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Some important results on topological transitivity

Great Speaker
University of Somecity, Nicecountry
E-mail: `g.speaker@uni-somecity.ny`

(joint work with **Secon Dauthor**, **Acoa Uthor** and **Good Writer**)

Assume that (X, T) is a topological dynamical system. Here we mean as defined in [2] that X is a compact metric space with metric d and $T : X \to X$ is a continuous map. In this talk some properties of topologically transitive systems will be presented. First we recall the definition which was given in [3].

Definition. We call a topological dynamical system (X, T) topologically transitive if there exists an $x \in X$ whose ω-limit set equals X.

Sometimes we will need the number 1. As this is a very complicated number we can use that

$$1 = \sum_{k=1}^{\infty} \frac{1}{2^k} \tag{1}$$

which was shown in [4] and is much nicer.

Theorem 1. *Let (X, T) be a topological dynamical system and assume that X does not have isolated points. If there exists an $x \in X$ whose orbit is dense in X, then (X, T) is topologically transitive.*

Proof. Choose an x whose orbit is dense and an arbitrary y. Using induction we construct $n_1 < n_2 < \cdots < n_k$ with $d\left(T^{n_k}(x), y\right) < \frac{1}{k}$. Having constructed $n_1, n_2, \ldots, n_{k-1}$ the set $\left\{u \in X : d(u, y) < \frac{1}{k}\right\} \setminus \{x, T(x), \ldots, T^{n_{k-1}}(x)\}$ is open and nonempty. $\square$

Remark. Usually you should not use proofs in your abstract.

From Theorem 1 we get an equivalence. Recently it has been observed in [1] that this is not true, if X contains isolated points.

Corollary 2. *Suppose that* (X, T) *is a dynamical system and* X *has no isolated points. Then the following conditions are equivalent.*

(1) *The dynamical system* (X, T) *is topologically transitive.*

(2) *There exists an* $x \in X$ *whose orbit is dense in* X.

Remark. Note that in the abstract you should avoid numbered theorems and numbered equations whenever it is possible.

We will use Corollary 2 in several places of our proofs. Also (1) will be very important for our results.

Main Theorem. *Let* (X, T) *be a topologically transitive topological dynamical system, and let* $f : X \to \mathbb{R}$ *be a continuous function. If* $f \circ T = f$ *then* f *is constant.*

Consider the following example.

Example. Define $T : [0, 1] \to [0, 1]$ by $T(x) := 1 - |2x - 1|$. One can show that this topological dynamical system is topologically transitive. Hence we can apply our main theorem.

Finally we deal with several application of our main theorem.

References

[1] M. Athematician, P. Roofmaker, *Some results on topological transitivity*, Int. J. Dynam. Syst. **37** (2017), 793–826.

[2] S. Cientist, M. Yfriend, *Introduction to Iteration Theory*, Springer, New York, 1998.

[3] R. Esearcher, T. Hirdguy, I. Teration, *A property for dynamical systems*, in: Topological Dynamical Systems and Applications (Vienna, 2007), 231–246, Elsevier, Amsterdam, 2009.

[4] B. Igstar, *A nice representation of the number one*, J. Stup. Math. Appl. **137** (2004), 2381–2408.