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Single-photon opto-mechanics in the strong coupling regime

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Abstract. We give a theoretical description of a coherently driven opto-mechanical system with a single added photon. The photon source is modeled as a cavity that initially contains one photon and that is irreversibly coupled to the opto-mechanical system. We show that the probability for the additional photon to be emitted by the opto-mechanical cavity will exhibit oscillations under a Lorentzian envelope, when the driven interaction with the mechanical resonator is strong enough. Our scheme provides a feasible route towards quantum state transfer between optical photons and micromechanical resonators.

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1. Introduction

A significant engineering discipline has been built around the ability to fabricate micron- and nano-scale opto-mechanical systems of extraordinary variety. The emerging field of quantum opto-mechanics extends this ability towards a fully quantum domain, enabling a new scientific
discipline that aims to establish mechanical resonators as novel systems for quantum science. In combination with quantum optics techniques and new fabrication methods, highly nonclassical states of motion, such as a vibrational energy eigenstate, squeezed states and even entangled states can be prepared and coherently manipulated [1]–[3]. This now provides a new approach for controlling the mutual interaction between light and mesoscopic structures, which is one of the eminent goals in quantum information science [4] and of importance for fundamental experiments at the quantum–classical boundary [5].

Almost all previous investigations in opto-mechanics have presupposed conventional optical sources, well described by statistical mixtures of coherent states. Some early theoretical work considered the possibility of using squeezed light in an opto-mechanical setting [6] and advanced LIGO may make some of these suggestions an engineering reality [7]. More generally speaking, most of the current proposals to achieve (opto-)mechanical quantum states are restricted to the class of Gaussian states. To go beyond this regime requires additional nonlinearities, either in the interaction or in the measurement process. One example is the use of single photons to prepare macroscopic mechanical superpositions [8, 9]. Current opto-mechanical systems, however, still exhibit couplings below the necessary single-photon interaction strength. In this paper, we propose a scheme that allows one to achieve single-photon opto-mechanics in presently available systems. The main idea is to enhance the single-photon coupling strength by the presence of a strong pump field. It has recently been shown in both theory [10, 11] and experiment [12] that this allows one to enter the strong coupling regime of an opto-mechanical system. We show that in such a case, even for small intrinsic single-photon coupling, a single-photon excitation of the cavity can be reversibly transferred to the vibrational motion of a mechanical resonator. We study the dynamics of this process and show that it can be detected as temporal oscillations in the cavity emission. This is in close analogy to optical three-wave mixing, where the pump field converts excitations in the optical signal mode (here, the cavity photons) into excitations in the optical idler mode (here, the vibrational phonons) and vice versa.

We consider a single-mode optical cavity of length $L$, frequency $\omega_c$ and linewidth $\kappa$, with a moving end mirror that is modeled as a simple mechanical resonator with mass $m$ and resonant frequency $\omega_m$. We also assume operation in the resolved-side-band regime, i.e. $\kappa < \omega_m$. The cavity is strongly driven with a coherent pump field at frequency $\omega_L$. We describe the interaction through a linearized treatment that is expanded around the steady state field amplitude in the cavity, which would arise in the absence of the opto-mechanical interaction. In addition to the coherent driving field, the cavity is also driven by a single-photon source. This is intended to be a sequence of pulses with one and only one photon per pulse; however, it will suffice to consider only a single pulse for the purposes of the calculation presented here.

To model the single-photon source we include a source cavity of frequency $\omega_s$ and of decay rate $\gamma$, which at $t = 0$ is prepared in a single-photon state (figure 1). The coupling between the source cavity and the opto-mechanical cavity is irreversible and can be described using the cascaded systems approach [13, 14]. In this way, we obtain a source that produces a single-photon pulse with a Lorentzian line shape. In the following, the single-photon source cavity is on resonance with the opto-mechanical cavity. The additional coherent laser driving field of frequency $\omega_L$ enhances the opto-mechanical radiation pressure coupling. This field is spectrally detuned from the cavity resonance by multiples of the mechanical resonance frequency so that motional side bands can be addressed. Our objective is to demonstrate coherent exchange of the added single-photon optical excitation with the vibrational excitation of the mechanical mirror. To this end we compute the dynamics of the mean added photon number in the cavity, $n_a$, which...
then determines the single-photon detection rate at detector $A$ as $\kappa n_a$ where the cavity damping rate is $\kappa$. We find that for strong coupling this detection probability oscillates due to the coherent exchange of the single photon with the mechanical phonon number.

The interaction between the cavity field of the source and the mechanical motion of the end mirror is via the standard radiation pressure coupling [15, 16]

$$H_{\text{rp}} = \hbar G a^\dagger a (b + b^\dagger)$$

(1)
with the coupling rate
\[ G = \frac{\omega_c}{L} \sqrt{\frac{\hbar}{m\omega_m}}, \]  
where \( m \) is the effective mass of the moving mirror. For current opto-mechanical systems \( G \) is small compared to the cavity linewidth \( \kappa \). However, a larger effective interaction is obtained by driving the cavity with a strong coherent laser field. In this case the amplitudes of both the cavity field and the mechanical motion will be displaced by their steady state amplitudes \( \alpha_0 \) and \( \beta_0 \), respectively. One can then linearize the radiation pressure force in the shifted reference frame to obtain
\[ H = \hbar \Delta a^\dagger a + \hbar \omega_m b^\dagger b + \hbar g (a + a^\dagger)(b + b^\dagger), \]  where \( g = G\alpha_0 \) (we have fixed the phase for the coherent drive to make this a real variable) is the effective coupling strength, \( \Delta = \omega_c - \omega_L + 2g \) is the effective cavity detuning, and \( a, a^\dagger \) and \( b, b^\dagger \) are the annihilation and creation operators for the displaced cavity field and for the motion of the mirror mechanical resonator, respectively. In the absence of damping, the Heisenberg equations of motion are linear and may be solved by the method of normal modes. The normal mode frequencies are given by
\[ \omega^2_{\pm} = \frac{1}{2} \left( \Delta^2 + \omega_m^2 \pm \sqrt{(\Delta^2 - \omega_m^2)^2 + 16g^2 \Delta \omega_m} \right). \]  

The normal mode splitting occurs for any \( g > 0 \) in the undamped case, but when damping is included a minimum coupling strength is required and given by \( g > \kappa \) (see the Supplementary Information in [12]). In the undamped case, oscillatory solutions will occur in the sought single-photon detection probability when \( \omega_{\pm} \) are real, i.e. \( 4g^2 \ll \Delta \omega_m \).

In the limit that \( 4g^2 \ll \omega_m \Delta \) we can chose the detuning \( \Delta \) to make particular resonant terms \( (\Delta = \pm \omega_m) \) dominate the interaction. To identify the resonant terms we first move to an interaction picture defined by the free Hamiltonian, \( H_0 = \hbar \Delta a^\dagger a + \hbar \omega_m b^\dagger b \). The resulting time-dependent Hamiltonian may be approximated by a time-independent Hamiltonian using the appropriate resonance condition [17]. For example, if \( \Delta = \omega_m \), the interaction Hamiltonian can be approximated by the red side-band coupling
\[ H_r = \hbar g (ab^\dagger + a^\dagger b), \]  which leads to cooling of the mechanical motion if the optical cavity is rapidly damped [18]–[20].

The validity of the rotating wave approximation implicit in this ‘side-band’ Hamiltonian, equation (5), depends on the ratio \( 2g/\Delta \ll 1 \). In general the coupling constant can be quite large [12], and one may have to take the full interaction, equation (3), into account, which includes counter-rotating (blue side band) terms like \( ab + a^\dagger b^\dagger \). In fact if one does tune the opto-mechanical system to the blue side band \( (\Delta = -\omega_m) \), amplification results. This can lead to entanglement between the optical and mechanical degrees of freedom [21], and also to heating of the mechanical motion [11] and eventually to an instability of the steady state resulting in self-sustained oscillation on a limit cycle [22].

The red side-band approximation, equation (5), describes a reversible swap of a single-photon excitation from the opto-mechanical cavity field to the mechanical system. In the general case, this exchange will not be perfect due to the counter-rotating terms \( ab \) and \( a^\dagger, b^\dagger \).
Before the single photon enters the cavity, the opto-mechanical system is in a dynamical steady state with a large circulating power in the cavity due to the coherent field $\alpha_0$. On top of this, the counter-rotating terms add additional photons by converting thermal phonons in the mechanical resonator into cavity excitations. If an additional single photon then enters the cavity at a random time, the system will move away from the steady state but return to it through damped oscillations (provided $4g^2 \leq \Delta \omega_m$) as energy is exchanged between the cavity and the mechanical resonator. This additional excitation can be lost either through mechanical damping or through the end mirror of the cavity. Note that in the latter case the emitted photon can in principle be detected via photon counting, provided it can be distinguished from the coherent component exiting the cavity (see section 3). If the coupling is large enough the excitation can be exchanged a number of times between the cavity and the mechanics before being lost. Such an oscillation will modulate the detection rate for photons leaving the cavity and will hence leave an unambiguous signature of the coherent exchange of energy between the cavity and the mechanical resonator. Eventually, this detection rate will return to zero as the opto-mechanical system returns to the steady state.

2. Master equation for cascaded systems

The interaction picture master equation describing the interaction between the system is

$$\frac{d\rho}{dt} = -i\Delta[a^\dagger a, \rho] - i\omega_m[b^\dagger b, \rho] - ig[(a + a^\dagger)(b + b^\dagger), \rho] + \kappa D[a]\rho + \gamma D[c]\rho$$

$$+ \mu(\tilde{n} + 1)D[b]\rho + \mu\tilde{n}D[b^\dagger]\rho - i\Delta[c^\dagger c, \rho] + \sqrt{\gamma \kappa} \left([c\rho, a^\dagger] + [a, \rho c^\dagger]\right),$$

(6)

where the interaction picture is defined by the coherent driving laser. Here, $\mu$ is the decay rate of the mechanical system resonator, $\tilde{n}$ is the mean thermal excitation of the mechanical environment at frequency $\omega_m$ and $c, c^\dagger$ are the annihilation and creation operators for the field of the source cavity. The damping superoperators are defined by

$$D[A]\rho = A\rho A^\dagger - \frac{1}{2}(A^\dagger A\rho + \rho A^\dagger A).$$

(7)

To demonstrate a successful state transfer, we calculate the value of $\langle a^\dagger a\rangle$ because the count rate for the photon emitted from the cavity is proportional to this quantity. As the Hamiltonian is at most quadratic in the field amplitude operators, a closed system of equations can be obtained for the second-order moments. To this end, we define the correlation matrix

$$C(t) = \langle \tilde{A}(t)\tilde{A}^\dagger(t) \rangle,$$

(8)

where $\tilde{A}^\dagger = (a(t), a^\dagger(t), b(t), b^\dagger(t))$. This obeys the following system of equations,

$$\frac{dC(t)}{dt} = KC(t) + C(t)^T K^T - \sqrt{\gamma \kappa} N(t),$$

(9)

where

$$K = \begin{pmatrix}
-\tilde{\kappa} & 0 & -ig & -ig \\
0 & -\tilde{\kappa}^* & ig & ig \\
-ig & -ig & -\tilde{\mu} & 0 \\
ig & ig & 0 & -\tilde{\mu}^*
\end{pmatrix}.$$
with $\tilde{\kappa} = i\Delta + \kappa/2$ and $\tilde{\mu} = i\omega_m + \mu/2$, and the noise matrix is defined by

$$N(t) = \begin{pmatrix}
\langle ac \rangle & \langle a^\dagger c + ac^\dagger \rangle & \langle cb \rangle & \langle cb^\dagger \rangle \\
\langle a^\dagger c + ac^\dagger \rangle & \langle a^\dagger e^\dagger \rangle & \langle be^\dagger \rangle & \langle c^\dagger b^\dagger \rangle \\
\langle cb \rangle & \langle be^\dagger \rangle & 0 & 0 \\
\langle cb^\dagger \rangle & \langle c^\dagger b^\dagger \rangle & 0 & 0 
\end{pmatrix}. \quad (11)$$

The noise matrix elements obey a separate set of equations given by

$$\frac{d}{dt} \begin{pmatrix}
\langle ac \rangle \\
\langle a^\dagger c \rangle \\
\langle bc \rangle \\
\langle b^\dagger c \rangle 
\end{pmatrix} = \begin{pmatrix}
-\sigma & 0 & -ig & -ig \\
0 & -(\kappa + \gamma)/2 & ig & ig \\
-ig & -ig & -\tau_+ & 0 \\
ig & ig & 0 & -\tau_* 
\end{pmatrix} \begin{pmatrix}
\langle ac \rangle \\
\langle a^\dagger c \rangle \\
\langle bc \rangle \\
\langle b^\dagger c \rangle 
\end{pmatrix} - \sqrt{\gamma}\kappa \begin{pmatrix}
\langle c^2 \rangle \\
\langle c^\dagger c \rangle \\
\langle b^\dagger c \rangle \\
\langle b^\dagger c \rangle 
\end{pmatrix} \quad (12)$$

(and the complex conjugate equations). We have defined $\sigma = 2i\Delta + (\kappa + \gamma)/2$ and $\tau_\pm = i(\omega_m \pm \Delta) + (\mu + \gamma)/2$. The equations of motion for the source cavity alone are

$$\frac{d\langle c^2 \rangle}{dt} = -(2i\Delta + \gamma)\langle c^2 \rangle, \quad (13)$$

$$\frac{d\langle c^\dagger c \rangle}{dt} = -\gamma\langle c^\dagger c \rangle. \quad (14)$$

If there is a number state, $|n\rangle_c$, prepared in the source cavity at $t = 0$, one immediately sees that $\langle c^2 \rangle(t)$ is zero for all time, while $\langle c^\dagger c \rangle(t) = n e^{-\gamma t}$. Note that the case for an initial coherent state, $|\beta\rangle$, in the source cavity is different, as for that case $\langle c^2 \rangle(t) = \beta^2 e^{-(2i\Delta+\gamma) t}$, $\langle c^\dagger c \rangle(t) = |\beta|^2 e^{-\gamma t}$, which makes the dynamics dependent on the phase of $\beta$. The number state case, in contrast, has no similar phase reference. The equation for the noise matrix may be solved directly in both cases and substituted into the equation for the correlation matrix $C$. Note that the intracavity photon number will depend on the time-dependent correlations between $a$ and $c$. This starts at zero, rises to a maximum as the single-photon excitation begins to grow in the cavity and then decays to zero.

We discuss our results in direct comparison with present experimental parameters. An experiment reported by Gröblacher et al [12] demonstrated the strong coupling regime with a linewidth of the optical cavity and the mechanical resonator of $\kappa = 2\pi \times 215$ kHz and $\mu = 2\pi \times 140$ Hz, respectively, and with an effective coupling strength of $g = 2\pi \times 325$ kHz. The mechanical resonator frequency was $2\pi \times 947$ kHz. In units such that $\kappa = 1$, these are equivalent to $\omega_m = 4.4$, $\mu = 6.5 \times 10^{-4}$, $g = 1.5$. In the following we set the detuning $\Delta = 1.02 \omega_m$ and we choose the source cavity to be nearly mode matched to the opto-mechanical cavity $\gamma = 0.9$. Before the single-photon source is turned on, we assume that the opto-mechanical system has reached a steady state, which then becomes the initial conditions when the source is turned on; i.e. we need to find the steady state solutions of equation (9) with $\gamma = 0$. The steady state solution for the opto-mechanical covariance matrix is given by $KC_\infty + C_\infty K^T = 0$.

We will first discuss the case of zero temperature, $\tilde{n} = 0$, as this provides a clear view of how the added single photon is exchanged with the mechanical resonator. We therefore set $\tilde{n} = 0$ and $\mu = 0.001$ in the equations of motion. One way to achieve this is to passively cool a high-frequency mechanical system directly into its ground state, for example inside a
Figure 2. Mean photon number in the opto-mechanical cavity versus time: $\tilde{n} = 0$, $\gamma = 0.9$, $\kappa = 1.0$, $\mu = 0.001$, $\omega_m = 4.4$, $\Delta = 1.02\omega_m$, $\omega_m = 4.4$ and four values of $g$.

dilution refrigerator, as has recently been demonstrated [24]. If the mechanical bath is not at zero temperature $\tilde{n} > 0$, the oscillations in the cavity photon number can still be seen on top of the constant thermal background as prior to the single photon pulse entering the cavity, the steady state cavity field is thermal, not vacuum. (It also has a coherent steady state component but we have linearised about this amplitude.) For experiments where the driving field allows for cooling close to the ground state of the resonator this offset is small.

We choose the single-photon linewidth as $\gamma = 0.9$. Figure 2 shows the time dependence of the mean cavity photon number for various coupling strengths $g < \kappa, \Delta$. For $g \ll \Delta$ we simply recover the statistics of the cavity decay as no significant opto-mechanical coupling takes place. Increasing $g$ such that $\kappa < g < \Delta$ we observe revivals in the detection probability, which arise because the single-photon excitation is exchanged coherently between the opto-mechanical cavity and the mechanical resonator. This can be seen directly in figure 3, where we plot the simultaneous evolution of both the intracavity mean photon number and the mean phonon number in the mechanical resonator for the case of $g = 1.5$. It might be noted that the photon number and phonon number oscillations are not $\pi$ out of phase in the first phase of the evolution, as one might expect if the cavity was started with exactly one photon at $t = 0$. For short times, this is due to the dynamics of the single-photon source excitation of the cavity, on top of the photon–phonon interactions: the dynamics in equation (9) depends explicitly on the correlations between the source and the opto-mechanical cavity, $\langle ac + a^\dagger c \rangle$. At later times, the mean cavity photon number does not peak at the same time as the minimum in mean mechanical phonon number because the decay rate of the cavity is very much greater than the mechanical decay rate. For $g > \kappa, \Delta$, see figure 4, the oscillations persist but additional frequencies appear due to normal mode splitting. In addition, the rotating wave approximation breaks down and there is an excitation of the dressed opto-mechanical system, analogous to heating.

As discussed above we expect the dynamics for an initial Fock state in the source cavity to differ from that for an initial coherent state with the same mean photon number. This is shown
Figure 3. (a) Mean photon number in the opto-mechanical cavity and the mean phonon number in the mechanical resonator versus time for $g = 1.5$, $\bar{n} = 0$, $\gamma = 0.9$, $\kappa = 1.0$, $\mu = 0.001$, $\omega_m = 4.4$, $\Delta = 1.02\omega_m$ and $\omega_m = 4.4$.

Figure 4. Mean photon number in the opto-mechanical cavity versus time: $\bar{n} = 0$, $\gamma = 0.9$, $\kappa = 1.0$, $\mu = 0.001$, $\omega_m = 4.4$, $\Delta = 1.02\omega_m$, $\omega_m = 4.4$ with $g = 2.0$.

in figure 5, where we compare the dynamics for a Fock state in the source cavity $n = 5$, and two coherent states, $|\alpha\rangle$ with $\alpha = \sqrt{5}$ and $\sqrt{5}i$.

We finally include coupling of the mechanical resonator to a nonzero temperature heat bath; here $\bar{n} = 1000$ (see figure 6). This can be accomplished by starting from different initial conditions that take into account that, prior to the single-photon injection, the optical cavity and
Figure 5. Mean photon number in the opto-mechanical cavity versus time, contrasting the case of an initial Fock state source and an initial coherent state source. In all cases $g = 1.5$. The mechanical resonator is at zero temperature, $\tilde{n} = 0$, and the opto-mechanical cavity starts with no photons. The source cavity is prepared in a Fock state $n = 5$ (solid line) or a coherent state with amplitude $\sqrt{5}$ (dashed-dot) and $\sqrt{5}i$ (dashed).

Figure 6. Mean photon number in the opto-mechanical cavity versus time showing the effect of the thermal driving of the nanomechanical resonator for $\tilde{n} = 1000$, $\gamma = 0.9$, $\kappa = 1.0$, $\mu = 0.001$ and $g = 1.5$. The case of $\tilde{n} = 0$ is shown for comparison in the dashed line.
the mechanical resonator are in thermal equilibrium. While the signal is of the same magnitude as in the zero-temperature case, there is an added noise that corresponds to the steady-state thermal occupation of the mechanical oscillator.

3. Discussion and conclusion

The calculation we have presented is based on a linearization of the intensity-dependent force acting on the mechanical element, around a strong coherent steady state field inside the cavity. This means that the average photon numbers we have calculated are in addition to a coherent steady state field. In order to detect the added photon number due to the linearized interaction, over and above the steady state coherent field inside the cavity, we need to subtract the known steady state field amplitude $\alpha_0$ by displacing the output field amplitude from the cavity before sending it to a photodetector. Such displacements can, for example, be done by mixing the output field from the cavity with a local oscillator coherent field (split off from the driving laser) on a beam splitter with very high reflectivity, see for example [25]–[27]. With the coherent amplitude displaced away, the photon detection rate is proportional to $\kappa$ times the mean photon number, as presented in figures 2–6.

With increasing amplitude of the coherent driving field the practical implementation of the background cancellation via displacement becomes more challenging. However, it is still possible to observe the wanted photon oscillations even for imperfect cancellation by averaging over sufficiently many measurement runs, which allows suppression of the background shot noise against the oscillations: for example we discussed here, $|\alpha|^2 \approx 10^{10}$. For a reasonable measurement time, we estimate averaging over a number of $10^9$ measurement runs (given from a single-photon source operating at a rate of approximately $10^4$ Hz and about 24 h ($10^5$ s) measurement time). In that case, an interference contrast for the displacement of about 99.99% is needed. These requirements are clearly very challenging. In the long run these conditions can be significantly relaxed in opto-mechanical systems with a higher single-photon coupling rate $G$, resulting in a lower intracavity field $\alpha = g/G$. For example in [28] an intracavity field of $|\alpha_0|^2 = 4 \cdot 10^4$ is sufficient to achieve $g = \kappa$. This implies that the presented scheme can be realized with an easily achievable interference contrast of 99.0% and averaging over $N = 10^6$ measurement runs.

We have modeled the single-photon source as a single cavity initialized with one photon. In order to sample the mean photon number in the cavity, the single-photon source cavity needs to be re-prepared. In reality, a single-photon source is either a pulsed or a heralded source with one and only one photon per trigger event [29]. The model we have used can apply to these cases, provided that the period between pulses is sufficiently long that the opto-mechanical system can return to steady state after detection of the single photon emitted from the cavity between each pulse. In addition, our cavity model assumes an exponential temporal pulse shape. These assumptions are however consistent with new narrow-linewidth single-photon sources that have been developed in the context of atom–light interfaces [30]. Yet, the actual pulse shape is not very important provided it is matched reasonably well to the opto-mechanical cavity linewidth. Finally, the timing information in heralding the single photon further helps to reduce noise in the experiment by a gated detection scheme.

We have proposed a novel scheme that allows the coherent exchange of single-photon excitations of an optical cavity with a micromechanical resonator. The single-photon coupling is enhanced by a strong pump field that mediates the state transfer, in close analogy to optical
three-wave mixing. A clear signature of the state transfer between light and mechanics is the oscillation of the added-photon emission probability from the cavity. The scheme can be realized with state-of-the-art opto-mechanical systems that operate sufficiently close to the quantum ground state. This provides the basis for storage and interacting of optical photons in/via mechanical structures. Note that a similar idea has recently been suggested to create mechanical superposition states [31] and non Gaussian states [32].

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