Exercises for nonlocality, entanglement und geometry of quantum systems
Sheet 4

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3.11.2010

Exercise 11

In this exercise we will look at the effective mass Hamiltonian, which is used to describe the decay of particles, for example k-mesons (kaons).
The eigenstates of H are

$$H|K_S\rangle = \lambda_S|K_S\rangle$$
$$H|K_L\rangle = \lambda_L|K_L\rangle$$

where $\lambda_{L,S} = m_{L,S} - \frac{i}{2}\Gamma_{L,S}$.
m describes the mass of the particle, and $\Gamma = \frac{\hbar}{\tau}$ where τ is the lifetime of the particle.
The indeces S, L stand for "short" and "long", because the lifetime of the two states differ. ($\tau_S = 10^{-10}s$, $\tau_L = 5 \cdot 10^{-8}s$) The state of a kaon is in general described by a density matrix

$$\rho(t) = \sum_{i,j=K_S,K_L} \rho_{ij}(t)|i\rangle\langle j|$$

A single element can be written as $\rho_{ij}(t) = \langle i|\rho(t)|j\rangle$.
The time evolution of this density matrix is given by the von Neumann equation.

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}\left(H\rho(t) - \rho(t)H^\dagger\right)$$

Calculate the time evolution of the matrix elements of $\rho(t)$ by solving the von Neumann equation.

Exercise 12

Consider a spin 1/2 particle in a static magnetic field. The Hamiltonian is given by

$$H = -\frac{\gamma B}{2}\sigma_z$$

where $\gamma = g \cdot \mu_B$, g is the gyromagnetic ratio (in this case $g = 2$) and μ_B is the Bohr magneton $\left(\mu_B = \frac{e\hbar}{2mc}\right)$.

The von Neumann equation is in this case

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}\left[H, \rho(t)\right]$$

Calculate the time evolution of the matrix elements of $\rho(t)$.