with equality if and only if \( f = 0 \) in \( D \) and
\[
\mathcal{M} = \mathcal{N} = \mathcal{M} + (x)\mathcal{M} \geq (x)\mathcal{N}.
\]
(02)

Then we have Young's inequality
\[
\mu(1) = \mu = \mu(1)\mathcal{N}
\]
(10)

where the derivative \( \mu \) is increasing. We set
\[
\mathcal{M}(x < 0) = 0
\]
for all \( x < 0 \). In particular, \( \mathcal{M} \) is a convex function with monotonically increasing derivative \( \mu \). We denote a two times differentiable function on \( \mathbb{R}^+ \) with derivative \( \mathcal{M} \).

**Preliminaries and Summary of Results**

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Received September 3, 1987

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Wigner Distribution and Bargmann Transform

Growth of Hermite Coefficients

Spaces of Type \( \mathcal{M} \).
Wigner distribution of any tempered distribution can be defined. Here we denote the Wigner distribution on \( \mathbb{R}^2 \) by \( \mathcal{W}(\phi) \), where \( \phi \) is a function in \( \mathcal{S}^* \). This \( \mathcal{W}(\phi) \) can be expressed as:

\[
\mathcal{W}(\phi)(x,y) = \int_{\mathbb{R}} e^{2\pi i (x\xi + y\xi^2)} \hat{\phi}(\xi) \, d\xi
\]

where \( \hat{\phi}(\xi) \) is the Fourier transform of \( \phi \).

The Wigner distribution is a generalization of the Fourier transform, and it can be used to analyze non-stationary signals in time and frequency domains. It provides a way to study the local properties of a function at different scales.

\[ \int_{\mathbb{R}} \mathcal{W}(\phi)(x,y) \, dx \, dy = \int_{\mathbb{R}} \phi(x) \, dx \]

is a constant.

The Wigner distribution is closely related to the information theory, where it is used to measure the uncertainty in a signal. It is also used in quantum mechanics to describe the distribution of a quantum state in phase space.
Throughout this paper, we deal with the following class of convex functions:
\[ \{ f(x) = \frac{1}{2} \| x \|_2^2 + \int_0^1 (\phi(t) x)^2 dt \mid \phi(t) \in C[0,1] \} \]

Here are the corresponding characterizations:

\[ \| x \|_2 \leq p \Rightarrow \sup_{\| \phi \|_H \leq 1} \int_0^1 (\phi(t) x)^2 dt \leq p^2 \]

Next, starting from square integrable functions with Wiener distribution, we derive integrals of square integrable functions with different distributions.

The plan of the paper is as follows. Let us consider a square integrable function of the form

\[ f(x) = \sum_{n=1}^{\infty} a_n x_n \]

The Bernstein-Gelfand-Fomin space is defined as

\[ \sum_{n=1}^{\infty} a_n x_n \in C \Rightarrow \sup_{\| \phi \|_H \leq 1} \int_0^1 (\phi(t) x)^2 dt \leq p^2 \]

For the Bernstein transformation, we refer to [1] where the Bernstein transformation is given.

Therefore, we obtain a unique operator \( A \) from \( (0,1) \to (1,0) \). In particular, we have

\[ A = \sum_{n=1}^{\infty} a_n x_n \]

The Bernstein transformation is defined by

\[ \sum_{n=1}^{\infty} a_n x_n \in C \Rightarrow \sup_{\| \phi \|_H \leq 1} \int_0^1 (\phi(t) x)^2 dt \leq p^2 \]

with the Bernstein-Szego-Fomin space consists of all continuous functions.

The Bernstein-Szego-Fomin space consists of all continuous functions:

\[ \{ f(x) = \frac{1}{2} \| x \|_2^2 + \int_0^1 (\phi(t) x)^2 dt \mid \phi(t) \in C[0,1] \} \]

Here, we prove the corresponding characterization:

\[ \| x \|_2 \leq p \Rightarrow \sup_{\| \phi \|_H \leq 1} \int_0^1 (\phi(t) x)^2 dt \leq p^2 \]

The Bernstein-Szego-Fomin space consists of all continuous functions in

\[ \{ f(x) = \frac{1}{2} \| x \|_2^2 + \int_0^1 (\phi(t) x)^2 dt \mid \phi(t) \in C[0,1] \} \]

The Bernstein-Szego-Fomin space consists of all continuous functions in

\[ \{ f(x) = \frac{1}{2} \| x \|_2^2 + \int_0^1 (\phi(t) x)^2 dt \mid \phi(t) \in C[0,1] \} \]

Furthermore, we prove the corresponding characterization:

\[ \| x \|_2 \leq p \Rightarrow \sup_{\| \phi \|_H \leq 1} \int_0^1 (\phi(t) x)^2 dt \leq p^2 \]
Now we proceed as follows: Let $0 < \gamma < \eta$. Then we have

$$(b) \mathcal{W} + (d) \mathcal{W} \geq (b + d) \mathcal{W}$$

is square integrable. For all $0 < b, d, \gamma, \eta$, we have

$$(x) \phi \left[ \int_{\mathbb{R}^d} (\nabla \phi || \phi ||^2) \right]$$

The conditions on $\mathcal{W}$ imply that for each $\eta$, the function $\gamma > \eta > 0$, the inequality $\gamma \frac{\partial}{\partial x} \mathcal{W} - \int_{\mathbb{R}^d} (\nabla \phi || \phi ||^2)$ holds.

Further, as a consequence of Theorem 1.5, we can replace equivalence (6.9).

$$(\int_{\mathbb{R}^d} (\nabla \phi || \phi ||^2)) \mathcal{W} = \mathcal{W}$$

and we have

$$(b) \mathcal{W} + (d) \mathcal{W} \geq (b + d) \mathcal{W}$$

Now let denote the self-adjoint operator of multiplication by $x$ in the space of all $\mathcal{W}$-isometric operators, such that

$$(x) \phi \left[ \int_{\mathbb{R}^d} (\nabla \phi || \phi ||^2) \right]$$

positive self-adjoint operators $\mathcal{W}$ we have

$$(0, \mathcal{W}) \subseteq \mathcal{W} \subseteq \mathcal{W}$$

for all $\mathcal{W}$-operator $x$ dependent on $\mathcal{W}$.
\[ \int_0^\infty - \int_a^b f \, dx \leq 0 \]

Then

\[ \int_0^\infty f \, dx < 0 \quad \text{for some } f \in L^\infty \]

Hence

\[ \int_0^1 \frac{x^2}{1 + x^4} \, dx \leq 0 \]

According to [4], Sec. 12.

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Further, since \( f, g \in L^\infty \), it follows that

\[ \int_0^1 \frac{x^2}{1 + x^4} \, dx < 0 \]

Hence

\[ \int_0^1 \frac{x^2}{1 + x^4} \, dx < 0 \]

Proof. Let \( 0 < q < p \), then the function \( x \mapsto x^q \) is convex. We have

\[ \int_0^1 \frac{x^2}{1 + x^4} \, dx \leq 0 \quad \text{since the function } x \mapsto x^q \]

Then

\[ \int_0^1 \frac{x^2}{1 + x^4} \, dx < 0 \]

Consider the following lemma.

The assertion now follows from the inequality

\[ \int_0^1 \frac{x^2}{1 + x^4} \, dx \leq 0 \]

because \( x \mapsto x^q \) is a convex function.

\[ \int_0^1 \frac{x^2}{1 + x^4} \, dx \leq 0 \]

Hence

\[ \int_0^1 \frac{x^2}{1 + x^4} \, dx < 0 \]

Now let \( q > p \). Then we have

\[ \int_0^1 \frac{x^2}{1 + x^4} \, dx < 0 \]

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\[
\begin{align*}
&\text{If } \xi \sim \mathcal{N}(0, 1) \text{ and } \gamma > 0 \text{, then} \\
&\quad \mathbb{E}[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-x^2/2} dx \\
&\text{and similarly, since } f(\xi) \sim \mathcal{N}(\mu, \sigma^2) \text{, we have} \\
&\quad \mathbb{E}[f(\xi)] = \int_{-\infty}^{\infty} f(x) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} dx.
\end{align*}
\]

Thus we have established a relation between the Hermite coefficients and the Gaussian distribution.

\section*{Characterization Based on Hermite Coefficients}

Now let us derive the Hermite polynomial.

1. \( |x| > \sqrt{2} \Rightarrow |\gamma(x)| \leq 2 \) and similarly, since \( f(\gamma(x)) \sim \mathcal{N}(\mu, \sigma^2) \), we have

\[
\begin{align*}
&\text{Let } \gamma > 0 \text{, then} \\
&\quad \mathbb{E}[f(\gamma(x))] = \int_{-\infty}^{\infty} f(x) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} dx.
\end{align*}
\]

Proof. For \( \gamma > 0 \), we have

\[
\begin{align*}
&\text{Lemma 1.3:} \\
&\quad \mathbb{E}[f(\gamma(x))] = \int_{-\infty}^{\infty} f(x) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2} dx.
\end{align*}
\]
where \( \gamma \in \mathbb{N} \), \( \delta \), and \( \omega \) are positive constants.

Thus the claim follows (the term with \( \ell = 0 \) is easily taken care of). Therefore

\[
\ell(i + f) \geq \frac{(f)^i}{\ell} \frac{f}{1 + f} 
\]

so by (11) we derive

\[
\ell(i + f) \frac{(f)^i}{\ell} \frac{f}{1 + f} \geq \frac{((f)^i)^{1/\ell}}{\ell} \frac{f}{1 + f} 
\]

Indeed we have

\[
\ell(i + f) \frac{(f)^i}{\ell} \frac{f}{1 + f} \geq \frac{(f)^i}{\ell} \frac{f}{1 + f} \]

Now we assert that there are \( c > 0 \) such that

\[
\ell(i + f) \frac{(f)^i}{\ell} \frac{f}{1 + f} \geq \frac{(f)^i}{\ell} \frac{f}{1 + f} \]

so that by (11) we obtain

\[
\ell(i + f) \frac{(f)^i}{\ell} \frac{f}{1 + f} \geq \frac{(f)^i}{\ell} \frac{f}{1 + f} \]

Further, we have the following crude estimate

\[
\ell(i + f) \frac{(f)^i}{\ell} \frac{f}{1 + f} \geq \frac{(f)^i}{\ell} \frac{f}{1 + f} \]

By assumption there exists \( \gamma > 0 \) such that

\[
\ell(i + f) \frac{(f)^i}{\ell} \frac{f}{1 + f} \geq \frac{(f)^i}{\ell} \frac{f}{1 + f} \]

We arrive at the following estimate

\[
\ell(i + f) \frac{(f)^i}{\ell} \frac{f}{1 + f} \geq \frac{(f)^i}{\ell} \frac{f}{1 + f} \]

Clearly, for \( 0 > \ell > 0 \),

\[
\ell(i + f) \frac{(f)^i}{\ell} \frac{f}{1 + f} \geq \frac{(f)^i}{\ell} \frac{f}{1 + f} \]
If the minimum is attained at \( x = d \), the function is continuous, and we have the equation

\[
0 < b \quad 0 = ((d + 1)v)_{\text{min}} + d.
\]

If there exists a constant \( q \) such that

\[
0 < b \quad ((d + 1)v)_{\text{min}} + d - b = \gamma.
\]

for all \( x \), there exists a constant \( q \) such that

\[
((d + 1)v)_{\text{min}} + d - b = \gamma
\]

By the mean value theorem, for some \( y \), we have

\[
((d + 1)v)_{\text{min}} + d - b = \gamma.
\]

Since \( y \) is continuous, we have

\[
((d + 1)v)_{\text{min}} + d - b = \gamma.
\]

Hence, by the mean value theorem, we have

\[
((d + 1)v)_{\text{min}} + d - b = \gamma.
\]

Toward an estimate, we consider the equation

\[
((d + 1)v)_{\text{min}} + d - b = \gamma
\]

For \( q > \gamma \), the largest integer between \( q \) and \( \gamma \) is

\[
((d + 1)v)_{\text{min}} + d - b = \gamma
\]

Therefore, for some constant \( q \) such that

\[
((d + 1)v)_{\text{min}} + d - b = \gamma
\]

we find

\[
((d + 1)v)_{\text{min}} + d - b = \gamma
\]

Since \( v \) is continuous, we have

\[
((d + 1)v)_{\text{min}} + d - b = \gamma
\]

Hence, by the mean value theorem, we have

\[
((d + 1)v)_{\text{min}} + d - b = \gamma
\]

We get

\[
0 < d \quad 0 < d
\]

Using that for some \( d \), we have

\[
0 < d
\]
\[
\frac{b_z}{(b) u} = (b) I + \frac{(b) W}{2} - \frac{(b) z}{d} = (b, d) \chi
\]

where the function is defined as

\[
[(b) W - \zeta] dx = \zeta - b \cdot \chi \wedge \zeta + (b, b) \chi
\]

When

\[
1 + (1) \leq b
\]

\[
\frac{3b_z}{(b) u} \leq \frac{(b) W}{2} - \frac{(b) z}{d} = (b, d) \chi
\]

we have

\[
[(i) \chi + \zeta b] dx = \frac{(i) \chi}{\chi} - \zeta = (i) \chi
\]

Now let us consider the function \((b, d) = (b, d) \chi\) is decreasing.

Since \(0 < b \leq (b) u\), we have

\[
(1 - b) \chi \leq \frac{(b) u}{(b) u - (b) u}
\]

Therefore we have

\[
sp \left( 1 - \frac{b_z}{d} \right) \frac{(b) u}{(b) u - (b) u} \leq
\]

\[
sp ((s) u - (s) u) \left( \frac{b_z}{d} - \frac{1}{d} \right) \frac{b_z}{d} \leq sp \left( 1 - \frac{b_z}{d} \right)
\]

and

\[
(1 - b) \chi \frac{(b) u}{(b) u - (b) u} = 1
\]

Thus we obtain

\[
(1) \chi = (i) \chi
\]

Since \(b \leq (b) u\), the function \(i\) is strictly decreasing. Hence it can be inverted.

\[
0 < b \quad \frac{b_z}{(b) u} = (b) I
\]

A straightforward analysis yields

\[
0 < b \quad (b) W = ((b) I)^{\chi} + (b) \chi b
\]

and

\[
\frac{b_z}{(b) u} = (b) I
\]
By Lemma (3.15), we have
\[
\int_{(1)^2} \left( (1)^2 \delta - \left( \frac{\delta}{x} \right) \frac{1}{1} \right) dx \, ds
\]
and by Lemma (3.15), we have
\[
\int \left[ (1)^2 \phi - \left( \frac{1}{1} \right) \frac{1}{1} \right] dx \, ds.
\]
By Lemma (3.15), we have
\[
\int ((1)^2 \delta - \left( \frac{1}{1} \right) \frac{1}{1}) dx \, ds.
\]
Finally, since \( x \) is such that \( x = 1 \), we get
\[
\int ((1)^2 \delta - \left( \frac{1}{1} \right) \frac{1}{1}) \, dx \, ds.
\]
The latter integral is estimated by
\[
\int ((1)^2 \delta - \left( \frac{1}{1} \right) \frac{1}{1}) \, dx \, ds.
\]
There exists a constant \( K \) such that for all \( x \leq t \),
the remaining estimations are based on Lemma (3.15).
\[
\int \left( \frac{1}{1} \right) \frac{1}{1} \, ds.
\]
for some $C' < 0$. Then
\[
[(\gamma'(\gamma - \gamma)) - \gamma]' \geq 0
\]
where
\[
\left(\frac{\mu^\gamma}{\omega^z}\right)' \geq 0
\]
defines the property that for certain positive constants $C, \theta, \eta > 0$, for every $\theta > \eta > 0$, the statement yields

**Lemma.** Let $\theta$ be a holomorphic function with the property that

\[
[(\gamma'(\gamma - \gamma)) - \gamma]' \geq 0
\]

then it follows that

\[
[(\gamma'(\gamma - \gamma)) - \gamma]' \geq 0
\]

and also that

\[
[(\gamma'(\gamma - \gamma)) - \gamma]' \geq 0
\]

that one of $\theta > \eta$ or $\eta > \theta$ is true. Since $\mu^\gamma$ is bounded, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. 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Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma$. Since $b^\gamma$ is also $\theta$, the set of $\theta$ is in the interior of $\mu^\gamma. \square$
REFERENCES

Consequently, for all \( a, b, c \in \mathbb{R} \) where \( a > 0, b > 0 \), there exists \( c \in (0, \infty) \) such that

\[
(\phi_a) x \in \mathbb{R} \exp([c x] + [c x] + [c x]) + \|x\|.
\]

Because of the inequality \( \|x\| \leq \|x\| + u \) follows that

\[
(\phi_{a + b} + \phi_c) x \in \mathbb{R} \exp([c x] + [c x] + [c x]) + \|x\|.
\]

For all \( u \in I, 1, 2, \ldots \). In particular, for \( u = 1, 2, \ldots \), we get

\[
(\phi_2) x \in \mathbb{R} \exp([c x] + [c x] + [c x]) + \|x\|.
\]

Proof: By Cauchy's formula,