MULTIPLICATION PROPERTIES IN PSEUDO-DIFFERENTIAL CALCULUS WITH SMALL REGULARITY ASSUMPTIONS ON THE SYMBOLS

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Abstract. There are four important products which appear on the symbol levels in pseudo-differential calculus. The composition of two Weyl quantizations corresponds on the symbol level on the so called Weyl product. The convolution appears when putting the theory of Toeplitz operators (the same as localization operators) on functions on $\mathbb{R}^d$ into the context of pseudo-differential calculus, using the fact that the Weyl symbol of a Toeplitz operator is a convolution by the Toeplitz symbol and an other convenient function. Finally, the twisted convolution and ordinary multiplication appear when applying the (symplectic) Fourier transform on Weyl products and ordinary convolutions, respectively.

In the talk we establish Young and Hölder relations for such products on Schatten-von Neumann symbols, Lebesgue spaces and modulation spaces. We use the results to extend the class of possible window functions in the definition of modulation spaces, and to prove that any Schatten-$p$ symbol in the Weyl calculus gives rise to a Schatten-$p$ Toeplitz operator. The Schatten-von Neumann classes here are of the form $\mathcal{S}_p(\mathcal{H}_1, \mathcal{H}_2)$, where $\mathcal{H}_1, \mathcal{H}_2$ are Hilbert spaces of (ultra-)modulation space type which stay between the Gelfand-Shilov space $\Sigma_1(\mathbb{R}^d)$ and its dual $\Sigma'_1(\mathbb{R}^d)$. Furthermore, the symbol classes of pseudo-differential operators stay between $\Sigma_1(\mathbb{R}^{2d})$ and its dual $\Sigma'_1(\mathbb{R}^{2d})$.

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