Tangled Up in Two
2D-Dilaton Gravity: A Toy Model for Gravity

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Outline

1 Motivation

2 Two-dimensional Dilaton Gravity

3 Black Hole Thermodynamics and the Euclidean Path Integral

4 Thermodynamics of 2D Dilaton Gravity

5 Conclusion and Outlook
Motivation

- Fundamental physics is plagued by a set of very deep and hard problems:
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Quantum Gravity
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- Unification of General Relativity with quantum theory
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**Quantum Gravity**  
Unification of General Relativity with quantum theory  
What to do when the problem is too hard?  
Study simple system = Toy models!
Motivation

- Fundamental physics is plagued by a set of very deep and hard problems: **Quantum Gravity**
- Unification of General Relativity with quantum theory
- What to do when the problem is too hard?
- Study simple system = Toy models!
- Toy models: 3D gravity, higher-spin gravity, 2D dilaton gravity, ...
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The action principle is given by:

$$I = \frac{1}{16\pi G} \int_{\mathcal{M}} d^2x \sqrt{|g|} \left[ X R - U(X)(\nabla X)^2 + 2 V(X) \right] + \frac{1}{8\pi G} \int_{\partial \mathcal{M}} d^x \sqrt{|\gamma|} X K$$

where $U(X)$ and $V(X)$ specify particular model
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Motivations for the study of dilaton gravity:

How can one obtain a theory of gravity in two dimension?

**Bulk action and GHY term for dilaton gravity**

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Why not \( I_{EH} = \int_{\mathcal{M}} d^Dx \sqrt{-g} R \) with \( D = 2 \)?

dilaton gravity describes spherically reduced Einstein gravity:
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ds^2 = g^{(D)}_{\mu\nu} dx^\mu dx^\nu = g_{\mu\nu} dx^\mu dx^\nu + \phi^2 (x^\mu) d\Omega^2_{D-2}
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inserted in \( D \)-dimensional EH action and integrated over \( S^{D-2} \).
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\[
X \propto (\lambda \phi)^{D-2}
\]

\[
U(X) = - \left( \frac{D-3}{D-2} \right) \frac{1}{X}
\]

\[
V(X) = -\frac{1}{2} (D-2)(D-3) \lambda^2 X^{\frac{D-4}{D-2}}
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\[ I = \int d^D x \sqrt{g} e^{-2\Phi} \left[ R + 4(\nabla \Phi)^2 + \frac{D - 26}{3\alpha'} \right] \]
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\( X = e^{-2\Phi} \) and \( D = 2 \) with appropriately chosen functions \( U, V \)
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Black Holes obey laws similar to thermodynamics

**Thermodynamics**

*Zeroth law:* $T = \text{const.}$ in equilibrium

*First law:* $dE = TdS + \mu dN + p dV$

*Second law:* $dS \geq 0$

*Third law:* $T \to 0$ not possible

**Black Hole Thermodynamics**

*Zeroth law:* $\kappa$ is constant on the horizon

*First law:* $dM = \kappa \frac{8\pi}{\kappa} dA + \Phi H dQ + \ldots$

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Black Hole Thermodynamics Reminder

Bekenstein (1972): Black holes have an entropy

\[ S_{BH} = \frac{c^3 A}{4 \pi G \hbar} \]

Hawking (1974): Black holes emit particles in a thermal spectrum of temperature

\[ T_H = \frac{\hbar \kappa}{2 \pi c k_B} \]

Black holes are thermodynamical systems. The laws of black hole mechanics are not a mere analogy but thermodynamics applied to black holes.

\[ dM = T_H dS_{BH} + \Phi_H dQ + ... \]
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Black Hole Thermodynamics Reminder

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$$dM = T_H dS_{BH} + \Phi_H dQ + ...$$
The Euclidean path integral provides an elegant way to BH thermodynamics:

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BH Thermodynamics and the Euclidean Path Integral

The Euclidean path integral provides an elegant way to BH thermodynamics:

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BH Thermodynamics and the Euclidean Path Integral I

The Euclidean path integral provides an elegant way to BH thermodynamics:

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- Specify boundary conditions for metric
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- Obtain thermodynamic potential \( Y \) from \( \mathcal{Z} \): \( \ln \mathcal{Z} = -\beta Y \)
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In semiclassical approximation:

\[ Z = \exp (-I[\bar{g}, \bar{X}]) \int \mathcal{D}\delta g \mathcal{D}\delta X \exp (-\delta I[\bar{g}, \bar{X}] - \delta^2 I[\bar{g}, \bar{X}] + ...) \]

\( \bar{g}, \bar{X} \) classical saddle-point of the action
Semiclassical approximation to Euclidean path integral of dilaton gravity:

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Variational principle/Thermodynamics well-defined if:

1. \[ \delta I = 0 \] for all variations compatible with boundary conditions
2. On-shell action \[ I[\bar{g}, \bar{X}] \] finite; then \[ I[\bar{g}, \bar{X}] = \beta Y \]
3. Thermodynamically stable if \[ \delta^2 I[\bar{g}, \bar{X}] > 0 \]; solved by putting BH in cavity

Euclidean dilaton action

\[ I = -\frac{1}{2} \int_M d^2x \sqrt{g} \left( XR - U(X) (\nabla X)^2 - 2 V(X) \right) - \int_{\partial M} d^x \sqrt{\gamma_X} K. \]
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\[ Z = \exp \left( -I[\bar{g}, \bar{X}] \right) \int \mathcal{D}\delta g \mathcal{D}\delta X \exp \left( -\delta I[\bar{g}, \bar{X}] - \delta^2 I[\bar{g}, \bar{X}] + \ldots \right) \]

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BH Thermodynamics and the Euclidean Path Integral II

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Semiclassical approximation to Euclidean path integral of dilaton gravity:

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dilaton gravity violates some conditions
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dilaton gravity violates some conditions
Add boundary term!
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1. Motivation

2. Two-dimensional Dilaton Gravity

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4. Thermodynamics of 2D Dilaton Gravity

5. Conclusion and Outlook
Euclidean Dilaton Action — Properties

→ identify correct boundary term!

$$I = -\frac{1}{2} \int_{\mathcal{M}} d^2x \sqrt{g} \left( R_X - U_X(\nabla_X)^2 - 2V_X \right) + \int_{\mathcal{M}} d^2x \sqrt{g_f} \left( \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} - \int_{\partial \mathcal{M}} d x \sqrt{g_X} K \right)$$

All solutions exhibit a Killing vector $\partial_{\tau}$; labelled by conserved quantity $Q_X = \int X \mathcal{Q}(y) dy \quad w_X = \int X e^{Q_X(y)} V(y) dy \quad h_X = \int X dy e^{Q_X(y)} f(y)$

Horizon encountered when $w_X h_X - 2M + q^2/4h_X = 0$
Euclidean Dilaton Action — Properties

→ identify correct boundary term!

Bulk action and GHY term for Maxwell-dilaton gravity

\[ I = -\frac{1}{2} \int_{\mathcal{M}} d^2x \sqrt{g} \left( X R - U(X)(\nabla X)^2 - 2V(X) \right) \]

\[ + \int_{\mathcal{M}} d^2x \sqrt{g} f(X) F^{\mu\nu} F_{\mu\nu} - \int_{\partial\mathcal{M}} dx \sqrt{\gamma} X K \]
identify correct boundary term!

**Bulk action and GHY term for Maxwell-dilaton gravity**

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\]

Solutions in diagonal gauge and \( X = X(r) \)

\[
ds^2 = \xi(X) d\tau^2 + \xi^{-1}(X) dr^2 \quad \partial_r X(r) = e^{-Q(X)}
\]
→ identify correct boundary term!

### Bulk action and GHY term for Maxwell-dilaton gravity

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Euclidean Dilaton Action — Properties

→ identify correct boundary term!

Bulk action and GHY term for Maxwell-dilaton gravity

\[ I = \frac{-1}{2} \int_{\mathcal{M}} d^2 x \sqrt{g} \left( X R - U(X)(\nabla X)^2 - 2V(X) \right) \]

\[ + \int_{\mathcal{M}} d^2 x \sqrt{g} f(X) F^{\mu\nu} F_{\mu\nu} - \int_{\partial\mathcal{M}} dx \sqrt{\gamma} X K \]

Solutions in diagonal gauge and \( X = X(r) \)

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Horizon encountered when \( w(X_h) - 2M + \frac{q^2}{4}h(X_h) = 0 \)
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Improved action given by \(^1\)

**Improved action**

\[
\Gamma = I + I_{ct} = I + \int_{\partial M} \, dx \sqrt{\gamma} \sqrt{e^{-Q(X)} w(X)}
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Thermodynamics of the improved action $\Gamma = I + I_{ct}$:

- fix periodicity of Euclidean time $\tau \sim \tau + \beta$ by regularity of metric at horizon; $\beta^{-1}$ coincides with Hawking temperature

calculate on-shell action $\Gamma[g_{cl}, X_{cl}]$ (now finite!)
calculate free energy $F(T, q) = -T \ln \Gamma[g_{cl}, X_{cl}]$
calculate thermodynamic quantities e.g. entropy $S$

$S = -\frac{\partial F}{\partial T} = 2\pi X h$

consistent with Wald’s entropy formula, coincides with spherically reduced Bekenstein–Hawking law

first law holds $\rightarrow$ reproduce correct thermodynamics!
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\(^2\text{M. Henneaux, C. Teitelboim} \textit{Phys. Lett.} B143 (1984)\)
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Works in any dimension with rank \( D \)-antisymmetric tensor field

Confining U(1) charge—Thermodynamics

Improved action for this class of models\(^3\)

\[^3\text{D. Grumiller, R. McNees, JS } \text{Phys.Rev. } \text{D90 (2014)}\]

\[^4\text{D. Grumiller } \text{J.Phys.Conf.Ser. } 33 (2006)\]
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c.c. and \( h(X_h) \) form \( p - V \) pair

BH thermodynamics in extended phase space

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Correct boundary term depends on specific model!

1. Asymptotic dilaton domination:
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Euclidean Dilaton Action

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   2D analogues of 3D flat-space cosmology solutions (FSCs)
Flat Space Cosmologies in 3D

Euclidean Einstein–Hilbert action in 3D

\[ I_{3D}^{EH} = - \frac{1}{2} \int_{\mathcal{M}} d^3x \sqrt{g} R \]

---


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\[ \Omega = r + r_0, \quad T = r^2 + 2\pi r_0 \]

\[ \text{phase transition between hot rotating flat space and FSC at } T_c = \frac{\Omega^2}{\pi}; \text{ similar to Hawking–Page phase transition AdS to BH} \]

---


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Flat Space Cosmologies in 2D dilaton gravity

Is this phase transition intrinsic to 3D?

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Flat Space Cosmologies in 2D dilaton gravity

Is this phase transition intrinsic to 3D? $\rightarrow$ Dimensional reduction

$\text{lim}_{X \rightarrow \infty} w(X) = 0 \quad \text{lim}_{X \rightarrow \infty} h(X) = 0$

e.g. dimensionally reduced from 3D

$ds^2 = (M - q X^2) d\tau^2 + (M - q X^2)^{-1} dr^2$

Study thermodynamics of this class of models

phase transitions between hot flat space and generalized FSCs occur fairly generically!

---

Is this phase transition intrinsic to 3D? → Dimensional reduction
generalized FSCs belong to class of asymptotic mass dominated models\(^7\):
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Outline

1 Motivation

2 Two-dimensional Dilaton Gravity

3 Black Hole Thermodynamics and the Euclidean Path Integral

4 Thermodynamics of 2D Dilaton Gravity

5 Conclusion and Outlook
Conclusion and Outlook

Dilaton gravity allows to study large class of different models in simple setting.

Complete study of thermodynamics of Maxwell-dilaton gravity, but not only interesting for thermodynamics!

Dilaton gravity allows non-perturbative path-integral quantisation! First order formulation:

\[ I[g, X] \rightarrow I[g[e^a, \omega^a, X], X] \]

Plus matter:

\[ Z = \int D\phi D[e D\omega D\phi] \]

Study semi-classical and quantum aspects of black holes in 2D!
Dilaton gravity allows to study large class of different models in simple setting.

\[ Z = \int D\phi D_e^a D\omega D^2X D^2X [\text{ghosts}] e^{i(I_g + I_m)} \]

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Thank you for your attention