In current brain machine interfaces, action potentials produced in the brain are typically transformed into digital signals using a uniform sampler, which ignore local structure in the signal and only depend on the maximum bandwidth of the input. In contrast, we present a new analog to digital converter which exploits the sparse features of neural recordings (action potentials).

We encode neural signals with the Bifasic Integrate-and-Fire sampler (BIF) [1] and are able to recover action potentials sampled at sub-Nyquist rates.

The output pulse train is defined over a discrete set of time stamps, usually nonuniformly spaced. Each of the intervals between the pulses satisfies:

\[ \theta_i = \int f(t) u_i(t) dt \text{ where } u_i(t) = \frac{1}{2} \left( e^{-t/\tau} - e^{-(t-\tau)/\tau} \right) \chi_{[i\tau, (i+1)\tau]} \]  

where \( \chi \) is the characteristic function over \( I \) (defined as one over the interval and zero everywhere else). The constants \( R \) and \( C \) represent the resistance and the capacitor values in the hardware implementation. The reconstruction algorithm is based on the framework developed in the area of nonuniform sampling [3, 4, 5]. The advantage of this method is that the size of the system matrix \( M \) is predefined based on the number of Fourier coefficients selected and does not increase with the number of samples.

We only consider finite length signals and a finite number of samples. Therefore \( p(t) \) can be approximated using \( 2M + 1 \) elements of the Fourier series. Our signal space is then:

\[ p_{\mathcal{M}} = \left\{ p : f \in \mathbb{C}^{2M+1} \right\} \]

Based on [4, 5], for \( p \) local averages, \( r \geq 2M + 1 \), the set of averaging functions \( u_\mathcal{M} \) satisfy the frame conditions [2] for all elements in \( p_{\mathcal{M}} \) and its projection into the bandlimited space \( u_\mathcal{M} \) is a frame for \( p_{\mathcal{M}} \). A frame operator is defined on the Fourier coefficients instead of the time signal. In matrix notation: \( Af = b \). Where \( A \) is the \( (2M+1) \times (2M+1) \) matrix with entries:

\[ (A)_{jk} = \sum_{\nu} \left( u_j(\nu) e^{2\nu j \beta} d\nu \right) \left( \int u_j(\nu) e^{-2\nu j \beta} d\nu \right) \]

and

\[ b_k = \sum_{\nu} \left( \langle p, u_j \rangle \int u_j(\nu) e^{-2\nu j \beta} d\nu \right) \]

The solution to this linear system can be found iteratively avoiding a direct inversion of the matrix \( A \). Once the coefficients are known, the time signal is easily computed by using the inverse Fourier transform.

The complete algorithm consists of the following steps:

1. Define \( u_{\nu,i} \) as in eq. 1.
2. Define the matrix \( A \) as in eq. 3.
3. Define the vector \( b \) from eq. 4, where \( (p, u_j) \) are determined from the samples.
4. Solve for \( \hat{y} \). This can be done using direct or iterative methods (Richardson method, conjugate gradients).
5. Reconstruct the time signal using the inverse Fourier transform.

The algorithm is linear in \( N \).

- \( N \): Number of samples.
- \( M \): Predefined number of Fourier coefficients.
- \( K \): Iterations used to determine \( \hat{y} \).
- \( S \): Number of points for the inverse Fourier transform.

**Synthetic input**

It consists of a finite sum of complex exponentials \( (M = 10) \) with randomly selected coefficients (only real signals are considered).

The original and reconstructed signals with the corresponding pulse train are shown. The Signal to Error Ratio (SER = \( 10^{-\log_{10}(\text{SER})} \) over the entire signal was 90 dB.

In real applications, we may only have an estimate of the input bandwidth. If the value is underestimated, perfect recovery is not possible. On the other hand if \( M > M \), where \( M \) is the estimate, it is still possible to find the solution but with an added cost. The following figure shows the effect on the SER (with 1000 iterations) and the condition number of the \( A \) matrix when overestimating the bandwidth.

**Results**

**Conclusions**

Using the BIF sampler and the proposed reconstruction algorithm, the pulse rate and the minimum sampling rate for neural signals can be reduced below the Nyquist bound and high SER can be achieved in the regions of interest. The structure of the matrix and the linear complexity in the number of samples make the algorithm appealing for real-time applications. However, the results show that the algorithm is sensitive to the sample distribution. Large gaps lead to an increase in the convergence rate and the condition number of the system matrix.

**References**


