BOUNDEDNESS OF FOURIER INTEGRAL OPERATORS ON
FOURIER LEBESGUE SPACES AND RELATED TOPICS

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(JOINT WORK WITH ELENA CORDERO AND LUIGI RODINO)

Abstract. Hörmander’s Fourier integral operators are, in a simplified local
form, integral operators in $\mathbb{R}^d$ of the type

$$Af(x) = \int e^{2\pi i \Phi(x,\eta)} \sigma(x,\eta) \hat{f}(\eta) \, d\eta.$$ 

The symbol $\sigma$ satisfies the growth estimate

$$|\partial_\alpha x \partial_\beta \eta \sigma(x,\eta)| \leq C_{\alpha,\beta}(1 + |\eta|)^{m-|\beta|}, \quad \forall (x,\eta) \in \mathbb{R}^{2d},$$

We also suppose that $\sigma$ has support with compact projection with respect to $x$. The phase $\Phi(x,\eta)$ is real-valued, positively homogeneous of degree 1 in $\eta$, smooth for $\eta \neq 0$, and non-degenerate. It is easy to see that such an operator maps the space $S(\mathbb{R}^d)$ of Schwartz functions into itself continuously. As basic results, the operator $A$ is $L^2$-bounded for $m = 0$, as well as $L^p$-bounded, $1 < p < \infty$, if the order $m$ of $\sigma$ is negative, satisfying

$$m \leq -(d-1) \left| \frac{1}{2} - \frac{1}{p} \right|,$$

as proved in [3].

Here we present some results from [1] where we studied the action of an operator $A$ as above on the spaces $F_L^p$ of temperate distributions whose Fourier transform is in $L^p$ (with the norm $\|f\|_{F_L^p} = \|\hat{f}\|_{L^p}$). We show that $A$ is bounded as an operator $(F_L^p)_{comp} \rightarrow (F_L^p)_{loc}$, $1 \leq p \leq \infty$, if $m \leq -d(1/2 - 1/p)$. This is similar to (1), but with the difference of one unit in the dimension. Surprisingly, this threshold is shown to be sharp in any dimension $d \geq 1$, even for phases linear with respect to $\eta$.

Another related problem is to investigate the global boundedness of such an operator, when $\sigma$ is no longer compactly supported with respect to $x$ but satisfies suitable decay estimates at infinity. This is the object of [2], which will be briefly discussed as well. In particular, we present some striking examples of failure of global boundedness on $L^p$. When dealing with such a global perspective, besides the spaces $L^p$ and $F_L^p$, the modulation spaces represent a natural framework for these problems.

References


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