2 [1 [t1 made Gene t2]]]]]]]

The problem is that scenario (42-a) also requires a cumulative reading of the two dogs relative to the two girls, since neither of the girls made Gene feed both dogs. There is no obvious way of interpreting
feed the two dogs in (43) that accounts for this fact. In particular, interpreting feed the two dogs as being true of plural individuals that cumulatively feed both of the dogs will not solve the problem, since the semantic argument of feed the two dogs in (43) is the non-plural individual Gene.

Intuitively, three plurals participate in this cumulative reading – the two girls, the two dogs and the predicate conjunction – but there is no way of deriving a relation that might form the input for a cumulation operator, since one of the three plurals syntactically contains another. If we move only two plural expressions in the syntax, as in (43), the resulting LF won’t give us the right semantics. But moving the two dogs out of the predicate conjunction won’t work either, given the syntactic proposal in Beck & Sauerland (2000): The predicate conjunction would contain an unbound trace, yielding an uninterpretable structure. We thus have a cumulative reading that cannot be reduced to a single cumulative relation. On our approach, there is no need to express the truth conditions of (42-b) in terms of such a relation: The derivations of cumulative sentences usually involve multiple applications of CC – one for each node on the ‘path’ between the plurals participating in cumulativity.

The relevant part of the derivation is sketched in (44), where the nodes labeled (i), (iii) and (iv) all involve CC. The part structure of the plural set [the two dogs] = [C + D] ‘projects’ to the denotation of the first VP. In step (ii), the VPs are conjoined via the operation ⊕, which, when applied to two singleton plural sets as in (44), returns another singleton set containing the sum of their elements. The crucial trait of this analysis that the predicate analysis cannot replicate is that the VP conjunction denotes a sum of three predicates in which the plural structure introduced by the two dogs and the plural structure introduced by conjunction are treated on a par. Although one plural expression occurs in the scope of another, we end up with a single ‘flat’ plurality that makes parts corresponding to both plurals accessible. We call this phenomenon the flattening effect.

(44)

In the next ‘projection’ step (iii), each of these predicates applies to the individual Gene. This yields a sum of three propositions, which combine with the matrix predicate via yet another application of CC. (Strictly speaking, since intensional verbs do not combine with their arguments via regular functional application, a cumulative version of intensional functional application is needed here (cf. Schmitt 2018).) We then end up with yet another sum of three predicates, which combines in the familiar way with the plural matrix subject.
2.4 Interim summary

The derivation in (44) illustrates two interesting properties of the system we introduced in this section. First, it permits a surface-compositional treatment of non-lexical cumulative relations like $[\lambda P, \lambda x. x \text{ made Gene do } P]$, and second, it naturally deals with configurations in which two plural expressions ‘participate’ in cumulativity even though one of them has scope over the other. This is made possible by CC, which is a cumulative version of functional application. To derive this rule, we had to determine what it means for an operation with arbitrary argument and result types to apply cumulatively. Two nonstandard ontological assumptions were necessary for this notion to make sense: First, we distinguish between ordinary sets (in the sense of unary predicates) and plural sets, which are treated differently by the composition rules. Plural sets can be viewed as a generalization of the alternative sets involved in a Hamblin semantics for indefinites (Kratzer & Shimoyama 2002). Second, we generalize the sum operation to all semantic domains, a move independently supported by analogies between conjunctions of arbitrary categories and plural definites.

3 A plural projection account of every DPs

But why introduce all this machinery for the analysis of ADUs? At the beginning of this paper, we saw that ADUs exhibit two distinctive traits. First, they can cumulate with syntactically higher plurals, but not with syntactically lower plurals, as the contrast in (45) (= (1-b), (2)) shows. Second, when they occur in Schein sentences like (46) (= (6)), the mixed cumulative/distributive reading cannot be analyzed via a single cumulative relation between individuals.

(45)  
a. *Every girl* in this town fed (the) two dogs.                    only distributive  
b. *(The) two girls* fed every dog in this town.                  cumulative reading possible  

(46)  
a. Scenario: There are two dogs, Carl and Dean. Ada taught Carl tricks 1 and 2. Ada taught Dean trick 3 and Bea taught Dean trick 2.  
b. Ada and Bea taught *every dog* two new tricks.                true in (46-a)  

It is the latter point that links the behavior of ADUs to the plural projection analysis, as there is a structural analogy between Schein sentences and the flattening effect: The important property of flattening sentences is that one plural expression – the VP conjunction – has another plural in its scope, but both plurals stand in a cumulative relation with a third plural higher up in the tree. This is why their truth conditions cannot be expressed in terms of a single cumulative relation between individuals. In Schein sentences, we find exactly the same configuration: The plural *two new tricks* occurs in the scope of the every DP, but still ‘participates’ in cumulativity in the sense that we need access to its part structure to formulate the cumulative truth conditions of the sentence. In Section 1.4 above, we sketched an intuitive take on this problem, namely that the subject in (46-b) must cumulatively satisfy at least one predicate sum in the plural set in (47).

(47) $[\text{taught}(T_1)(C) + \text{taught}(T_2)(C) + \text{taught}(T_1)(D) + \text{taught}(T_2)(D), \ldots]$  
$tought(T_1)(C) + \text{taught}(T_2)(C) + \text{taught}(T_2)(D) + \text{taught}(T_3)(D), \ldots]$  
$tought(T_1)(C) + \text{taught}(T_3)(C) + \text{taught}(T_1)(D) + \text{taught}(T_2)(D), \ldots]$  

The predicate pluralities in (47) can be split into parts corresponding to the individual dogs, but also have accessible parts corresponding to the individual tricks they were taught. The denotation in (47) therefore preserves both the part structure introduced by *two new tricks* and the part structure corresponding to the *every* DP and treats them on a par. This is analogous to the ‘flat’ denotation we derived for the VP in
The only missing component we need to derive (47) in our system is the lexical entry for *every*, to be discussed in Section 3.1. In combination with the plural projection mechanism, it will account for cumulativity asymmetries and the behavior of *every* DPs in Schein sentences. Thus our approach to the flattening problem illustrated by (42-b) will be general enough to extend to Schein sentences, while Section 6 will show that not all existing analyses of Schein sentences extend to the flattening problem.

3.1 The lexical meaning of *every*

In our analysis, *every* DPs are distributive w.r.t. material in their scope, but once they combine with their scope argument, the result is a plural set of values. How is this implemented? Like conjunction and plural determiners, *every* directly manipulates plural sets of predicates, thus blocking application of CC. The atomic individuals satisfying the restrictor of *every* are matched up with pluralities of predicates from its nuclear scope. For instance, when we combine *every girl* with the plural set [P + Q, R + S], each individual girl is associated with at least one predicate sum in this set, e.g. Ada with P+Q and Bea with R+S. This will give us the ‘distributive effect’. Finally, by ‘summing up’ the results again, we obtain a plural set of values – in our example, [P(A) + Q(A) + R(B) + S(B)]. Crucially, the pluralities in this set preserve the part structure of the scope argument of *every* and can be used for cumulation with syntactically higher pluralities.

The operation defined in (48-a) (repeated from (25)) takes a plural set of predicates and returns the set of all atomic individuals satisfying some part of a predicate sum in the set. (48-b) introduces another supplementary operation, \( D \), which, given a plurality \( P \) of functions with argument type \( a \) and any object \( x \) of type \( a \), applies each atomic part of \( P \) to \( x \) and returns the sum of all the values. (48-c) states the lexical entry for *every*: Its restrictor argument is a plural set of NP denotations, its nuclear scope a plural set of functions of type \( (e,a) \), for an arbitrary type \( a \). Hence, *every* DPs can combine with predicates of any arity. The idea behind (48-c) is that when *every* combines with an NP and a plural set \( R \) of predicates, we consider different functions that map each individual in the NP extension to an element of \( R \). For each such function, we take every NP individual, apply all the predicates in the predicate sum it is mapped to and then sum up the results over all the individuals. The sums obtained in this way are collected into a plural set.

(48)

a. \( \mathcal{A}(P^x_{(e,a)}) = \lambda x_e (\exists P_{(e,a)} P \in p_t^{-1}(P^x) \land \exists P'_{(e,a)} P' \leq_a P \land P'(x)) \)

b. For any \( P(a,b), x_a: D(P,x) = \{ Q(x) \mid Q \leq_a P \}) \)

c. \[ \text{[every}\langle e,a'\rangle_{(e',a''a')}] = \lambda P_{(e',a''a')} \lambda R_{(e,a)}. [\mathcal{A}(D(f(x),x) \mid x \in \mathcal{A}(P'))] \]

(48) states the lexical entry for *every*: Its restrictor argument is a plural set of NP denotations, its nuclear scope a plural set of functions of type \( (e,a) \), for an arbitrary type \( a \). Hence, *every* DPs can combine with predicates of any arity. The idea behind (48-c) is that when *every* combines with an NP and a plural set \( R \) of predicates, we consider different functions that map each individual in the NP extension to an element of \( R \). For each such function, we take every NP individual, apply all the predicates in the predicate sum it is mapped to and then sum up the results over all the individuals. The sums obtained in this way are collected into a plural set.

3.2 Deriving cumulativity asymmetries

We first derive the cumulativity asymmetry illustrated in (49), a slightly simplified version of (45) (49-a) only has a distributive reading, (49-b) permits a cumulative reading. We consider a scenario with two girls, Ada and Bea, and three pets, Carl, Dean and Eric.

(49)

a. *Every girl fed two pets.*

b. *Two girls fed every pet.*

We start with (49-a). Given our assumptions from Section 2, the VP denotes the plural set in (50). The structure of the plural set \( \llbracket \text{two pets} \rrbracket \) projects via CC.

(50) \[ \llbracket \text{fed two pets} \rrbracket = [\text{feed}(C) + \text{feed}(D), \text{feed}(C) + \text{feed}(E), \text{feed}(D) + \text{feed}(E)] \]
(51) gives the denotation of the *every* DP: *every* combines with its restrictor, a singleton plural set containing only the predicate *girl*, yielding the function in (51-a), which reduces to (51-b).

\[
\langle \text{every girl} \rangle = \lambda R^*_{(e_d)} \cdot \left[ + \langle \mathcal{D}(f(x), x) \mid x \in \Lambda[\text{girl}] \rangle \right]
\]

| a. | \lambda R^*_{(e_d)} \cdot [ + \langle \mathcal{D}(f(x), x) \mid x \in \{A, B\} \rangle ] = \lambda R^*_{(e_d)} \cdot [ \mathcal{D}(f(A), A) + \mathcal{D}(f(B), B) \mid f \text{ is a function from } \{A, B\} \text{ to } p_{f^{-1}(R')}^* ]
| b. | \lambda R^*_{(e_d)} \cdot [ + \langle \mathcal{D}(f(x), x) \mid x \in \{A, B\} \rangle ] = \lambda R^*_{(e_d)} \cdot [ \mathcal{D}(P, A) + \mathcal{D}(Q, B) \mid P, Q \in p_{f^{-1}(R')}^* ]

To apply this meaning to the VP denotation in (50), we must consider all possible different functions from \{A, B\} – the set of atomic girls – to the plural set \{50\} (52) provides two examples of such assignments.

\[
\{ (A, \text{fed}(C) + \text{fed}(D)), (B, \text{fed}(C) + \text{fed}(E)) \}, \{ (A, \text{fed}(C) + \text{fed}(E)), (B, \text{fed}(D) + \text{fed}(E)) \}, \ldots
\]

The next steps required by [48-c] are as follows: (i) For each pair \((P, x)\) in such an assignment, we add up the values that result from applying all atomic parts of \(P\) to \(x\) (53-a). (ii) For each assignment, we sum up all the value pluralities corresponding to pairs in the assignment (53-b). (iii) We collect all these pluralities into the plural set in (53-c).

\[
\{ \text{fed}(C)(A) + \text{fed}(D)(A), \text{fed}(C)(B) + \text{fed}(E)(B) \}, \{ \text{fed}(C)(A) + \text{fed}(E)(A), \text{fed}(D)(B) + \text{fed}(E)(B) \}, \ldots
\]

\[
\text{fed}(C)(A) + \text{fed}(D)(A) + \text{fed}(C)(B) + \text{fed}(E)(B) \}, \{ \text{fed}(C)(A) + \text{fed}(E)(A) + \text{fed}(D)(B) + \text{fed}(E)(B) \}, \ldots
\]

\[
\text{fed}(C)(A) + \text{fed}(D)(A) + \text{fed}(C)(B) + \text{fed}(E)(B), \text{fed}(C)(A) + \text{fed}(E)(A) + \text{fed}(D)(B) + \text{fed}(E)(B) \ldots\}
\]

We end up with (54) as our denotation for [49-a]. This plural set counts as true iff at least one of the pluralities contains only true propositions – i.e., iff Ada and Bea each fed at least two pets.

\[
\langle \text{every girl} \rangle ([50]) = \text{fed}(C)(A) + \text{fed}(D)(A) + \text{fed}(C)(B) + \text{fed}(E)(B), \text{fed}(C)(A) + \text{fed}(E)(A) + \text{fed}(C)(B) + \text{fed}(D)(B), \text{fed}(C)(A) + \text{fed}(E)(A) + \text{fed}(D)(B) + \text{fed}(E)(B) \ldots
\]

This correctly predicts that [49-a] only has a distributive reading, since we assign a separate predicate sum to each individual girl and apply the predicate sums ‘distributively’ to the corresponding girls via the operator \(\mathcal{D}\) in [48-b]. Since each girl must satisfy all the predicates in the sum assigned to her, and each predicate sum amounts to feeding a certain plurality of two pets, each girl is related to two pets. Thus, we don’t get pluralities like \text{fed}(D)(A) + \text{fed}(C)(B), and the cumulative reading of [49-a] is blocked. While the entire sentence denotes a set of pluralities of propositions, [49-a] does not contain any higher plural that could combine with this set via CC.

Next, we derive the cumulative reading of [49-b] where the *every* DP occurs in object position. First, we apply \(\langle \text{every pet} \rangle\) to the singleton set \{fed\}. As we have only one predicate to assign, there is only one assignment of predicate sums to the individual pets. Definition [48] thus yields a singleton set [55-a]. Importantly, since this set contains a plurality, it can combine with the subject plurality via CC. We obtain a set, partially given in [55-b] which contains all sums of propositions of the form \text{fed}(x)(y) that ‘cover’ every pet and also ‘cover’ both Ada and Bea. This is exactly what we need for the cumulative reading.
The plural set in (57-b) then combines with Ada corresponds to our basic idea for Schein sentences from Section 1. Each satisfy at least one atomic part of Accordingly, the sentence is true if there is a predicate plurality \( P \) in (58).

\[
C([\text{taught two new tricks}]) = [\text{taught}(T1) + \text{taught}(T2), \text{taught}(T1) + \text{taught}(T3), \text{taught}(T2) + \text{taught}(T3)]
\]

The present analysis of every therefore predicts cumulativity asymmetries: Since every DPs directly take plural sets as their arguments, CC is blocked when they combine with their nuclear scope, and the distributive effect can be built into the lexical entry. Since the result of combining every with its arguments is a plural set, CC with higher plurals is not blocked.

3.3 Deriving Schein sentences

We now consider how this approach derives the interaction between distributivity and cumulativity in Schein sentences. Recall that (56) \((46-b)\) is true in scenario \((46-a)\) above.

(56) Ada and Bea taught every dog two new tricks. true in \((46-a)\)

The predicate taught two new tricks denotes the plural set in (57-a) due to CC and our analysis of plural indefinites. When combining this set with every dog, we must consider all possible assignments that map each dog to a sum in the set. As opposed to (55) above, where the every DP combined with a singleton plural set, we now have a plural set with more than one element. Therefore, Carl and Dean may be mapped to different elements of (57-a) – and since each dog is combined with all the predicates in his corresponding sum, we obtain a 'distributive' interpretation of every dog relative to two new tricks. For each assignment of predicate pluralities to the two dogs, the results of functional application are summed up, yielding the plural set in (57-b). This plural set matches our earlier intuitions: Each of the predicate pluralities encodes ‘distributivity’ in the sense that every dog is related to two tricks, but also has accessible parts corresponding to the individual dog-trick pairs, reflecting the part structure of two tricks.

(57) a. \([\text{taught two new tricks}] = C([\text{taught}], [\text{two new tricks}]) = [\text{taught}(T1) + \text{taught}(T2), \text{taught}(T1) + \text{taught}(T3), \text{taught}(T2) + \text{taught}(T3)]\)

b. \([\text{every dog} \, \text{taught two new tricks}] = [\text{taught}(T1)(C) + \text{taught}(T2)(C) + \text{taught}(T3)(D), \text{taught}(T1)(D) + \text{taught}(T2)(D) + \text{taught}(T2)(C) + \text{taught}(T3)(C), \ldots ]\)

The plural set in (57-b) then combines with [Ada + Bea] via CC, resulting in the plural set of propositions in (58).

(58) \(C([\text{every dog} \, \text{taught two new tricks}]) = [\text{taught}(T1)(C)(A) + \text{taught}(T2)(C)(A) + \text{taught}(T2)(D)(B) + \text{taught}(T3)(D)(A), \text{taught}(T1)(D)(B) + \text{taught}(T2)(D)(B) + \text{taught}(T2)(C)(A) + \text{taught}(T3)(C)(A), \ldots ]\)

Accordingly, the sentence is true if there is a predicate plurality \( P \) in (57-b) such that Ada and Bea each satisfy at least one atomic part of \( P \), and each atomic part of \( P \) is satisfied by Ada or Bea. This corresponds to our basic idea for Schein sentences from Section 1.
3.4 Interim summary

We supplemented the plural projection framework with a lexical entry for every: every DPs are distributive regarding material in their scope, but the result of combining them with their nuclear scope is a plural set. This plural set reflects the part structure of the pluralities in the scope of the every DP and makes them accessible for further cumulative composition. This proposal correctly predicts cumulativity asymmetries and also derives the particular readings of Schein sentences.

4 A plural projection account of D-conjunctions

We now turn to another type of ADU, the distributive conjunctions mentioned in Section 1. After a brief discussion of the empirical situation, we show how our account can capture these cases as well.

4.1 D-conjunctions: Empirical background

Many languages have more than one conjunction strategy for conjuncts of type e (see e.g. Szabolcsi 2015, Flor et al. 2017, to appear). German, for instance, has A und B (59-b) as well as sowohl A als auch B (59-c) (= (4-b) from above), Hungarian (discussed by Szabolcsi 2015) has A és B (60-a) as well as A és és B is (60-b), and Polish has A i B (61-b) alongside i A i B (61-c).

(59) German

a. scenario: Two skiing races took place today. Ada and Bea were the only German participants. Ada competed in the downhill and won. Bea competed in the slalom and won.

b. Heute haben die Ada und die Bea die zwei Rennen gewonnen!
   ‘Today, Ada and Bea won the two races.’ true in (59-a)

c. Heute haben sowohl die Ada als auch die Bea die zwei Rennen gewonnen!
   false in (59-a)

(60) Hungarian (Dóra Kata Takács, p.c.)

a. scenario: Sára called ‘Express Catering’. Marcsi called ‘Star Catering’.

b. Sára és Marcsi időben felhívta a két kiszállító céget.
   Sára and Marcsi on-time called the two catering company.acc
   ‘Sára and Marcsi called the two catering companies ahead of time.’ true in (60-a)

c. Sára is és Marcsi is időben felhívta a két kiszállító céget.
   Sára is and Marcsi is on-time called the two catering company.acc
   ‘Both Sára and Marcsi called the two catering companies ahead of time.’ false in (60-a)

(61) Polish (Magdalena Roszkowski and Marcin Wągiel, p.c.)

a. scenario: Sabina called ‘Express Catering’. Magda called ‘Star Catering’.

b. Sabina i Magda dostatecznie wcześniej zadzwoniły do tych dwóch restauracji.
   Sabina i Magda enough early called to these two restaurants
   ‘Sabina and Magda called these two restaurants early enough.’ true in (60-a)

c. I Sabina i Magda dostatecznie wcześniej zadzwoniły do tych dwóch restauracji.
   I Sabina i Magda enough early called to these two restaurants
   ‘Both Sabina and Magda called these two restaurants early enough.’ false in (60-a)
The data in (59) – (61) show that in each language the two strategies differ semantically: While what we call the ‘simple conjunction’ strategies in the (b)-examples permit a cumulative reading in the syntactic contexts provided, the strategies in the (c)-cases are restricted to a distributive reading in the same syntactic contexts. That’s why we call the latter strategy ‘distributive conjunctions’, following Szabolcsi 2015 for the Hungarian case. In Hungarian and Polish, the D-conjunction strategies morphosyntactically include the simple conjunction strategies, as they contain the markers found in the simple conjunction strategies plus additional material. The semantic difference between the strategies is thus often attributed to these additional ‘conjunction particles’ (Szabolcsi 2015, Mitrović & Sauerland 2014, 2016). Although German patterns differently – the relation between the two strategies is not morphosyntactically transparent – we here assume the same underlying structure for all languages: While simple conjunctions have the structure in (62-a), where ‘AND’ represents conjunction in the respective language, (62-b) schematizes D-conjunctions, where an additional node ‘µ’ (spelled out as a conjunction particle) attaches to each conjunct (Mitrović & Sauerland 2014, 2016, Flor et al. to appear).

(62) a. A AND B

b. µ A AND µ B

Since simple conjunctions display the plural-like behavior discussed in Section 2.1, ‘AND’ will be analyzed as in (30) above, i.e. as recursive sum formation. The difference in interpretation between simple conjunctions and D-conjunctions must thus be due to the conjunction particles, more specifically to the µ nodes in (62-b).

A previously unnoticed fact is that even D-conjunctions sometimes permit cumulative construals. As mentioned in Section 1, their behavior mirrors that of singular universals: The cumulative reading is available whenever they occur below another plural expression. Thus, as opposed to (59-c), (60-c) and (61-c), the examples in (63-b) (= (4-c)), (64-b) and (65-b) can be true in ‘cumulative’ scenarios.

(63) German

a. scenario: Two German participants, Ada and Bea. Ada won the downhill. Bea won the slalom.

b. *Heute haben die zwei Deutschen sowohl die Abfahrt als auch den Slalom gewonnen!*

‘Today, the two Germans won both the downhill and the slalom.’ true in (63-a)

(64) Hungarian (Dóra Kata Takács, p.c.)

a. scenario: Sára called Bálint. Marcsi called Péter.

b. *Szerencsére a két szervező időben felhívta Bálintot és Pétert is.*

‘Fortunately, the two organizers called both Bálint and Péter ahead of time.’ true in (64-a)

(65) Polish (Magdalena Roszkowski and Marcin Wągiel, p.c.)

a. scenario: Sabina called Adam. Magda called Piotr.

b. *Na szczęście dwie organizatorki dostatecznie wcześnie poinformowały i Adama i Piotra.*

‘Fortunately, the two organizers informed both Adam and Piotr early enough.’ true in (65-a)

---

10Existing analyses of D-conjunctions with conjunction particles (Szabolcsi 2015, Mitrović & Sauerland 2014, 2016) fail to derive this.
For D-conjunctions in German and Polish (but apparently not in Hungarian) we can reproduce the infinitival embedding data from (5) above that showed that the asymmetry cannot be captured by appealing (exclusively) to thematic roles. Since (66-b) from German and (66-c) from Polish have cumulative readings, we can assume that the asymmetry is tied to scope.

(66)  
   a. scenario: Yesterday, detectives Mia and Kai were observing two suspects, Peter and Anna. Mia saw Peter sell crack. Kai saw Anna sell pot.
   b. *Die zwei Detektive haben sowohl den Peter als auch die Anna Drogen verkaufen*
      The two detectives have seen the Peter and also the Anna drugs sell
      ‘Ada and Bea saw both Carl and Dean smoke a cigar.’  German; true in (66-a)
   c. *Wczoraj dwaj detektywi widzieli i Petera i Anne sprzedających*
      yesterday two.nom detective.nom.pl see.pst.3pl i Peter.acc 1 Anne.acc sell.pTCP.acc.pl
      narkotyki.
      drug.acc.pl
      Polish (Magdalena Roszkowski, p.c.); true in (66-a)

Finally, D-conjunctions behave like every DPs in Schein sentences: (67) from German is analogous to (6) above except that the every DP has been replaced by a D-conjunction in (67-b). The sentence is true in scenario (67-a).

(67)  
   a. *Scenario:* There are two dogs, Carl and Dean. Ada taught Carl tricks 1 and 2. Ada taught Dean trick 3 and Bea taught Dean trick 2.
   b. *Die Ada und die Bea haben sowohl dem Carl als auch dem Dean zwei neue Tricks*
      The Ada and the Bea have taught the Carl and also the Dean two new tricks
      ‘Ada and Bea taught both Carl and Dean two new tricks’  true in (67-a)

Accordingly, the analytical problem is analogous to that posed by every DPs. However, D-conjunctions have two additional features that need to be explained. First, we want to derive the correct meaning from the structure (62-b), where AND has the meaning assumed in Section 2.2. Distributivity must therefore be attributed to the semantic contribution of the conjunction particles. Second, every DPs required the application of predicate pluralities to the atomic individuals in the NP denotation, but in the case of D-conjunctions, the ‘units’ of distribution are not necessarily atomic individuals, but rather the denotations of the individual conjuncts, which may be pluralities. This is illustrated by (68-b) from German. Since the sentence cannot express that the plurality made up of the boys and the girls fed two dogs in total, the predicate must distribute over the individual conjuncts. This is to be expected, as the conjunction asymmetrically c-commands the indefinite. But, crucially, the sentence can be true in the scenario in (68-a). Here, it is not the case that each girl fed two dogs and each boy fed two dogs. Therefore, the pluralities denoted by the individual conjuncts – [the girls] and [the boys] – can each be in a cumulative relation with the predicate.

(68)  
   b. *Sowohl die Mädchen als auch die Buben haben zwei Hunde gefüttert*
      The girls also the boys have two dogs fed
      ‘Both the girls and the boys fed two dogs.’  true in (68-a)
4.2 Analysis of D-conjunctions

The empirical properties of D-conjunctions suggest that their analysis should mirror that of every DPs: They should be distributive relative to material in their scope, but once they combine with that material, the result should be a plural set of values which preserves the part structure introduced by scopally lower plural expressions. This plural set is then accessible for further cumulative composition. Yet, their analysis must also deviate from that of every DPs in some respects. First, the semantic workload must be allocated differently: The aforementioned properties should follow from a structure where the individual conjuncts are modified by the $\mu$ particles and then combined by means of our ‘sum-forming’ conjunction operation $\oplus$. The challenge here is that conjunctions involving $\oplus$ usually have a cumulative reading relative to lower plurals, and the semantics of $\mu$ has to block this reading. Furthermore, examples like (68) show that the ‘units’ we distribute over are not always atomic individuals: If the conjuncts are plural, each individual conjunct may cumulate with syntactically lower plurals, even though cumulative readings for the conjunction as a whole are unavailable.

We start with the denotation in (69) for conjunction particles. Just like every, the particles directly manipulate plural sets of predicates. They take two arguments – a plural set of individuals (which will be the denotation of the conjunct they modify) and a plural set of predicates (which represents their scope). They then require that their two arguments ‘cumulate’ via the Cumulation Composition operation $C$. This latter step will be crucial to account for data like (68), as it will permit a cumulative relation between the individual conjuncts and material in the scope of the conjunction.

\[ (69) \quad \text{Conjunction particles} \]
\[ \mu_{(e',(e,a)^*)} = \lambda x^* \cdot \lambda P^* \cdot \langle \langle e, a \rangle^*, C(P^*, x^*) \rangle \]

(70) illustrates how these particles combine with the conjuncts they modify: A semantically singular expression like Carl is first shifted to a singleton plural set (this is analogous to what happens in the simple conjunction structures discussed in Section 2.2). The result of applying the particle denotation to this plural set is a function that maps a plural set $P^*$ of predicates to the plural set of values obtained via applying CC to $P^*$ and [Carl].

\[ (70) \quad \mu_{[\uparrow \text{Carl}]} = \mu_{[\text{Carl}]} = \lambda P^* \cdot C(P^*, [\text{Carl}]) \]

As the next step, we consider the denotation of the entire coordinate structure, e.g. (71). (We discuss examples from German.)

\[ (71) \quad \text{sowohl Carl als auch Dean} \ ('\text{both Carl and Dean}') \]

Given our semantics for conjunction from Section 2.2 above, AND operates on plural sets. Each of the modified conjuncts must therefore be shifted to a plural set. This gives us the LF and corresponding semantic derivation in (72) for (71). For the entire D-conjunction, we obtain a plural set containing a single plurality of functions. Importantly, the atoms of this plurality are functions on plural sets that perform cumulation with the sets [Carl] and [Dean], respectively. Since each atom of this function plurality requires an argument of type $\langle e, a \rangle^*$ – a plural set – rather than an ordinary predicate of type $\langle e, a \rangle$, the conjunction as a whole can no longer cumulate with the plural set in its scope.
To show how this D-conjunction combines with other elements in the clause, we consider the Schein sentence in (73) (= (67-a)). Crucially, (73-b) is true in scenario (73-a). To derive the relevant reading, we assign the LF in (73-b) to this sentence. The semantic types of the individual nodes and the conditions on the type-shifts $\uparrow$ and $\downarrow$ will become clear right below.

(73)  

a. **Scenario**: There are two dogs, Carl and Dean. Ada taught Carl tricks 1 and 2. Ada taught Dean trick 3 and Bea taught Dean trick 2.

b. *Die Ada und die Bea haben sowohl dem Carl als auch dem Dean zwei neue Tricks*  
The Ada and the Bea have **prt** the Carl **prt** also the Dean two new tricks *beigebracht*.  
  
  ‘Ada and Bea taught both Carl and Dean two new tricks’

(74)

For node 1, we derive the denotation in (75-a), a plural set of transitive predicates, via CC: Each predicate sum in this set reflects the part structure of a plurality of two tricks. But we cannot directly combine this plural set with the denotation of the D-conjunction in (75-b) since the plurality in the singleton set...
Since we built cumulation into the lexical meaning of the material, they return a plurality of values that reflects the part-structure of their scope; and this plurality in [75-b] - a plural set of predicates - to each atom of the function plurality in [75-b]. We must therefore apply each atom of the function plurality to the plural set of predicates. Since the lexical meaning of the conjunction particle involves the cumulation operation C, this amounts to cumulating the plural set of predicates with each of the conjunct denotations [Carl] and [Dean]. For each conjunct, we obtain a plural set of one-place predicates (those in the first set encode that Carl was taught two tricks, while those in the second set encode that Dean was taught two tricks). Finally, these two plural sets are combined by the sum-operation ⊕, which forms pluralities by picking an element from each set and summing up the chosen elements, as shown in [75-d-iii]. What we have now is a plural set containing a plural set of one-place predicates. Each plurality in this latter set contains at least two atoms relating to Carl being taught a trick and at least two atoms relating to Dean being taught a trick. This higher-type denotation is now ‘reduced’ to a simple plural set via ⌫. This gives us the right input for CC with the subject plurality [Ada + Bea], the result of which is shown in [75-f].

\[1\] C([[beibringen]], [[zwei Tricks]]) = C([[teach], [t1 + t2, t2 + t3, t1 + t3]]) = [teach(t1) + teach(t2), teach(t2) + teach(t3), teach(t2) + teach(t3)]

\[2\] = [AP\(P \ominus P\) ∘ C(P\(P \ominus P\)), [Carl]] ⊕ AP\(P \ominus P\) ∘ C(P\(P \ominus P\)), [Dean]]

\[3\] = ⌫([[1]]) = [[teach(t1) + teach(t2), teach(t2) + teach(t3), teach(t2) + teach(t3)]

\[4\] = C([[2]], [[3]])

(i) = [C([[teach(t1) + teach(t2), teach(t1) + teach(t3), teach(t2) + teach(t3)], [Carl]] ⊕ C([[teach(t1) + teach(t2), teach(t1) + teach(t3), teach(t2) + teach(t3)], [Dean]])]

(ii) = [[teach(t1)(C) + teach(t2)(C), teach(t1)(C) + teach(t3)(C), teach(t2)(C) + teach(t3)(C)] ⊕ [teach(t2)(D) + teach(t2)(D), teach(t2)(D) + teach(t3)(D), teach(t3)(D)]]

(iii) = [[teach(t1)(C) + teach(t2)(C) + teach(t1)(D) + teach(t2)(D), teach(t2)(C) + teach(t3)(C) + teach(t1)(D) + teach(t2)(D), teach(t1)(C) + teach(t2)(C) + teach(t2)(D) + teach(t3)(D), . . . ]

\[5\] = ⌫([[4]])

= [teach(t1)(C) + teach(t2)(C) + teach(t1)(D) + teach(t2)(D), teach(t2)(C) + teach(t3)(C) + teach(t1)(D) + teach(t2)(D), teach(t1)(C) + teach(t2)(C) + teach(t2)(D) + teach(t3)(D), . . . ]

\[6\] = C([[5]], [Ada und Bea]) = C([[5]], [Ada + Bea])

= [[teach(t1)(C)(A) + teach(t2)(C)(A) + teach(t1)(D)(B) + teach(t2)(D)(B), teach(t2)(C)(A) + teach(t3)(C)(A) + teach(t1)(D)(B) + teach(t2)(D)(A), teach(t1)(C)(A) + teach(t2)(C)(A) + teach(t2)(D)(B) + teach(t3)(D)(A), . . . ]

The sentence is thus true iff Carl was taught two tricks, Dean was taught two tricks and Ada and Bea cumulatively did the teaching. This makes the correct prediction for scenario [73-a] (73-a) Just like every DPs, D-conjunctions are distributive relative to material in their scope, but once they combine with this material, they return a plurality of values that reflects the part-structure of their scope; and this plurality of values can then be used for further cumulative composition.

Since we built cumulation into the lexical meaning of the η particles, the analysis naturally extends

11Since this shift can be used to derive distributive interpretations of standard conjunctions, which do not seem to be freely available cross-linguistically (Flor et al. to appear), its use must be constrained. We here assume that the ‘trivial’ typeshift ⌫ can only be applied to a node α if a would be unable to combine with its syntactic sister without it. In spirit, this resembles Partee & Rooth’s [1983] assumptions, but it differs on the technical level since in our system, the application of ⌫ is not always forced by type mismatches.
to examples (76) (eqn:68-a) where the individual conjuncts denote pluralities. Such examples permit cumulation between the predicate and the individual conjuncts, even though the conjunction ‘as a whole’ remains distributive, i.e. (76) can be true if the girls *between them* fed two dogs and the boys *between them* also fed two dogs.

(76) *Sowohl die Mädchen als auch die Buben haben zwei Hunde gefüttert*

‘Both the girls and the boys fed two dogs.’


We assume the scenario in (eqn:77). The denotation of the D-conjunction is given in (eqn:78-a). Combining this denotation with the shifted VP-denotation in (eqn:78-b) via $C$ yields (eqn:78-c). We again ‘pair’ parts of the functor plurality with the only element of (eqn:78-b) a plural set of predicates (eqn:78-c-ii). At this point, the second application of $C$ – contributed by the lexical meaning of the conjunction particles $\mu$ – ‘kicks in’, as shown in (eqn:78-c-iii). We form the sum for each cover of $[\text{Ada} + \text{Bea}]$ and some element of the predicate plurality. Likewise, we form the sum for each cover of $[\text{Gene} + \text{Ivo}]$ and some element of the predicate plurality. This has the effect that the pluralities contributed by the conjuncts cumulate with the plural indefinite. Finally, we ‘sum up’ the two resulting plural sets via $\uplus$ (eqn:78-d). We end up with propositional pluralities that amount to the girls feeding two dogs between them and the boys feeding two dogs between them. This already indicates that while the individual conjuncts can cumulate with the predicate pluralities, the conjunction as a whole will remain distributive. The final step is again to reduce this ‘higher-order’ plural set to a simple plural set of propositions via $\downarrow$. Since the truth definition requires one of the elements of (eqn:78-d) to consist exclusively of true propositions, we derive the correct truth-conditions.\[12]

(78) a. $\llbracket [\llbracket [\text{die Mädchen}] \land [\text{die Buben}] \rrbracket] \rrbracket = [\llbracket [\text{Ada} + \text{Bea}] \rrbracket \uplus [\llbracket [\text{Gene} + \text{Ivo}] \rrbracket]$

b. $\llbracket [\text{zwei Hunde gefüttert}] \rrbracket = [[\text{feed}(C) + \text{feed}(D), \text{feed}(C) + \text{feed}(P), \text{feed}(D) + \text{feed}(P)]]$

c. $C([\llbracket (78-a)] \llbracket, [\llbracket (78-b)] \rrbracket)\rrbracket$

(i) $= [C[[\text{feed}(C) + \text{feed}(D), \text{feed}(C) + \text{feed}(P), \text{feed}(D) + \text{feed}(P)], [\text{Ada} + \text{Bea}] \uplus C([\text{feed}(C) + \text{feed}(D), \text{feed}(C) + \text{feed}(P), \text{feed}(D) + \text{feed}(P)], [\text{Gene} + \text{Ivo}])]$

(ii) $= [[\text{feed}(C)(A) + \text{feed}(D)(B), \text{feed}(C)(A) + \text{feed}(P)(B), \text{feed}(D)(A) + \text{feed}(P)(B), \text{feed}(C)(B) + \text{feed}(D)(A), \ldots ] \uplus [\text{feed}(C)(G) + \text{feed}(D)(I), \text{feed}(C)(G) + \text{feed}(P)(I), \text{feed}(D)(G) + \text{feed}(P)(I), \text{feed}(C)(I) + \text{feed}(D)(G), \ldots ]$

(iii) $= [[\text{feed}(C)(A) + \text{feed}(D)(B) + \text{feed}(C)(I) + \text{feed}(D)(G), \text{feed}(C)(A) + \text{feed}(P)(B) + \text{feed}(D)(G) + \text{feed}(P)(I), \text{feed}(D)(A) + \text{feed}(P)(B) + \text{feed}(C)(G) + \text{feed}(D)(I), \text{feed}(C)(B) + \text{feed}(D)(A) + \text{feed}(C)(G) + \text{feed}(D)(I) \ldots ]$

d. $\llbracket \llbracket [\llbracket (78-c) \rrbracket \rrbracket \rrbracket = [\text{feed}(C)(A) + \text{feed}(D)(B) + \text{feed}(C)(I) + \text{feed}(D)(G), \text{feed}(C)(A) + \text{feed}(P)(B) + \text{feed}(D)(G) + \text{feed}(P)(I), \text{feed}(D)(A) + \text{feed}(P)(B) + \text{feed}(C)(G) + \text{feed}(D)(I), \text{feed}(C)(B) + \text{feed}(D)(A) + \text{feed}(C)(G) + \text{feed}(D)(I) \ldots ]$

In sum, we derive the correct results for conjunctions with plural conjuncts as well: Informally speaking, the conjunction as a whole will be distributive, because each conjunct is combined via the operation $C$ with the entire plural set of predicates in the scope of the conjunction. But since $C$ encodes cumulativity,

\[12\text{Assuming the analysis of every DPs from Section 3 above, our proposal also correctly derives the fact that when two singular universals are conjoined by a D-conjunction, we get ‘full distribution’ w.r.t. the material in the scope of the conjunction.}]}
each individual conjunct can cumulate with this plural set.

5 Revisiting the scope generalization

Having provided a plural projection analysis of the asymmetric behavior of *every* DPs and D-conjunctions, we return to an empirical issue: We assumed that cumulativity asymmetries are tied to scope, following Champollion 2010. Our plural projection account is sensitive to such scope relations, if the latter are modeled as c-command relations at LF. Without further assumptions, it therefore makes the following prediction:

(79) An ADU can have a cumulative reading relative to another plural expression if and only if that plural expression may c-command the ADU at LF.

Since our theory does not posit any covert syntactic operations that are specific to cumulative sentences, (79) relates the possibility of cumulative readings for an ADU to the scope options it has on a distributive reading: In configurations where an ADU cannot scope below another plural on its distributive reading, a cumulative reading relative to that plural should be unavailable.

We will now discuss data from German which show that, while scope does influence the distribution of the cumulative reading, generalization (79) is too strong – there seem to be additional factors blocking cumulativity. The data are surprising for any existing theory of cumulativity and show that further research is needed. However, they also suggest that our scope-related theory of cumulativity is a plausible starting point.

5.1 Scope vs. other syntactic asymmetries

There are two alternatives to the scope-based view of cumulativity asymmetries. First, thematic-role asymmetries: For instance, Kratzer 2003 takes the agent role to correlate with the lack of a cumulative reading. Second, grammatical function asymmetries: The cumulative reading could be blocked in (matrix) subject positions, but available in non-subject positions, as claimed by Drozd et al. 2017, who connect the lack of a cumulative reading to singular verb agreement.

We thus tested the scope-based generalization by examining configurations in German that allow us to dissociate wide scope from agenthood, subjecthood and singular verb agreement.

To collect preliminary data, we carried out an informal survey of 30 speakers using an online questionnaire. (This was not a controlled experiment, but an informal ‘pre-test’ for future work.) We presented German sentences containing a DP headed by *jed-* ‘every’ and a numeral-modified plural indefinite, together with a short text describing a cumulative scenario. Speakers had to judge how adequately the sentence describes the scenario on a scale from 1 (‘not at all adequate’) to 5 (‘completely adequate’). As fillers, we used sentences with a *jed-* ‘every’) DP and a singular indefinite, and cumulative sentences with two numeral-modified plural indefinites.\(^{13}\)

\(^{13}\)Not all speakers in our survey had access to the cumulative reading, even in syntactic configurations that should uncontroversially permit it. Since our design did not tell us whether a reading was unavailable for grammatical reasons or simply dispreferred, we leave the conditions on this variation to future study and concentrate on the aggregated data.
5.2 The status of the scope generalization

Embedded subjects  The first test configuration involved subjects of embedded infinitives, which are asymmetrically c-commanded by the matrix subject, but may still be agents. The thematic-role hypothesis therefore predicts cumulation between the matrix subject and the embedded subject to be blocked, while the other generalizations do not. For most of our consultants, DPs with jed- ‘every’ permit a cumulative reading in this position (80-a), while a jed- DP in matrix subject position lacks a cumulative reading relative to the embedded subject (80-b) (see Figure 1 for results).

(80) a. *Gestern haben zwei Detektive jeden von diesen Kriminellen Drogen yesterday have two.nom detectives.nom every.acc of these criminals drugs verkaufenesehen.
sell seen
‘Yesterday, two detectives saw each of these criminals sell drugs.’
NUM NP (matrix) > every NP (embedded) ✓
cumulative

b. Gestern hat jeder Detektiv zwei von diesen Kriminellen Drogen verkauften
seen
‘Yesterday, every detective saw two of these criminals sell drugs.’
every NP (matrix) > NUM NP (embedded) *cumulative

This contrast provides straightforward evidence against a thematic-role asymmetry: In (80), both the matrix subjects and the embedded subjects are agents. A simple subject/object asymmetry cannot account for this contrast either. However, the data do not yet distinguish between a scope asymmetry and the claim that matrix subjects are exceptional in blocking cumulative readings.

Scrambling  The latter two hypotheses can be disentangled by considering scrambling in German (movement to one of various positions that follow the derived position of finite verbs). Since scrambling may affect scope (see Frey [1993], Beck [1996], Büring [1997], Lechner [1998], Heck [2001], Pafel [2005], Wurmbrand [2008] for discussion), but not thematic roles or subject/object asymmetries, any effects of scrambling on the availability of cumulative readings would support the scope-based account.

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14Topicalization has been claimed to permit scope reconstruction more easily than scrambling (Höhle [1991], Beck [1996], Büring [1997], Heck [2001], Pafel [2005] a.o.), so the latter should reveal the putative effects of scope on cumulativity more clearly.
Most syntacticians agree that German sentences where the arguments appear in their base positions – with the subject preceding the object and no topicalization – disallow scope inversion (but see Pafel 2005, Frey 1993, Heck 2001 for dissenting views). Sentences where the object is scrambled over the subject, however, may permit scope reconstruction, but the conditions on reconstruction are still debated. For instance, Frey (1993) claims that scrambling structures are generally ambiguous between a surface-scope reading and a reconstructed reading, while Wurmbrand (2008) argues that reconstruction is only possible if the scrambled phrase has the prosodic and pragmatic properties of a topic.\footnote{Another complication is that scope reconstruction has been claimed to be less restricted than binding reconstruction, suggesting that both syntactic and semantic reconstruction are available [Frey 1993, Lechner 1998].}

The pattern we obtained for scrambling in Figure 2 is surprising regardless of one’s views on reconstruction. First, as expected, the majority of our consultants accessed the cumulative reading for a non-scrambled jed- DP in direct object position (81-a). Further, scrambling of the jed- DP clearly affects the availability of this reading: In cumulative scenarios, (81-b) is rejected more often than (81-a), although we find inter-speaker variation. This contrast suggests that scope relations influence the availability of a cumulative reading.

A second prediction of (79) is not borne out, however: In sentences with a jed- DP in subject position and a plural indefinite object, scrambling of the indefinite should license a previously unavailable cumulative reading. But as Figure 2 shows, sentences with an ADU subject and a plural indefinite object were rejected in cumulative scenarios by almost all speakers, regardless of scrambling.
Theoretical consequences: A more complex scope-based hypothesis  

There is no plausible hypothesis about scope reconstruction in scrambling sentences which, together with generalization (79), predicts the entire data set. Given the mostly negative judgments for (81-b), the generalization suggests that for scrambling of the direct object over the subject, scope reconstruction is sometimes available, but dispreferred. If so, however, generalization (79) falsely predicts (81-d) to have a cumulative reading for many speakers. In fact, given our data set, generalization (79) leads us to expect that scrambled direct objects obligatorily reconstruct, which is not just incompatible with other observations concerning scope in German, but also fails to account for (81-b).

At the same time, the contrast in (81-a,b) shows that scope has an impact on cumulativity. We therefore hypothesize that the pattern in (81) follows from a restriction on cumulativity sensitive to the base positions of the plural expressions (or, if scrambling involves $A'$-movement, their highest A-positions) in addition to their surface positions. More specifically, we tentatively submit the assumptions in (82). While the syntactic assumption (82-a) can be motivated independently (cf. Frey 1993, Wurmbrand 2008), (82-b), if correct, would have to be built into an extension of our Plural Projection semantics.

(82)  

a. Arguments in the German ‘Mittelfeld’ do not undergo QR. Any scope ambiguities between co-arguments of a verb in German are the result of overt movement plus optional reconstruction. Reconstruction takes place in the syntax.  

b. A plural expression $\alpha$ has a cumulative reading relative to an ADU only if the ADU does not c-command any element of $\alpha$’s chain at LF.  

Let us see how this derives the data: If there is no obligatory QR in German (cf. e.g. Beck 1996, Büring 1997), (81-a) may have an LF without any movement of the object (83-a). Generalization (82-b) licenses a cumulative reading since the $\text{jed-}$ DP fails to c-command the subject. In (81-b), the $\text{jed-}$ DP has moved overtly, giving rise to the additional LF in (83-b). Here, the surface position of the $\text{jed-}$ DP c-commands the subject, so (82-b) predicts cumulativity to be impossible. However, if syntactic reconstruction is possible in scrambling configurations, we generate a second LF for (81-b), namely (83-a), which allows for cumulativity. If reconstruction of scrambled phrases is restricted, e.g. by information-structural conditions, this predicts the observed contrast between (81-a) and (81-b).

(83)  

a. [[zwei Jäger] [[jeden Hirsch in diesem Wald] erschossen]]  

b. [[jeden Hirsch in diesem Wald] [[zwei Jäger] [t$_1$ erschossen]]]  

[82] would also account for the lack of a clear contrast between (81-c) and (81-d). In both cases, the $\text{jed-}$ DP c-commands the base position of the plural indefinite. Therefore, generalization (82-b) can never be

16 We assume that if $\alpha$ has not undergone movement, the only element of its chain is $\alpha$ itself.
met regardless of movement and both of the possible LFs in (84) should lack the cumulative reading.

(84)  a. [[[jeder Jäger in diesem Ort] [[fünf Hirsche] erschossen]]
    b. [[[fünf Hirsche] [[jeder Jäger in diesem Ort] [t₁ erschossen]]]]

**Double object constructions** Finally, we tested when cumulative readings are available in double object constructions (see Schein 1993 for English). A simple contrast between subjects and non-subjects or agents and non-agents would predict cumulative readings for ADUs in direct object and indirect object position, regardless of the surface word order. Generalization ([79]) predicts that the cumulative reading should be dispreferred whenever the ADU precedes the plural indefinite in the surface order, but available otherwise. Finally, the more complex hypothesis in ([82]) predicts that a cumulative reading should be possible only if the ADU has a lower base position than the indefinite and no scrambling has taken place.

Our data do not clearly confirm any of these hypotheses. The examples involved the verbs *zuweisen* ‘assign’ and *zeigen* ‘show’. We manipulated both the base position of the ADU (direct vs. indirect object) and the surface word order (scrambled vs. non-scrambled). As Figure 3 shows, examples with an ADU in indirect object position and a plural indefinite in direct object position (85-c,d) were rejected by almost everybody. This is predicted by our hypothesis in ([82]) but not by any of the other hypotheses: If cumulativity were sensitive to subjecthood or agenthood, but not to scope, the indirect objects in (85-c,d) should both have cumulative readings.

This by itself does not show that the crucial factor is a structural scope relation, rather than a thematic-role hierarchy where RECIPIENT is ranked higher than THEME, or a an analogous grammatical-function hierarchy ranking indirect objects and direct objects. However, the pattern in (81) suggests we can disentangle these options using sentences with the *jed- DP in direct object position: ([82]) predicts that such sentences should have a cumulative reading if the base order is preserved, but not if the *jed- DP is scrambled over the indirect object. In contrast, the thematic rule and grammatical function hypotheses predict a cumulative reading of the direct object to be available across the board.

Neither of these two predictions is clearly borne out: Most of our consultants rejected sentences with ADUs in direct object position and plural indefinites in indirect object position, like (85-a,b), regardless of scrambling, although a notable minority found them acceptable (Figure 3). There was no clear contrast between the non-scrambled structure in (85-a) and the scrambled structure in (85-b), contrary to the predictions of ([82]). However, the data do not support a hypothesis based on a ranking of grammatical functions or thematic roles either, as such hypotheses predict cumulation to be possible in both (85-a) and (85-b).

(85)  a. *Heute habe ich zwei Kindern jede Aufgabe auf dieser Liste zugewiesen.*
    today have I two.DAT children.DAT every.ACC task.ACC on this list assigned ‘Today, I assigned two children every task on this list.’
    NUM NP (IO) > every NP (DO)
    b. *Heute habe ich jede Aufgabe auf dieser Liste zwei Kindern zugewiesen.*
    every NP (DO) > NUM NP (IO)
    c. *Heute habe ich jedem Kind zwei Aufgaben von dieser Liste zugewiesen.*
    today have I every.DAT child.DAT two.ACC tasks.ACC from this list assigned

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37Since ([82]) predicts ([81-b]) to permit a cumulative reading if and only if reconstruction is available, it might help explain why the contrast between ([81-a]) and ([81-b]) is not more clear-cut. According to Wurmbrand (2008), scrambled phrases may reconstruct only if they have the pragmatic and prosodic properties of a contrastive topic. Since we used a written questionnaire, we could not control the prosody. Hence, some of our consultants might have assigned a contrastive topic interpretation to the scrambled object, thus licensing reconstruction. If so, a more clear-cut contrast should emerge if the prosody were controlled in a way that makes a contrastive topic interpretation unlikely.
Figure 3: Survey data on cumulative readings of ADUs in indirect object vs. direct object position, with and without scrambling

‘Today, I assigned every child two tasks on this list.’

\[ \text{every NP (IO)} \rightarrow \text{NUM NP (DO)} \]

d. \text{Heute habe ich zwei Aufgaben von dieser Liste jedem Kind zugewiesen.}

\[ \text{NUM NP (DO)} \rightarrow \text{every NP (IO)} \]

The fact that there was more inter-speaker variation for the sentences with ADUs in direct object position supports our conclusion that the base positions of the plural expressions matter for cumulativity. However, since double-object sentences were mostly rejected in cumulative scenarios regardless of the syntactic configuration, we did not find the effect of scrambling predicted by (82). Experimental work is needed to determine the reasons for this low acceptability of the cumulative reading (e.g. prosody, lexical properties of predicates) and to see whether scrambling has an effect once these factors are controlled for.

**Theoretical consequences** If some version of our hypothesis [82-b] turns out to be tenable, it raises the question how the ‘blocking’ of cumulativity by antecedent-trace relations should be implemented. In Section 2, we intentionally ignored the question how traces, and variable binding more generally, should be integrated into the Plural Projection framework, in order to circumvent the technical problems that arise when Hamblin/Rooth-style Alternative Semantics is combined with an ultimately non-compositional mechanism like the standard predicate abstraction rule. But generalization [82-b] suggests that the issue is not merely technical: If correct, this generalization would entail that traces interact with the semantic mechanism responsible for cumulativity in a previously unnoticed, non-trivial way. More specifically, the semantic rules applying to traces would have to be such that if the trace of a plural
expression is c-commanded by an ADU, the ADU distributes over pluralities ‘introduced’ by the trace, blocking a non-distributive reading of the antecedent of the trace. We leave the implementation of this idea as well as the investigation of relevant empirical questions to future work.

5.3 Interim summary

Our preliminary survey concerning the scope hypothesis for German jed- yielded the following results: First, neither a thematic-role asymmetry nor a simple subject/object asymmetry can explain the full pattern, suggesting that scope indeed influences the availability of cumulative readings. Second, the simple scope generalization is too strong: Certain configurations disallow the cumulative reading for most speakers even though their scopal properties should permit it. We thus tentatively suggested that the base positions of the plural expressions are also relevant.

6 Comparison to previous analyses of ADUs

We developed a new approach to the semantics of ADUs that accounts for the peculiar truth conditions of Schein sentences and takes the interpretation of ADUs to depend on their structural position relative to other pluralities. These features are shared by two existing approaches to cumulativity asymmetries: the event-based approach pioneered by Schein (1993) and extended by Kratzer (2003), Ferreira (2005), Zweig (2008) a.o., and the event-less, predicate-based analysis in Champollion (2010). Like our analysis, these approaches build on Schein’s 1993 insight that Schein sentences cannot be analyzed in terms of a single cumulative relation. However, they make different assumptions about the nature of the multiple cumulative operations at work: While our analysis of Schein sentences requires repeated application of the Cumulative Composition rule, Champollion (2010) assigns them an LF with two different cumulative operators, and the event-based approach relies on cumulative thematic-role relations that relate event pluralities introduced by the verb to pluralities of individuals.

The two existing analyses make relatively conservative assumptions concerning the question which semantic domains contain pluralities, raising the question whether the expressive power of the Plural Projection system is really needed. In this section, we show that both existing approaches fail to extend to some of the more ‘complex’ instances of cumulativity discussed above.

6.1 The event-based approach

Several authors have explored the idea that cumulativity is inherently connected to event semantics (e.g. Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008). In a neo-Davidsonian semantics, verbs denote sets of events and combine with their type e arguments via thematic-role relations of type \(\langle e, \langle l, t \rangle \rangle\) (with \(l\) the type of events). Given a sum operation for events, these thematic-role relations are cumulative relations connecting pluralities of individuals to corresponding sums of events (cf. Kratzer 2003 for discussion). The event-based approach thus resembles our system in that it builds cumulativity into the standard mechanism for function-argument composition, but since the role of our predicate pluralities is taken over by plural events, it does not require a sum operation for non-primitive types.

Here, we present a version of the event-based approach inspired by Ferreira (2005). Arguments combine with thematic-role predicates, which map an individual to a one-place predicate of events (86). The effect of the ** operator is encoded in the lexical entries of these thematic-role predicates: As (87-b) illustrates, the agent relation \([AG]\) holds of an individual \(x\) and an event \(e\) iff there is a partition \(P\) of \(e\) into disjoint
subevents such that the atomic parts of \( x \) are cumulatively agents of the subevents in \( P \).

(86) a. The two girls fed (the) two dogs.
    b. \([\text{AG}[\text{the two girls}]] \text{[fed] [TH[\text{the two dogs}]}}\)  

(87) a. \( \text{partition}(P)(e) \) holds for a set \( P \) of events and an event \( e \) if \( P \) is a set of disjoint subevents of \( e \) whose sum is \( e \)
    b. \( \lbrack \text{AG} \rbrack = \lambda x_e . \lambda e_t . \lambda P_{(d)} \lbrack \text{partition}(P)(e) \land \forall x' [x' \subseteq a \ x \rightarrow \exists e'[P(e') \land \text{agent}(x')(e')]] \land \forall e'[P(e') \rightarrow \exists x'[x' \subseteq a \ x \land \text{agent}(x')(e')]]\)

The thematic-role predicates map the argument denotations to sets of events, which then combine interactively with the verb meaning (88). This predicts (86-a) to be true if there is an event that is (i) a sum of disjoint feeding events, (ii) a sum of a set of disjoint subevents that cumulatively stand in the agent relation to \( a + b \), and (iii) a sum of a set of disjoint subevents that cumulatively stand in the theme relation to \( c + d \).

(88) a. \( \lbrack \text{fed} \rbrack = \lambda e_t . \lambda P_{(d)} \lbrack \text{partition}(P)(e) \land \forall e'[P(e') \rightarrow \text{feeding}(e')]\)
    b. \( \lbrack \text{TH[the two dogs]} \rbrack = \lambda e_t . \lbrack \text{TH}[c + d](e)\)
    c. \( \lbrack \text{AG[the two girls]} \rbrack = \lambda e_t . \lbrack \text{AG}[a + b](e)\)
    d. \( \lbrack \text{fed[TH[the two dogs]} \rbrack = \lambda e_t . \lbrack \text{partition}(P)(e) \land \forall e'[P(e') \rightarrow \text{feeding}(e')]\rbrack \land \lbrack \text{TH}[c + d](e)\rbrack \land \lbrack \text{AG}[a + b](e)\rbrack\)
    e. \( [[\text{AG[the two girls]}]] \lbrack \text{fed[TH[the two dogs]} \rbrack\]

There is a clear analogy between this analysis and our plural projection approach: Since each event in the set (88-d) can be partitioned into a part whose theme is \( c \) and a part whose theme is \( d \), the part structure of these events ‘preserves’ the part structure of the plurality \( c + d \). These plural events are then cumulated with the atomic parts of the subject plurality, yielding another set of plural events. Thus, the merology of events ensures that the denotations of complex expressions containing a plurality of individuals provide the part structure necessary to indirectly form a cumulative relation with these individuals, even if the complex expression is not itself individual-denoting. The Plural Projection system implements the same intuition more directly, since the mechanism by which complex constituents inherit the part structure of their plural subconstituents is not specific to events.

It is therefore unsurprising that the event-based approach provides an account of Schein sentences that is analogous to ours. (89-a) receives the LF in (89-b), where each DP combines with a thematic-role relation. To model the distributive effect of every DPs, we assume that they do not compose with predicates of events in the same way as other plural DPs: In (89-a) Ada and Bea should be the (cumulative) agent of an event which, for each dog, has a subevent in which that dog is taught two tricks. We thus give every DPs a higher-type denotation that takes a thematic-role relation \( R \) and a predicate \( P \) of events as its arguments (90). It returns the set of those events that can be partitioned in such a way that every event in the partition satisfies \( P \), and every individual in the NP denotation stands in relation \( R \) to some element of the partition.

\[^{18}\]The requirement that each subevent in the partition must have an atomic agent is not a necessary feature of event-based analyses – we introduce it here to emphasize the parallels with the Plural Projection approach.
(89)  a. Ada and Bea taught every dog two new tricks.  
    (= [6-b] adapted from Schein 1993)
    b. [[AG [Ada and Bea]] [[RE [every dog]] [taught [TH [two new tricks]]]]]

(90)  a. \[
    \forall e \in \text{every} = \lambda P_{(e,e)}, \forall R_{(e,e)}, \lambda e_1, \lambda e_2, \exists Q_{(e,e)}, \text{PARTITION}(Q(e)) \land \forall e' [Q(e') \rightarrow P(e')]
    \land \forall x [P(x) \rightarrow \exists e' [Q(e') \land R(x)(e')]]
    \land \forall e' [Q(e') \rightarrow \exists x [\text{dog}(x) \land \forall e'' [\text{RE}(x)(e'')]]
\]
    b. \[
    \forall e \in \text{RE} = \lambda P_{(e,e)}, \forall R_{(e,e)}, \lambda e_1, \lambda e_2, \exists Q_{(e,e)}, \text{PARTITION}(Q(e)) \land \forall e' [Q(e') \rightarrow P(e')]
    \land \forall x [\text{dog}(x) \rightarrow \exists e' [Q(e') \land \forall e'' [\text{RE}(x)(e'')]]
    \land \forall e' [Q(e') \rightarrow \exists x [\text{dog}(x) \land \forall e'' [\text{RE}(x)(e'')]]
\]

In (89), two new tricks combines in the usual way with its thematic-role relation and the verb meaning, yielding the set of all sums of teaching events which have a plurality of two new tricks as their theme. Applying the meaning of the every DP yields a new set of events (91-b): It contains all events that can be partitioned into subevents in such a way that every dog stands in the relation [RE] (‘recipient’) to one of the subevents, every subevent in the partition has a dog as its recipient and every subevent involves the teaching of two tricks. We thus predict (89-a) to be true iff the sum of Ada and Bea cumulatively stands in the agent relation to at least one event in the set (91-b).

(91)  a. \[
    \forall e \in \text{taught} = \lambda e_1, \lambda e_2, \exists P_{(e,e)}, \exists Q_{(e,e)}, \text{PARTITION}(P(e)) \land \forall e' [P(e') \rightarrow \text{teaching}(e')]
    \land \exists x [\text{dog}(x) \land |x| = 2 \land \forall e'' [\text{RE}(x)(e'')]]
\]
    b. \[
    \forall e \in \text{taught} = \lambda e_1, \lambda e_2, \exists P_{(e,e)}, \exists Q_{(e,e)}, \text{PARTITION}(P(e)) \land \forall e' [Q(e') \rightarrow P(e') \land \forall e'' [\text{teaching}(e'')]]
    \land \exists x [\text{dog}(x) \land |x| = 2 \land \forall e'' [\text{RE}(x)(e'')]]
\]

The compositional interpretation of (89-a) therefore involves several different cumulative relations: The theme relation applies cumulatively to events and pluralities of two tricks, which ensures that every dog is related to a ‘plural’ event involving two tricks. The lexical entry of every in (90-a) encodes another cumulative relation between individuals and events: Each dog is the recipient of a subevent in the relevant partition, and each subevent in the partition has a dog as its recipient. Finally, the ‘plural’ events with this property cumulatively stand in the agent relation to the subject plurality ada + bea.

This analysis therefore meets the challenge posed by (89-a) without recourse to pluralities of higher types, but crucially relies on the mereology of events. So do its empirical predictions differ from those of our theory, on which cumulativity and events are unrelated?

The first argument against the event-based account is that cumulative readings of predicates do not reflect any distinction between predicates with and without an eventuality argument. Therefore, it is incompatible with several attempts to draw such a distinction on the basis of independent linguistic arguments (Higginbotham 1983, Kratzer 1995 a.o.). For instance, Higginbotham (1983) claims that only predicates with an event argument can occur in bare infinitival complements of perception verbs: A predicate like German zusammenhängen ‘be connected’ that resists embedding in such complements would lack an event argument. However, zusammenhängen permits a cumulative interpretation of ADUs in its lower argument position, (92). So, cumulative readings and embedding under perception verbs cannot both be diagnostics for an event argument.

(92)  Hans hat gesehen, dass zwei Annahmen mit jeder von den Behauptungen auf Seite 15 zusammenhingen.

Hans has seen that two assumptions with every of the claims on page 15 zusammenhingen.

be.connected
‘Hans saw that two assumptions are connected to each of the claims on page 15.’

This problem can be avoided by denying that Higginbotham’s test is informative about the presence of an event argument (see Maienborn 2011 for discussion). Indeed, Schein (1993) assumes a neo-Davidsonian
semantics for all lexical predicates, motivated mainly by data involving cumulativity. In his system, the claim that cumulativity requires an event argument is no longer independently testable.

A stronger argument against the event-based approach derives from Schmitt’s 2018 observation that in certain contexts, cumulative relations can ‘reach inside’ the complements of attitude verbs, which are standardly assumed to denote neither individuals nor events. (93) gives an example from German.

(93)  
   a. context: It is not true that Abe’s friends were shocked by his behavior at the party last night . . .
   b. Sie glauben nur, dass er gestern viel getrunken und gekiff hat, aber harte Drogen haben sie keine gesehen.
   c. scenario: Half of Abe’s friends believe he drank a lot and the other half believe he smoked weed.

An event-based approach to cumulativity is insufficient here, since it attributes all instances of cumulativity to cumulative thematic-role relations. If all natural-language predicates combine with their arguments in a neo-Davidsonian manner, we need a thematic-role relation that relates the propositional argument of believe to its eventuality argument, a belief state (cf. Kratzer 2006, Moulton 2015, Elliott 2017 a.o. for relevant discussion). This relation must be cumulative, since we need pluralities of belief states. For (93), this would mean that Abe’s friends stand in a cumulative relation to a sum of belief states containing some parts with the propositional content that Abe drank a lot, and some parts with the propositional content that he smoked weed. In order to make sense of the notion of a cumulative relation between states and propositions, however, we need a notion of parthood for propositions. In particular, the proposition expressed by the embedded clause must be decomposable into parts with the content [[that he drank and smoked weed yesterday]] = \lambda w . \lambda e_1, e_2 [\text{drink(abe)}(e_1)(w) \land \text{smoke-weed(abe)}(e_2)(w) \land e = e_1 + e_2 \land \text{yesterday(e)}]

6.2 Champollion (2010)

Both our approach and the event-based analysis attribute the cumulativity asymmetry associated with every to its lexical semantics. Champollion (2010) develops a fundamentally different approach, on which every DPs are run-of-the-mill plural expressions: every boy denotes the sum of all boys, written as +boy here. Cumulative readings relative to higher plural expressions are therefore expected regardless

19 It might seem that examples like (93-b) can be captured by an event-based analysis, if we let the attitude verb combine with a property of events rather than a proposition. (93-b) would then involve combining believe with the property in (i), which maps every world w to the set of sums of an event of Abe drinking in w and an event of Abe smoking weed in w. These ‘plural’ events reflect the part structure of the embedded conjunction in (93-a). So, if we could find a way of cumulating the belief states of Abe’s friends with the parts of these plural events, we would not need pluralities of higher-type objects. The problem is that for each of their beliefs, the events that would make it true belong to many different possible worlds. We do not know of any semantic theory that would encode the contents of their beliefs in a single event anchored to a particular world. Therefore, the truth conditions of (93-b) cannot be paraphrased in terms of a cumulative ‘belief’ relation holding between Abe’s friends and a sum of events.

(i) \text{[[that he drank and smoked weed yesterday]]} = \lambda w . \lambda e_1, e_2 [\text{drink(abe)}(e_1)(w) \land \text{smoke-weed(abe)}(e_2)(w) \land e = e_1 + e_2 \land \text{yesterday(e)}]
of the particular theory of cumulativity. The requirement that every DPs must be distributive w.r.t. their nuclear scope, however, must be encoded in additional assumptions about the syntax-semantics interface. These assumptions allow Champollion to maintain a predicate analysis of cumulativity that relies on cumulative relations between individuals, which are derived by syntactically adjoining a cumulation operator (**, ***, etc.). If there is no surface constituent denoting the required relation, a suitable LF constituent is created via movement (see Section 2). So unlike our analysis or the event-based approach, this analysis does not build cumulativity into the basic mechanism for predicate-argument composition.

The LF Champollion (2010) assigns to a Schein sentence like (94-a) is given in (94-b): the two girls and every dog move in order to derive an adequate input relation for **.

(94) a. The two girls taught every dog two new tricks.
   b. [[[the two girls] [every dog] [** [2 [1 [[two tricks] [3 [t1 [[the2 dog] [[*** taught] t3]]]]]]]]]]

What distinguishes Champollion’s approach from other versions of the predicate analysis is that the interpretation of this relation blocks a cumulative reading of every relative to the lower plural two new tricks: The trace of an every DP contains a copy of the NP complement of every, which combines with an indexed definite determiner (95) (see Fox 2002, Sauerland 2004a). This NP copy is interpreted as a predicate that is true only of atomic individuals. The definite determiner within the trace thus imposes an atomicity requirement on the values of the variable bound by the every DP.

(95) \[ \text{the2 dog} \] = g(2) if g(2) is an atomic dog; undefined otherwise

This atomicity restriction holds even if we interpret taught cumulatively, as indicated by the *** operator attached to it in (94-b). The argument of two tricks relative to an assignment g is a property which is true of those sums of tricks that were cumulatively taught to an atomic dog, g(2), by a possibly plural individual, g(1) (96). To create a binary relation, we abstract over the indices 1 and 2 which correspond to the subject and the every DP (96-b). This abstraction preserves the atomicity presupposition: If Ada and Bea cumulatively taught Dean trick 1 and trick 2, this relation is true of the pair \( (a + b, d) \). However, if Ada taught Carl trick 1 and Bea taught Dean trick 2, the relation does not hold of the pair \( (a + b, c + d) \), since \( c + d \) is not an atomic dog. Hence, every dog must have learned two tricks.

(96) a. \[ [3 [t1 [[the2 dog] [[*** taught] t3]]]] = \lambda z_e. [***] (taught)(z)(g(2))(g(1)) if \text{dog}(g(2)) = 1, undefined otherwise
   b. \[ [2 [1 [[two tricks] [3 [t1 [[the2 dog] [[*** taught] t3]]]]]]] = \lambda x_e : \text{dog}(x). \lambda y_e. \exists z_e. [\forall z' \leq a \text{trick}(z') \land |z| = 2 \land [***](taught)(z)(x)(y)]

In order to felicitously combine the relation in (96-b) with the sum \[ every dog \] in spite of its atomicity presupposition, another cumulation operator is attached. Importantly, Champollion’s cumulation operators do not quantify over atomic parts of the plural individuals involved, but close a relation between individuals under ‘pointwise sum’ (97). This is crucial for the interpretation of (94-a) since the argument of ** here is a relation that connects each atomic dog to possibly plural individuals that cumulatively taught two tricks to the dog (96-b). The effect of ** in our example is to ‘add up’ pairs of individuals that stand in this relation.

(97) \[ [***] = \lambda R_{e,(e,d)}. \lambda x_e. \lambda y_e. \exists R'[R' \subseteq R \land x = + \{x' \mid \exists y'. R'(x')(y')\} \land \exists y [y' \mid \exists x'. R'(x')(y')]] \]

Since the presupposition of (96-b) does not project above the ** operator, we end up with a relation between pluralities of dogs and (possibly plural) individuals cumulatively responsible for teaching each
dog two tricks

Champollion (2010) thus manages to analyze Schein sentences without recourse to events or higher-type pluralities. However, additional syntactic restrictions are needed to account for the correlation between the availability of cumulative readings for ADUs and surface c-command. One necessary assumption is that the surface c-command relations between plural expressions are preserved at LF. Further, LF movement of plurals must be obligatory, since we could otherwise interpret every DPs in situ, circumventing the atomicity requirement imposed by their traces.

But these restrictions are still not enough: A sentence like (98-a), where a cumulative reading of every is impossible, could be assigned the LF in (98-b). Since the atomicity presupposition does not project above the ** operator, the cumulative relation in (98-b) is true of any pair \((x, y)\) where \(x\) is a girl or plurality of girls that cumulatively fed \(y\). Therefore, (98-a) is predicted to be true whenever the girls cumulatively fed the two dogs.

(98) a. Every girl fed the two dogs.
   b. \[\{every girl\} \{the two dogs\} \{** [2 \{[the1 girl] [fed t2]\}]\}]\]

Champollion (2010) addresses this problem by requiring that every DPs are interpreted above a distributivity or cumulation operator, but does not state the precise restrictions on the syntactic position of this operator that would exclude cumulativity in (98-a). One possibility is that the every DP has to minimally c-command the operator. But this would not block the LF in (99-a). Given Champollion’s view of cumulation operators – in particular that their lexical semantics does not reach ‘all the way down’ to the atomic parts of the pluralities – the * operator in (99-a) must be interpreted as in (99-b). If the girls cumulatively fed the two dogs, their sum will already satisfy the predicate * combines with. Since the only effect of the operator in (99-b) is to close a predicate extension under sum, the cumulative reading of the every DP is not blocked, in spite of its syntactic position.

(99) a. \[\{every girl\} \{* [the two dogs]\} \{** [2 \{[the1 girl] [fed t2]\}]\}]\]
   b. \[\{*\} = \lambda P_{(e,t)}.Ax.3P'[P' \subseteq P \land x = +P']\]

The problem is that given Champollion’s semantics, inserting a cumulation operator or * operator between an every DP and another plural is not enough to block cumulative relations between them. In addition, the binder index that abstracts over the trace of the every DP has to c-command the other plural. If both the every DP and the other plural end up c-commanding that binder index as a result of ‘tucking-in’ movement, a cumulative reading can be derived regardless of their relative syntactic positions. So, to derive the asymmetric behavior of every DPs, all other plurals would have to be banned from ‘tucking in’ between an every DP and its associated binder index, as the two dogs does in (98-b) and (99-b). This amounts to directly translating the scope-related restrictions on cumulativity into restrictions on syntactic movement.

In sum, Champollion (2010) illustrates that at the cost of a rather complex LF syntax, Schein sentences like (94-a) can be analyzed without event semantics or the Plural Projection machinery. However, two classes of more complex examples cannot easily be accommodated in his framework (but are unproblematic for our analysis and the event-based approach).

The first case involves D-conjunctions like German sowohl A als auch B, which is also found in Schein sentences. As noted in Section 2 when the conjuncts of a D-conjunction are plural as in (76) above,
a cumulative reading of the whole conjunction relative to a plural in its scope is still ruled out, but each individual conjunct can cumulate with the lower plural. In Champollion’s system, the distributivity requirement of sowohl . . . als auch would have to be implemented via a restriction on the traces left by LF movement of the conjunction, but it is unclear how. Scenário (100-b) shows that the trace should be permitted to range over pluralities (we don’t want to require that every child fed two dogs). At the same time, it cannot range over arbitrary pluralities, since we would lose the distributive effect of sowohl . . . als auch and predict (76) to be true in scenario (100-a). Intuitively, the definite determiner within the complex trace should apply to a predicate that is true of the conjunct denotations, and false of all other pluralities. Since (76) does not seem to contain any subconstituent denoting this predicate, this would require a complication of the theory of traces or a more complex syntax for D-conjunction.

(100) a. scenario: The girls all fed Carl, and the boys all fed Dean. (76) not true
b. scenario: Some of the girls fed Carl and the others fed Dean. Some of the boys fed Gene and the others fed Ivo. (76) true

The second problem relates to our general criticism of the predicate analysis in Section 3 (cf. Schmitt 2018). We showed that any analysis based on syntactically derived cumulative relations has a problem with the ‘flattening effect’. (101) shows that this effect extends to every DPs and other ADUs within a conjunction.

    the neighbours have Gene every of their cats feed and the dog brush.
    ‘The neighbours let Gene feed each of their cats and brush their dog.’

b. Scenario: Gene’s neighbours, Ada and Bea, are usually very protective of their pets. But today, Ada let him feed her two cats and Bea let him feed her cat and brush her dog.

As discussed in Section 3, scenario (101-b) requires an interpretation in which die Nachbarinnen cumulates both with the predicate conjunction and with the DP headed by jede ‘every’, but this reading cannot be derived via LF movement.

Summing up, Champollion (2010) shows that Schein sentences do not necessarily require building cumulativity into the mechanism for function-argument composition. However, he needs certain assumptions about LF syntax that are hard to maintain in the light of more complex cases. Thus, while we agree with Champollion that there is no intrinsic connection between cumulativity and event semantics, the consequences of his syntactic assumptions actually strengthen the case for a purely semantic approach to Schein’s puzzle.

7 Conclusion and open problems

In this paper, we considered expressions that can cumulate with plural expressions in some, but not all syntactic contexts. We showed that in addition to English every DPs (Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008, Champollion 2010) and German jed- DPs, this class of asymmetrically distributive universals (ADUs) also contains distributive conjunctions in several languages. We furthermore presented data suggesting that this asymmetry is at least partially related to scope, rather than thematic roles (following Champollion 2010 contra Kratzer 2003).

The behavior of ADUs was shown to impose several constraints on theories of cumulativity: First, since cumulativity asymmetries correlate with syntactic asymmetries, they cannot be reduced to properties of
lexical elements. The data thus lend independent support to Beck & Sauerland's 2000 claim that the mechanism responsible for cumulativity must be able to ‘span’ larger chunks of syntactic structure. In addition, the observation that cumulativity is sensitive to syntactic asymmetries seems to conflict with the traditional view that cumulativity is inherently symmetric. The second point goes back to Schein 1993: The behavior of ADUs in Schein sentences shows that the part structure of plural expressions in the scope of ADUs must still be accessible for cumulative relations between the ADU and higher plurals. Consequently, Schein sentences cannot be analyzed in terms of a single cumulative relation between all pluralities that ‘partake’ in cumulativity.

To account for these observations, existing proposals either assume that cumulation targets thematic-role relations (Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008), or limit cumulation to predicates of individuals while supplementing it with a complex LF syntax (Champollion 2010). We argued that both types of proposals undergenerate and presented a novel approach to ADUs and cumulativity that circumvents their problems.

The core idea of the plural projection mechanism we adopted (Schmitt 2018, Haslinger & Schmitt 2018b) is that all semantic domains contain pluralities and that the notion of cumulativity applies to any binary operation, regardless of its type. This allows us to derive cumulativity via a compositional rule operating on so-called plural sets which applies at every node intervening between the plurals that ‘partake’ in cumulativity. In this system, the parts of pluralities embedded in another plural expression correspond to parts of the denotation of the embedding plural expression. This accounts for the parallels between the so-called flattening effect and Schein sentences with ADUs.

We then analyzed every DPs and distributive conjunctions as functions that operate directly on plural sets. Each of them combines elements of the plural set in its scope with (i) each atom of its restrictor (every DP) or (ii) each individual conjunct (D-conjunctions), which enforces distributivity w.r.t. the nuclear scope. Since the output of this operation is a plural set which reflects the part structure of scopally dependent material, it is available for cumulation with syntactically higher plural expressions.

Since we already mentioned several unresolved problems above, we conclude by drawing attention to two further issues that arise when our proposal is viewed in a broader research context.

On the empirical side, all the existing compositional analyses of ADUs, including ours, fail to generalize to cumulative readings of non-upward-monotonic indefinites like exactly two girls. Since our truth definition involves existential quantification over a plural set at the sentence level, we fail to derive the ‘upper-boundedness’ of exactly two girls in (102).

(102) Exactly two girls fed every dog in this town.

A common response to the problem raised by non-upward-monotonic indefinites is to use a two-dimensional semantics, where plural indefinites introduce a separate semantic dimension responsible for the maximality conditions associated with modified numerals (e.g. Krifka 1999, Landman 2000, Brasoveanu 2013; but see Buccola & Spector 2016 for a different approach). Both dimensions are computed in parallel and combined at the sentence level, which allows the maximality conditions to ‘take scope’ over the cumulation operation. In Haslinger & Schmitt 2018a we show how this approach can be combined with the plural projection system.

On the theoretical side, there is one particularly pressing question (Buccola & Spector 2016, Haslinger & Schmitt 2017): The system employed here uses non-classical meanings for logical expressions like conjunction or universal determiners. These meanings are at odds with the assumptions of all analyses that rely on entailment relations between such lexical elements to derive linguistic phenomena like scalar implicatures (see e.g. Horn 1989, Sauerland 2004b).
References


