Interferometric Bell-state analysis

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We present the experimental demonstration of a Bell-state analyzer employing two-photon interference effects. Photon pairs produced by parametric down-conversion allowed us to generate momentum-entangled Bell states and to demonstrate the properties of this device. The performance obtained indicates its readiness for use with quantum communication schemes and in experiments on the foundations of quantum mechanics.

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The study of correlations and of entanglement between quantum systems was initiated by Einstein, Podolsky, and Rosen (EPR) as a means of inquiring about the completeness of quantum mechanics [1]. And for a long time entangled states were merely seen as a tool to study fundamental issues in quantum mechanics [2]. Only recently was the potential use of quantum correlations for quantum computation [3] and for a variety of new schemes in quantum communication discovered. Besides an alternative technique for quantum cryptography [4], several proposals increase and extend the performance of standard communication techniques by employing various kinds of entanglement manipulation. Quantum dense coding allows the transmission of two bits of information in a single two-state particle, and the transfer of an unknown quantum state becomes possible by quantum teleportation [5]. More recent proposals already use these techniques for state preparation in extended Bell-type experiments and for the manipulation of general entangled states [6,7].

Central to all these new ideas is the combination of two, in the simplest case, two-state quantum systems and a measurement in the so-called Bell-state basis defined by the four basis states

\[
|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|A\rangle|C\rangle \pm |B\rangle|D\rangle),
\]

\[
|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|A\rangle|D\rangle \pm |B\rangle|C\rangle),
\]

where \(|A\rangle\), \(|B\rangle\) represent the two states of particle one and \(|C\rangle\), \(|D\rangle\) those of particle two, respectively [7]. Up to now, only two-particle interactions are known to be able to perform unique measurements on the two-particle system, using, for example, the strong coupling between an atom and a cavity field in proposals for teleporting quantum states [8]. The progress in cavity-QED experiments [9] is likely to make such measurements possible.

However, two-particle interference enables one to realize an efficient analysis of Bell states much easier and allows at least a partial solution to the problem [10,11]. In our case, different interference effects produce three different results, identifying two out of the four Bell states with the other two states giving the same, third, measurement signal. This already can be used to encode 1.58 (\log_2 3) bits of information in a two-state photon (as compared with the classical limit of one bit) or to teleport 50% of the quantum particles. However, the method fully suffices to entangle independent particles or to manipulate and to transform different kinds of entanglement. In this Rapid Communication we report an experimental realization of a Bell-state analyzer using bosonic interference effects between two momentum-correlated particles.

The states \(|A\rangle\) and \(|B\rangle\) for the first particle and \(|C\rangle\) and \(|D\rangle\) for the second particle, respectively, are represented by two spatial modes (or beams) in which the particle can be found. Combining the modes of the two particles at the beam splitter configuration shown in Fig. 1 gives rise to two-particle interference, causing various output probabilities for the different Bell states. In fact, if the state is either \(|\Phi^+\rangle\) or \(|\Phi^-\rangle\), i.e., if both particles are incident on the same beam splitter, then both particles will be registered by any of the four detectors \(D_1,\ldots,D_4\) with equal probability \(P_{ii} = 25\%\) (\(i=1,\ldots,4\)) [12]. This interference is independent of any phase imparted to any of the beams \(A, B, C,\) or \(D\). As a consequence, such detections do not allow one to distinguish between \(|\Phi^+\rangle\) and \(|\Phi^-\rangle\). Contrarily, if the two particles are in the states \(|\Psi^+\rangle\) or \(|\Psi^-\rangle\) they fall onto different beam splitters and nonlocal and phase sensitive interference results in different coincidence detection events. More generally, for the state \(|\phi\rangle = 1/\sqrt{2}\,(|A\rangle|C\rangle + e^{i\chi}|B\rangle|D\rangle)\) the coincidence detection probabilities are given by \(P_{13}(\chi) = P_{24}(\chi) = \frac{1}{2}(1 + \cos \chi)\), and \(P_{14}(\chi) = P_{23}(\chi) = \frac{1}{2}(1 - \cos \chi)\), where, e.g., \(P_{13}\) is the probability that detectors \(D_1\) and \(D_3\) fire in

![FIG. 1. Layout of a Bell-state analyzer: The two two-state particles are momentum correlated; their states are represented by the modes A and B (C and D).](image)
FIG. 2. Setup of the experiment: The photon pair was created by parametric down-conversion in a nonlinear KDP crystal. The state preparation was performed by beam splitters BS’ and BS” and the piezocontrolled delay line $\Delta_2$. Bell-state analysis was done by monitoring two-photon interference behind the beam splitters BSA’ and BSA” by a sixfold coincidence logic (not shown).

We conclude that we can uniquely distinguish the three different classes [13]:

$$D_1 \wedge D_3 \text{ or } D_2 \wedge D_4 \Rightarrow |\Psi^+\rangle,$$

$$D_1 \wedge D_4 \text{ or } D_2 \wedge D_3 \Rightarrow |\Psi^-\rangle,$$

2 particles at either $D_1$, $D_2$, $D_3$ or $D_4$

$$\Rightarrow |\Phi^+\rangle \text{ or } |\Phi^-\rangle$$

(where, e.g., $\wedge$ stands for a coincident detection in these detectors [15]).

In the experiment, the Bell states have been realized with pairs of entangled photons [14]. Degenerate parametric down-conversion was used to generate from the 351.1-nm UV line of an argon-ion laser a pair of photons with a center wavelength of 702.2 nm. As shown in Fig. 2, each of the two photons was directed to a beam splitter where the photons have been independently split into modes $A$ and $B$ or $C$ and $D$, respectively. Thereby a superposition of the states $|\Psi^+\rangle$ and $|\Phi^+\rangle$ was generated. Mirrors were used to direct the four different modes to the beam splitter configuration of Fig. 1, where the actual two-particle interference occurred. Irises and interference filters in front of the single-photon detectors (Si-avalanche diodes operated in the Geiger mode) defined the modes and also the wavelength bandwidth to about 2 nm; fast logic circuits registered all six possible coincidence detection events. Path length alignment to within the coherence length ($\ell_c \approx 120 \mu m$) of the down-converted photons was done with dc-motor-driven delay lines in the arm towards the beam splitter BS” ($\Delta_1$) and in mode $D$ ($\Delta_2$). To make the alignment easier, similar delay lines were installed around BS’. For phase scans on the wavelength scale, $\Delta_2$ was equipped with an additional piezocontrolled translation stage. This allowed the change of the general two-photon state from a superposition of $|\Psi^+\rangle$ and $|\Phi^+\rangle$ to a superposition of $|\Psi^-\rangle$ and $|\Phi^-\rangle$.

For the path length adjustment, first the two-photon interference at BSA’ was searched by monitoring the coincidence rate $C_{12}$ (i.e., the rate of coincidence counts between detectors $D_1 \wedge D_2$) when varying $\Delta_1$. When interference occurs the coincidence rate shows a minimum, since both photons will be detected by only one of the detectors [15]. The same method was then used to equalize path lengths in modes $A$ and $D$ by scanning $\Delta_2$ and monitoring $C_{34}$. Figure 3 demonstrates the resulting dependency of the two coincidence rates on the variation of $\Delta_1$. The anticoincidences are the unique signature of the states $|\Phi^-\rangle$. Their observation therefore constitutes the projection onto a plane spanned by these Bell states.

At this optimal position of $\Delta_1$ and $\Delta_2$ the nonlocal interference also occurs at the beam splitters BSA’ and BSA”. Figure 4 shows the four coincidence rates analyzing the states $|\Psi^-\rangle$ and $|\Psi^-\rangle$. The maxima in the coincidence rates $C_{13}$ and $C_{24}$ indicate the registration of $|\Psi^+\rangle$. At the same time, the anticoincidences, i.e., the rate of coincidence counts between detectors $D_1 \wedge D_3$ and $D_2 \wedge D_4$, while the rates $D_1 \wedge D_4$ and $D_2 \wedge D_3$ vanish.

FIG. 3. Coincidence rates $C_{12}$ and $C_{34}$ as a function of the path length difference $\Delta_1$. Destructive interference of these rates at $\Delta_1 = 0$ is a unique signature of the Bell states $|\Phi^+\rangle$ and $|\Phi^-\rangle$.

FIG. 4. Measurement of the nonlocal coincidence rates characterizing the Bell states $|\Psi^\pm\rangle$. Nonlocal interference at $\Delta_1 = 0$ enables one to analyze the incoming two-photon state. For $|\Psi^+\rangle$ as shown, constructive interference occurs for the coincidences between $D_1 \wedge D_3$ and $D_2 \wedge D_4$, while the rates $D_1 \wedge D_4$ and $D_2 \wedge D_3$ vanish.
time the rates $C_{14}$ and $C_{23}$, which are significant for $|\Psi^+\rangle$, have to reach a minimum. A piezoscan around $\Delta_2 = 0$, thus varying the phase $\chi$, allows a continuous change from $|\Psi^+\rangle$ to $|\Psi^-\rangle$ and back. This results in conjugate oscillations of the respective coincidence rates, alternately indicating those Bell states (Fig. 5).

What now determines the quality and the precision of this Bell-state analyzer? The interference visibility, i.e., the signal-to-noise ratio of the analysis, is determined by the indistinguishability of the interfering modes. Here a compromise has to be found between low intensity due to very fine filtering and a poor interference contrast in the case of coarser selectivity. The interference of the states $|\Phi^-\rangle$ involves the overlap of only two modes at a beam splitter. With irises of 2 mm in front of each detector (at roughly a 1.5- and 2-m distance from the down-conversion point in the crystal), a visibility of 85% was obtained. However, the measurement of the states $|\Psi^+\rangle$ and $|\Psi^-\rangle$ depends on the overlap of two pairs of modes ($A$ and $D$, and $B$ and $C$). This results in the slightly lower visibilities (of 76.9%, 79.8%, 77.7%, and 77.1%) for the respective coincidence rates (see Fig. 5).

Narrowing the irises did not increase these visibility values, mainly due to the a priori divergence of the pump beam. Only a significantly improved $k$ selection of the down-converted photons, most efficiently obtained if they were emitted at small angles and coupled into single-mode fibers (and replacing the beam splitters by fiber couplers), would help to improve the visibility. A clear improvement can be expected when using spin entangled Bell states, as can now be produced by a new type-II down-conversion source [16]. There, the analysis of all four Bell states necessitates only the overlap of two beams at a single beam splitter with successive polarization analyses.

Only coincidence detection allows the projection onto the Bell states. This consequently reduces the two-photon detection efficiency to $\eta^2$, as compared to the single photon detection efficiency of $\eta$. Currently the best Si-avalanche diodes available reach an efficiency of $\eta = 71\%$. In principle, there are other techniques with a reported efficiency of $\eta = 85\%$; they need, however, cooling to below 10 K and, above all, are still classified [17].

The output of Si-avalanche detectors is the same whether one or more photons are absorbed simultaneously. This actually results in an inability to distinguish $|\Phi^-\rangle$ states from single-detection background pulses. The standard two-photon registration technique therefore uses, instead of one detector, an additional beam splitter and two detectors to register at least half of the two-photon events [18]. Then, with 50% probability, a two-photon state falling onto the beam splitter will give rise to one photon in each of the outputs that can be registered by a coincidence count there. Because one has to replace all four detectors by such a configuration, altogether eight detectors and an increasingly complex coincidence logic would be necessary.

However, another solution becomes possible if one avoids interference for the states $|\Phi^+\rangle$ and $|\Phi^-\rangle$ entirely. The two-photon interference effect at one beam splitter necessitates equal path lengths to the respective beam splitter ($l_A = l_D$ and $l_B = l_C$, where $l_M$ is the path length from the crystal to the beam splitter via the mode $M$). If these conditions are not fulfilled the two photons behave independently at a beam splitter and are detected at the two different output detectors in 50% of the events. Since only the states $|\Phi^+\rangle$ and $|\Phi^-\rangle$ can cause such coincidences [19], we still can distinguish them from $|\Psi^+\rangle$ and $|\Psi^-\rangle$, now with no additional equipment.

Yet, even as the local two-photon interference at each beam splitter vanishes, there is still the possibility to observe the nonlocal two-photon interference effect used to distinguish between $|\Psi^+\rangle$ and $|\Psi^-\rangle$. For these, interference occurs, even when the two photons pass the beam splitters BS$A'$ and BS$A''$ at different times, as long as it is undecidable which of them came via BS$'$ or BS$''$. The path length condition then is $l_A - l_C = l_D - l_B$. Therefore, starting from the initial alignment, we can change the path lengths $l_A$ and $l_C$ by the same amount without losing the availability of analyzing $|\Psi^+\rangle$ and $|\Psi^-\rangle$, but with the additional possibility of detecting 50% of the $|\Phi^-\rangle$ states via coincidences.
between detectors $D_1 \setminus D_2$ and $D_3 \setminus D_4$. Figure 6 shows these coincidence counts as a function of the path length difference $\Delta_1$ after enlarging the paths in both modes A and C by 1.6 mm. At a position reduced by this distance from the initial alignment position $\Delta_1 = 0$, the coincidence minimum in $C_{12}$ can be observed, whereas the minimum in $C_{34}$ occurs at a path length difference $\Delta_1 = 1.6$ mm. In between, the nonlocal interference in $C_{13}$ at $\Delta_1 = 0$ indicates the state $|\Psi^+\rangle$, but at this position the coincidences in $C_{12}$ and $C_{34}$ can be used to detect the states $|\Phi^\pm\rangle$.

In this Rapid Communication we report an experimental realization of a Bell-state analyzer, in our case for mode encoded two-photon states. Such an analyzer will be of immediate use in all further experiments in the new field of quantum communication and in a number of tests on the foundations of quantum mechanics.

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[13] Another combination of modes allows one to distinguish $|\Phi^+\rangle$ from to $|\Phi^-\rangle$, but not $|\Psi^\pm\rangle$ [10].
[15] This is true for bosons; fermions show the opposite behavior leaving the beam splitter via different outputs.
[18] For photons, only special photomultipliers allow one to distinguish between one or more particle events, yet the efficiency is forbiddingly low for the wavelengths used.
[19] The coincidence windows ($\sim 5$ ns) are much larger than any time delays due to different path lengths.