BOUNDEDNESS OF FOURIER INTEGRAL OPERATORS ON FOURIER LEBESGUE SPACES AND RELATED TOPICS

FABIO NICOLA
(JOINT WORK WITH ELENA CORDERO AND LUIGI RODINO)

Abstract. Hörmander's Fourier integral operators are, in a simplified local form, integral operators in $\mathbb{R}^d$ of the type

$$Af(x) = \int e^{2\pi i\Phi(x,\eta)}\sigma(x,\eta)\hat{f}(\eta)\,d\eta.$$

The symbol σ satisfies the growth estimate

$$|\partial_x^\alpha\partial_\eta^\beta\sigma(x,\eta)| \le C_{\alpha,\beta}(1+|\eta|)^{m-|\beta|}, \quad \forall (x,\eta)\in\mathbb{R}^{2d},$$

We also suppose that σ has support with compact projection with respect to x. The phase $\Phi(x,\eta)$ is real-valued, positively homogeneous of degree 1 in η, smooth for $\eta \neq 0$, and non-degenerate. It is easy to see that such an operator maps the space $\mathcal{S}(\mathbb{R}^d)$ of Schwartz functions into itself continuously. As basic results, the operator A is L^2-bounded for $m = 0$, as well as L^p-bounded, $1 < p < \infty$, if the order m of σ is negative, satisfying

$$m \le -(d-1)\left|\frac{1}{2}-\frac{1}{p}\right|, \tag{1}$$

as proved in [3].

Here we present some results from [1] where we studied the action of an operator A as above on the spaces $\mathcal{F}L^p$ of temperate distributions whose Fourier transform is in L^p (with the norm $\|f\|_{\mathcal{F}L^p} = \|\hat{f}\|_{L^p}$). We show that A is bounded as an operator $(\mathcal{F}L^p)_{comp} \to (\mathcal{F}L^p)_{loc}$, $1 \le p \le \infty$, if $m \le -d\,|1/2-1/p|$. This is similar to (1), but with the difference of one unit in the dimension. Surprisingly, this threshold is shown to be sharp in any dimension $d \ge 1$, even for phases linear with respect to η.

Another related problem is to investigate the *global* boundedness of such an operator, when σ is no longer compactly supported with respect to x but satisfies suitable decay estimates at infinity. This is the object of [2], which will be briefly discussed as well. In particular, we present some striking examples of failure of global boundedness on L^p. When dealing with such a global perspective, besides the spaces L^p and $\mathcal{F}L^p$, the modulation spaces represent a natural framework for these problems.

References

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[3] A. Seeger, C. D. Sogge and E. M. Stein., *Regularity properties of Fourier integral operators*, Ann. of Math. (2) **134** (1991), 231–251.

Dipartimento di Matematica, Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy

E-mail address: `fabio.nicola@polito.it`