Grading Modality
A New Approach to Modal Concord and its Relatives

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Abstract

This paper addresses modal matching configurations, i.e. constructions where two modal elements co-occur in a sentence but modal force is only expressed once. It shows that the co-occurring elements only need to match in logical strength (e.g. necessity combines with impossibility) and do not have to match in modal force. It argues that previous approaches to modal matching (specifically: to modal concord), such as Geurts & Huitink (2006) and Zeijlstra (2008), cannot account for such matching in strength. A new approach to modal matching is presented, which treats it as degree modification over gradable modals, solving this puzzle: The matching requirement is argued to be a polarity presupposition on the modal degree modifier. The paper concludes by extending the analysis to the German particles ruhig, bloß and JA (see Grosz to appear).

1 The Puzzle

1.1 Background: Modal Matching

I use the theory-neutral term modal matching to refer to configurations where two modal elements co-occur in a sentence but modal force is only expressed once. It subsumes modal concord, (1)\(^1\), but also covers the distribution of certain modal particles (shown in section 3).

\(1\) Modal Concord (Geurts & Huitink 2006, Zeijlstra 2008, Huitink 2008, to appear)

Visitors {must / #may} mandatorily sign this form.

\(\approx\) Visitors must sign this form. / It is mandatory that visitors sign this form.

Crucially, we also find modal-concord-like configurations that involve matching in logical strength (by which I mean that necessity matches impossibility, illustrated in (2a), and possibility matches non-necessity, discussed in section 3). Crucially, the context in (2) rules out any sensible reading where modal force is expressed twice (i.e. we have to require

\(\approx\) 1 For expository reasons, section 1 and 2 focus on deontic necessity statements. However, the proposal covers other types of modality, as illustrated for non-dynamic root necessity and possibility in section 3.
you to). Therefore, (2b) is infelicitous for speakers who do not permit modal matching across clausal boundaries. (Note that there are some native speakers who do permit such matching.)

(2) Context: “You are now entering a secure area of this public facility. We would now like to specify the rules that make up our special security standards. Nothing forces us to have these security standards, yet we have them, as we are special.”

a. In view of our special security standards, you mandatorily may not cross the yellow line without a special permit issued by the facility head quarters.
≈ You may not cross … / It is mandatory that you do not cross …

b. # In view of our special security standards, it is mandatory that you may not cross the yellow line without a special permit issued by the facility head quarters.

Generalizing from (1) and (2), the adverb mandatorily can enter a modal matching configuration with a necessity modal (must) and with a negated possibility modal (may). In contrast, it cannot combine with non-negated possibility (3a) or negated necessity (3b).

(3) a. # You mandatorily may / may mandatorily stay in this area.
≠ You may stay in this area.

b. # You mandatorily need not / need not mandatorily stay in this area.
≠ You need not stay in this area.

Modal Matching exhibits three core properties (cf. Geurts & Huitink 2006, Zeijlstra 2008, Grosz to appear). First, the two elements must match in logical strength (illustrated in (1)-(3) above). Second, the range of ordering sources that the modal statement selects is restricted. Third, the modal force is felt to be strengthened (or, rarely, weakened). Examples (4) and (5) illustrate the second property: mandatorily requires a deontic ordering source (e.g. what the law prescribes) and cannot combine with a bouletic ordering source (e.g. what I want).

(4) Modal Concord
a. In view of what the law prescribes, visitors must mandatorily sign this form.
b. # In view of what I want, you must mandatorily clean my room once a day.

(5) Modal Concord-like Matching for Strength
a. In view of our special security standards, you mandatorily cannot enter sector 7 without a member of our facility.
b. # In view of what I want, you mandatorily cannot enter my room.

Example (6) and (7) illustrate the strengthening effect in terms of informal paraphrases\(^2\).

(6) Modal Concord
In view of what the law prescribes, visitors must mandatorily sign this form.
≈ In view of what the law prescribes, it is necessary to a high degree that visitors sign this form.

\(^2\) Zeijlstra (2008) and Huitink (2008, to appear) call this “the emphatic effect” of modal concord.
Modal Concord-like Matching for Strength

In view of our special security standards, you **mandatorily cannot** enter sector 7 without a member of our facility.

Approximately, in view of our special security standards, it is necessary to a high degree that you do not enter sector 7 without a member of our facility.

A uniform theory of modal matching should account for these three properties: the strengthening effect (here: the degree of necessity is raised), the matching effect (i.e. the modal expressions must match in logical strength) and the restrictions on the modal type of the utterance. After discussing a puzzle for current approaches to modal matching in section 1.2, I propose an analysis in section 2. While the new analysis is motivated by the fact that it can account for a problem that neither of the previous analyses can account for, a novelty of my proposal is that it builds on the strengthening effect, rather than on the matching effect (contra Geurts & Huitink 2006, Zeijlstra 2008). On the one hand, this accounts for Zeijlstra’s (2008) observation that modal concord always leads to emphatic strengthening of the modified modal statement. On the other hand, it accounts for the German particles *ja*, *bloß* and *ruhig*, which clearly qualify as modal matching elements (see section 3 and Grosz to appear); these are typically perceived as strengtheners of sorts (e.g. Thurmair 1989 argues that *ja* and *bloß* strengthen a command, and *ruhig* makes a permission more encouraging).

1.2 A Problem for Previous Theories

Geurts & Huitink (2006) propose that one of the modal elements has a type-shifted ‘functional’ meaning under which it merely checks the type of the other modal element; this is modeled in terms of an operator ©, as illustrated in (8).

\[
\begin{align*}
(8) & \quad a. \quad [[[© mandatorily] must] visitors sign this form before entering the facility] \\
& \quad b. \quad |mandatorily|=|must| = \lambda p.\lambda w. \exists w' [R(w)(w') \rightarrow p(w')] \\
& \quad c. \quad |©| = \lambda p.\lambda Q : P = Q . P \\
& \quad \text{(my rendering of Geurts & Huitink 2006:18-19)}
\end{align*}
\]

The main problem for Geurts & Huitink’s account is that it cannot derive modal matching in strength, (9a), as the identity condition is too strong. While possibility under negation is equivalent to impossibility (i.e. “necessity that not”), yielding universal force, it is not the case that *may not* and *mandatorily* end up being synonymous. This is shown in (10). Therefore, the definedness conditions for © are not satisfied in (9b).

\[
\begin{align*}
(9) & \quad a. \quad \text{You **mandatorily may not** cross the yellow line without a special permit} \\
& \quad \text{issued by the facility head quarters} \\
& \quad \approx \quad \text{You **may not** cross the line. / It is **mandatory** that you do **not** cross the line.} \\
& \quad b. \quad [[[© mandatorily] [not may]] you cross the yellow line without a special permit]
\end{align*}
\]

\[
\begin{align*}
(10) & \quad a. \quad |\text{not may}| = \lambda p.\lambda w. \exists w' [R(w')(w') & p(w')] \\
& \quad \equiv \lambda p.\lambda w. \exists w' [R(w)(w') \rightarrow \neg p(w')] \\
& \quad b. \quad |\text{mandatorily}| = \lambda p.\lambda w. \forall w' [R(w)(w') \rightarrow p(w')] \\
& \quad c. \quad |\text{not may}| \neq |\text{mandatorily}|
\end{align*}
\]
Zeijlstra’s (2008) syntactic account faces the same problem. He treats modal concord as syntactic agreement between an item with an interpretable [iMOD] feature and one with a matching uninterpretable feature [uMOD]. To account for the matching requirement, he assumes two sets of modal features: [i∃-MOD] / [u∃-MOD] and [i∀-MOD] / [u∀-MOD].

(11) Visitors **must mandatorily** sign this form.
⇒ **mandatorily**[i∀-MOD] visitors **must**[u∀-MOD] sign this form.  

Modal matching that involves negation does not provide the relevant feature configurations for Zeijlstra’s analysis, illustrated in (12) (contrast with (11)).

(12) Visitors **mandatorily may not** cross the yellow line.
⇒ **mandatorily**[i∀-MOD] visitors **may**[u∃-MOD] **not** cross the yellow line.  

It might be argued for English that in these cases **may not** forms one lexical item specified for universal modality (i.e. [u∀-MOD]), as it expresses impossibility (Hedde Zeijlstra, p.c.). However, this explanation does not carry over to German, where **verpflichtend** ‘obligatory’, (13a), and **müssen** ‘must’, (13b), correspond to English obligatory/mandatory and **must**.

(13) *at the hot springs*

a. Es ist **verpflichtend**, dass Badegäste vor dem Baden duschen.  
it is obligatory that bath.guests before the bathing shower  
‘It is **obligatory** that bathers shower before entering the pool.’

b. Badegäste **müssen** vor dem Baden duschen.  
bath.guests must before the bathing shower  
‘Bathers **must** shower before entering the pool.’

In German, **müssen** ‘must’ and **verpflichtend** ‘obligatory’ can enter a modal matching relationship like English **must** and obligatorily, shown in (14).

(14) Badegäste **müssen** vor dem Baden **verpflichtend** duschen.  
bath.guests must before the bathing obligatorily shower  
‘Bathers **must obligatorily** shower before entering the pool.’

≈ ‘Bathers must shower before entering the pool.’

Such modal matching is subject to the usual constraints on matching in logical strength, as shown in (15).

(15) Badegäste **dürfen** während des Badebesuches (**#verpflichtend**).  
bath.guests may during the bath.visit obligatorily  
a **dressing gown** of the baths use  
‘Bathers **may** (**#obligatorily**) (borrow and) use a dressing gown from the bath administration.’

Crucially, (16) shows that **verpflichtend** ‘obligatorily’ enters a modal matching relation with negated **dürfen** ‘may’. What is significant is that **nicht dürfen** ‘may not’ cannot be analyzed as one lexical item with a [u∀-MOD] feature in such configurations.
We can thus conclude that matching in logical strength, as illustrated for matching between necessity and impossibility, cannot be explained by previous accounts of modal concord. In section 2, I propose a novel analysis of modal matching, which naturally accounts for these data. The main claim of my analysis is that modal matching is degree modification over degrees of modality. I argue that the requirement on matching in logical strength and the restrictions on modal type are definedness conditions on degree modifiers like mandatorily. The posited truth conditions are given in (17) and (18).

(17)  Matching in Modal Force (here: necessity and necessity)

a. Visitors must mandatorily sign this form.

b. LF: \[\langle\text{mandatorily must}\rangle [\text{visitors sign this form}]\]

c. truth conditions:

\[
[[\text{mandatorily must}] [\text{visitors sign this form}]]
\]

is defined iff the first argument of mandatorily (here: must) expresses universal deontic modality, and if defined, it is true iff it is necessary to a high degree \(d\) that visitors sign this form and false otherwise.

(18)  Matching in Logical Strength (here: necessity and impossibility)

a. Visitors mandatorily may not cross this yellow line.

b. LF: \[\langle\text{mandatorily not may}\rangle [\text{visitors cross this yellow line}]\]

c. truth conditions:

\[
[[\text{mandatorily not may}] [\text{visitors cross this yellow line}]]
\]

is defined iff the first argument of mandatorily (here: not may, equivalent to must not) expresses universal deontic modality, and if defined, it is true iff it is necessary to a high degree \(d\) that visitors do not cross the yellow line and false otherwise.

2  The Analysis

This section formalizes the proposal in (17) and (18). Section 2.1 provides an analysis of graded modality, based on a proposal by Portner (2008). Section 2.2 shows how to formalize degree modification over modals. Section 2.3 proposes an analysis of the matching requirement. Section 2.4 summarizes and illustrates the complete proposal.

2.1  Grading Modality

The overarching claim in this section is that possibility, necessity, impossibility and non-necessity can be graded, and degrees of necessity are based on the weight of a proposition with respect to a contextually salient ordering source. Consider the data in (19), which are naturally occurring instances of graded modality, found by way of the google search engine and verified with native speakers.
Graded modality in comparative constructions

a. It is more necessary than anything else to arm one’s self.
b. Some equipment is more mandatory than other pieces.
c. Sometimes a chain of events is more possible than a single event.
d. Why is it more impossible to believe that the universe created itself than that god created it?
e. We’re looking for films that are even more unnecessary than Predator 2.

Graded modality in “how”-questions

a. How necessary is human resource training?
b. How mandatory is the EU biofuel directive?
c. How possible is it to escape your own ideology?
d. How impossible is it to get a mortgage?
e. How bad is idling vehicles, and how unnecessary is it?

To account for such examples, I propose an analysis based on Portner’s (2008) approach to graded possibility. Portner assumes that scales of possibility are construed by considering alternative ordering sources with respect to which different propositions are possible. This models the weight of a proposition in a global ordering source. Assume that the three propositions in (21) are all contained in some deontic ordering source. Clearly, the weight of a proposition correlates with the height of punishment when it is violated; i.e. punishment is most severe for killing, less severe for stealing, and least severe for parking in driveways.

Propositions of different weights in a deontic ordering source g

a. k = You do not kill. = highest weight in g
b. s = You do not steal. = medium weight in g
c. p = You do not park in driveways. = lowest weight in g

The statements in (22) show that Portner’s insight that weight correlates with the degree of possibility or necessity holds for the situation in (21). As (22a) and (22d) show, is more necessary roughly corresponds to carries more weight, and is more unnecessary / less necessary roughly corresponds to carries less weight. In accord with treating necessity and possibility as duals (i.e. ¬p is possible if p is not necessary), it follows that a proposition ¬p is more possible than a proposition ¬k with respect to an ordering source if k has more weight than p, as confirmed by the statement in (22b). Correspondingly, in the same situation, ¬k is more impossible than ¬p, shown in (22c).

a. In view of the law, it is more necessary [k that you do not kill] than [p that you do not park in driveways].
b. In view of the law, it is more possible [¬p that you park in driveways] than [¬k that you kill].
c. In view of the law, it is more impossible [¬k that you kill] than [¬p that you park in driveways].

3 Note that the utterances in (22) sound a bit stilted, and there is clearly inter-speaker variation as to how natural they are judged to be. However, the intuition is shared by those speakers who accept them that the statements in (22) most appropriately capture the situation described in (21).

4 The reason that more unnecessary is somewhat odd is that unnecessary seems to be evaluative in some sense: p is more unnecessary than k intuitively entails both p and k are unnecessary.

5 Alternatively, one might introduce these types of utterances by in view of what you have to do in order to pass as a law-abiding citizen.
d. In view of the law, it is “more unnecessary” (i.e. less necessary) \([p\) that you do not park in driveways\] than \([s,\) that you do not kill\].

In Portner’s system, we can model the weight of the propositions in (21) as follows. Consider three alternative ordering sources\(^6\) that are salient in the context: \(g_1, g_2\) and \(g_3\). The least inclusive ordering source \((g_1\) in Table 1\) includes only the rules that carry most weight; the most inclusive ordering source \((g_3\) in Table 1\) includes all rules mandated by the law in this context. I assume that each ordering source under consideration contains a finite number of propositions, including the most restrictive and least restrictive ordering sources. This has the consequence that the scales of necessity and possibility are totally closed, a correct prediction, as shown in section 2.2.

Table 1: An example scale of necessity / possibility (based on Portner 2008 and adapted)

<table>
<thead>
<tr>
<th>Ordering Source</th>
<th>Necessities</th>
<th>Possibilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g_3 = {k, s, p})</td>
<td>(k, s, p)</td>
<td>(k, s, p)</td>
</tr>
<tr>
<td>(g_2 = {k, s})</td>
<td>(k, s)</td>
<td>(k, s, p, \neg p)</td>
</tr>
<tr>
<td>(g_1 = {k})</td>
<td>(k)</td>
<td>(k, s, \neg s, p, \neg p)</td>
</tr>
</tbody>
</table>

We can now define a formal relation of being more necessary than, written as \(>_{\text{NEC}}\), using the subset-superset relations between the alternative ordering sources, which represent weight of a proposition with respect to the global ordering source \(g\). The definition is given in (23), adapting Portner’s (2008:6) definition of more possible than. (The difference is based on expository choice, as I focus on necessity, whereas Portner focuses on possibility.)

\[
(23) \quad \text{The relation } >_{\text{NEC}} \text{ (‘is more necessary than’) is defined as follows}
\]

For any \(p, q,\) and contextually given set of alternative ordering sources \(X, p >_{\text{NEC}} q\)

\[
\text{iff } \exists g [g \in X \land p \text{ is necessary with respect to } g \land \forall h [(h \in X \land q \text{ is necessary with respect to } h) \rightarrow g \subset h]]
\]

\[\text{in words: “p is more necessary than q with respect to a contextually given set of alternative ordering sources X iff some g in X with respect to which p is necessary is properly included in every h in X with respect to which q is necessary.”}\]

Applying this definition to Table 1 above, \(k\) is more necessary than \(s\), because \(k\) is necessary with respect to some ordering source in \(X\) (namely \(g_1\)), which is properly included in every ordering source in \(X\) (here: \(g_2\) and \(g_3\)) with respect to which \(s\) is necessary.

We can now define the scale of necessity and its degrees in terms of equivalence classes. Define the Equivalence Relation \(\equiv_{\text{NEC}}\) as in (24) (adapted from Villalta 2006, following Cresswell 1976).

\[
(24) \quad p \equiv_{\text{NEC}} q \text{ iff } \forall z: (p >_{\text{NEC}} z \text{ iff } q >_{\text{NEC}} z) \land (z >_{\text{NEC}} p \text{ iff } z >_{\text{NEC}} q)
\]

\[\text{in words: “p is as necessary as q iff any proposition z that is less necessary than p is also less necessary than q, and any proposition z that is more necessary than p is also more necessary than q.”}\]

---

\(^6\) For ease of exposition, I treat ordering sources as sets of propositions in this section, rather than, for instance, as functions from possible worlds into sets of propositions.
The degree \( p \) to which a proposition \( p \) is necessary can then be assumed to equal the set of all propositions that are in an equivalence relation with \( p \). I adopt Villalta’s (2006) rendering of Cresswell’s (1976) idea; I use \( F \) to refer to a field of a relation (i.e. the set of things that are related to other things by this relation). We can now define degrees and relations between degrees as given in (25a) and (25b) respectively.

(25)  
\[ \begin{align*}  
\text{a. } & \text{ } p \in F(>_{\text{NEC}}) \iff \exists p \in F(>_{\text{NEC}}) : p = \{ z : z \equiv_{\text{NEC}} p \}  
\text{in words: } & \text{“A degree } p \text{ is on the scale of necessity iff there is a proposition } p \text{ which is related to other propositions by the necessity relation and } p \text{ equals the set of propositions that are in an equivalence relation with } p.\text{”} 
\text{b. } & \text{ } p >_{\text{NEC}} q \text{ iff } p >_{\text{NEC}} q 
\text{in words: } & \text{“A degree } p \text{ is higher on the scale of necessity than a degree } q \text{ iff any proposition } p \text{ which has the necessity degree } p \text{ is more necessary than any proposition } q \text{ which has the necessity degree } q.\text{”} 
\end{align*} \]

Based on the definitions in (25), we can now write the full meaning of a simple necessity modal as in (26).

(26) \( \| \text{must} \| = \| \text{necessary} \| = \lambda d \lambda p \lambda w . \text{NECESSITY}(p)(w) \geq d \) 
where \( d \in F(>_{\text{NEC}}) \)

To have lexical entries for all four corners of the Square of Opposition\textsuperscript{7}, (27), I define possible, impossible and unnecessary below in (28) and (29).

(27) \[ \begin{align*}  
\text{necessity } (\boxdot p) & \quad \text{contrariness} \quad \text{impossibility } (\neg \neg p, \neg \boxdot p) \quad \text{strong}  
\text{entailment} & \quad \text{contradictoriness} \quad \text{entailment}  
\text{possibility } (\lozenge p) & \quad \text{subcontrariness} \quad \text{non-necessity } (\neg \boxdot p, \diamond \neg p) \quad \text{weak}  
\end{align*} \]

Given that unnecessary can be assumed to be the antonym of necessary, we can derive its meaning based on the negation theory of antonymy (see Heim 2008 for a recent version); treating negative antonyms as the negation of their positive counterparts, we posit (28).

(28) \( \| \text{need not} \| = \| \text{unnecessary} \| = - \lambda d \lambda p \lambda w . \text{NECESSITY}(p)(w) \geq d \] = 
= \lambda d \lambda p \lambda w . \text{NECESSITY}(p)(w) < d 
where \( d \in F(>_{\text{NEC}}) \)

Making the further uncontroversial assumption that necessity and possibility are duals in natural language (i.e. \( \neg \Box \neg \equiv \square \) and \( \neg \Diamond \neg \equiv \lozenge \)), we can propose the entry in (29a) for possible. In words, possible (to the degree \( d \)) that \( p \) translates to not necessary (to the degree \( d \)) that not \( p \). Correspondingly, impossible, as the antonym of possible, is defined in (29b). Given the lexical entry in (29b), impossible that \( p \) correctly translates to necessary that not \( p \).

\textsuperscript{7} The Square of Opposition dates back to Aristotle, cf. Parsons (2008) for an overview.
2.2 Degree Modification over Modals

As outlined above, I assume that the core semantics of \textit{mandatorily} is that of a degree modifier over modal elements. In addition, I assume that the matching requirement in logical strength and the restrictions on the modal type are definedness conditions. Below, I give a first approximation of the meaning of \textit{mandatorily} in modal matching contexts.

First of all, to capture the core meaning of \textit{mandatorily}, I adapt Kennedy & McNally’s (2005:369) semantics for \textit{completely} to modals, given in (33a). Compositionally, \textit{completely necessary} and \textit{completely impossible} have the meanings in (33b) and (33c) respectively.

\begin{align*}
(33) & \quad \text{a. } ||\text{completely}|| = \lambda p \lambda w. \exists d [d = \max(S_M) \land M(d)(p)(w)] \\
& \quad \text{where } \max(S_M) \text{ is the maximum of the (upper or totally closed) scale of } M \\
& \quad \text{b. } ||\text{completely necessary}|| = \lambda p \lambda w. \exists d [d = \max(S_M) \land \text{NECESSITY}(p)(w) \geq d] \\
& \quad \text{in words: } \text{“There is a degree } d \text{ to which } p \text{ is necessary in } w \text{ and } d \text{ is the maximum of the scale of necessity.”} \\
& \quad \text{c. } ||\text{completely impossible}|| = \lambda p \lambda w. \exists d [d = \max(S_M) \land \text{NECESSITY}(\neg p)(w) \geq d] \\
& \quad \text{in words: } \text{“There is a degree } d \text{ to which } \neg p \text{ is necessary in } w \text{ and } d \text{ is the maximum of the scale of necessity.”}
\end{align*}

If we approximate the meaning of \textit{mandatorily} in terms of degree maximization\footnote{Some native speakers feel that (i) is equally strong as (ii) and (iii); others intuit that (i) is weaker. For expository ease, I give an analysis for the former group; an analysis for the latter might be framed as in (iv), based on Kennedy & McNally’s (2005:353) definition of \textit{most of the way}.}, we can assume that its core meaning component is the same as that of \textit{completely}, with additional definedness conditions that account for the matching requirement and restrictions on modal type. For a first sketch, we can assume that \textit{mandatorily} is lexically specified as an element that only combines with constructions that express deontic necessity. This will be further refined in section 2.3. A first approximation of the meaning of \textit{mandatorily} is given in (34).

\begin{align*}
(34) & \quad \text{First sketch of the formalization of “mandatorily” (to be revised in section 2.3)} \\
& \quad ||\text{mandatorily}|| = \lambda M \lambda p \lambda w : M \text{ expresses deontic necessity} . \\
& \quad \exists d [d = \max(S_M) \land M(d)(p)(w)] \\
& \quad \text{where } \max(S_M) \text{ is the maximum of the (upper or totally closed) scale of } M
\end{align*}
This analysis posits modal modifiers that refer to endpoints on the scale, rather than to relative standards. In (34), *mandatorily* refers to the maximum; later we will see that other modifiers (such as German *ruhig*) seem to refer to the minimum. This analysis thus assumes that the scale of necessity/possibility is a totally closed scale (i.e. it has both a minimum and a maximum). The argument for the assumption that gradable modals such as *(un-)necessary* and *(im-)possible* use totally closed scales (i.e. scales that have both a minimum and a maximum) can be based exactly on the behavior of endpoint modifiers like *completely*.

Kennedy & McNally (2005) show that only totally closed scales allow endpoint modifiers like *completely* to modify both the positive and the negative elements of antonym pairs.

(35) a. *Open scale (neither maximum nor minimum)*
   Her brother is completely *???tall / ???short*.

b. *Lower-closed scale (no maximum)*
   The author is completely *???famous / unknown*.

c. *Upper-closed scale (no minimum)*
   The treatment is completely *safe / ???dangerous*.

d. *Totally closed scale (both maximum and minimum)*
   The glass was completely full / empty.  
   (Kennedy & McNally 2005:355)

It can be shown for German (avoiding idiosyncracies of English that rule out some of the English counterparts) that modal elements generally exhibit compatibility with an endpoint modifier (here: *vollkommen* ‘completely’).

(36) a. Meiner Meinung nach ist es vollkommen möglich, dass jemand mit diesen Fähigkeiten geboren wird. With these abilities born is
   ‘In my opinion, it is completely possible that someone is born with these abilities’
   (http://www.blairwitch.de/index.php?seitenid=20&specialid=41)

b. Es ist vollkommen unmöglich, kostenlose DVD-Programme zu finden, denn es müssen Lizenzgebühren für die Implementierung des MPEG-2-Encodings gezahlt werden. ‘It is completely impossible to find free DVD-programs, as one has to pay license fees for the implementation of the MPEG2-encoding.’
   (http://forum.de.selfhtml.org/archiv/2005/12/t120372/)

c. Diese Schlappe war vollkommen unnötig. ‘This failure was completely unnecessary.’
   (http://www.ngz-online.de/public/article/nachrichten/235800/Diese-Schlappe-war-vollkommen-unnoetig.html)
d. Beeindruckend sind zu Anfang die doppelten Fenster, die aber
impressive are to start the double windows that but
vollkommen notwendig sind, was man vor allem bei schlechtem
completely necessary are what one before all at bad
Wetter merkt.

‘To begin with, the double windows are impressive, but they are completely
necessary, which becomes clear during the bad weather periods.’

(http://www.staff.uni-mainz.de/kamphus/jfj.html)

We can thus conclude that (un-)necessary and (im-)possible make use of a totally closed
scale, motivating an analysis of modal matching as degree maximization / minimization.

2.3 How to Derive the Matching Requirement

This section further refines the definedness conditions on modal matching elements such as
mandatorily. Rather than positing a global definedness condition that \( M \) expresses deontic necessity as given in (34), I propose to decompose it into \( M \) expresses deontic modality and
\( M \) is positive (meaning that \( M \) is the positive element of an antonym pair).

\[
\begin{align*}
\text{(37)} & \quad \text{Final analysis of “mandatorily”} \\
& \quad |\text{mandatorily}| = \lambda M \lambda p \lambda w : \text{M expresses deontic modality} \land M \text{ is positive} \\
& \quad \exists d = \max(S_M) \land M(d)(p)(w)
\end{align*}
\]

where \( \max(S_M) \) is the maximum of the (upper or totally closed) scale of \( M \)
is positive means that \text{NECESSITY}(p)(w) exceeds the degree that \( M \) combines with

Recall that both necessary and impossible (i.e. the strong modals) have been defined as positive, see (38), whereas possible and unnecessary (i.e. the weak modals) have been
defined as negative, see (39). (Examples (38) and (39) are repeated from (26), (28) and (29)).

\[
\begin{align*}
\text{(38) a.} & \quad |\text{must}| = |\text{necessary}| = \lambda d \lambda p \lambda w . \text{NECESSITY}(p)(w) \geq d \\
\text{b.} & \quad |\text{must not}| = |\text{impossible}| = \lambda d \lambda p \lambda w . \text{NECESSITY}(\neg p)(w) \geq d
\end{align*}
\]

where \( d \in F(\geq \text{NEC}) \)

\[
\begin{align*}
\text{(39) a.} & \quad |\text{need not}| = |\text{unnecessary}| = \lambda d \lambda p \lambda w . \text{NECESSITY}(p)(w) < d \\
\text{b.} & \quad |\text{may}| = |\text{possible}| = \lambda d \lambda p \lambda w . \text{NECESSITY}(\neg p)(w) < d
\end{align*}
\]

where \( d \in F(\geq \text{NEC}) \)

By decomposing the definedness conditions on modal modifiers, we arrive at two
definedness conditions that are independently motivated. On the one hand, the condition \( M \) expresses deontic modality reflects the fact that modal type is often lexically encoded, as in
the more restricted modal dürfen ‘may’ versus the more liberal können ‘can’. On the other
hand, the condition \( M \) is positive seems to reflect a more pervasive type of definedness
condition on certain degree modifiers.

Specifically, it can be shown that modal matching is not the only instance of such a
matching requirement. Other types of degree modification are analogous, cf. (40) and (41)\(^9\).

\(^9\) Many thanks also to Pranav Anand for pointing out supporting evidence from Hoeksema (1997), who
The fly was really big. / The fly was gigantic.

The elephant was really small. / The elephant was tiny.

The princess was really ugly (lit. terribly ugly).

The princess was really beautiful (lit. enchantingly beautiful).

What unites the German constructions in (40) and (41) is that the adjectival counterparts of the degree modifiers have the same directionality on the scale as the adjective that they modify. Specifically, riesig ‘gigantic’ / bezaubernd ‘enchanting’ / mandatory are positive and their adverbial counterparts combine with positive groß ‘big’ / schön ‘beautiful’ / must10. I conjecture that certain adjectives (like riesig ‘gigantic’) undergo a derivational process (in English marked by the derivational affix -ly) that turns them into degree modifiers with polarity presuppositions11.

2.4 Summary of the Analysis

I proposed that mandatorily (and similar elements) in modal matching configuration has as its core meaning component the meaning of a degree modifier over modals. It differs from regular degree modifiers in that it also has definedness conditions, which require its complement M to express deontic modality and to be positive with respect to the scale of necessity. The formal analysis is repeated in (42) from (37).

\begin{equation}
\mathrm{||mandatorily||} = \lambda M \rho \lambda p : M \text{ expresses deontic modality} \land M \text{ is positive}.
\end{equation}

\begin{equation}
\exists d [d = \max(S_M) \land M(d)(p)(w)]
\end{equation}

where \( \max(S_M) \) is the maximum of the (upper or totally closed) scale of M is positive means that necessity(p)(w) exceeds the degree that M combines with.

Derivations of the compositional semantics of two statements that involve modal matching are given in (43). Example (43a) illustrates matching between mandatorily and must, whereas (43b) illustrates matching between mandatorily and negated may. In both cases, the arrows indicate how the matching requirement is implemented.

documents adverbs in Dutch that are sensitive to scale-orientation. For instance, the Dutch degree adverb knap ‘pretty’ only seems to modify negative evaluative adjectives, such as vervelend ‘annoying’, and beroerd ‘lousy’; furthermore, the PPI bar ‘very’ only seems to modify negative members of antonym pairs, whereas the NPI bijster ‘very’ only modifies positive members.

10 A different question concerns the fact that the more extreme adjective must become a degree adverb, i.e. we find bezaubernd schön ‘enchantingly beautiful’ but not schön bezaubernd ‘beautifully enchanting’. Further research should address which adjectives can become degree adverbs of this type. This is beyond the scope of this project.

11 The idea that (certain) adverbials are linked to degrees and scales has a precedent in Nilsen (2004).
(43) a. **Necessity** (i.e. □)

\[ \lambda p \lambda w . \exists d [ d = \text{max}(S_M) \land \text{NECESSITY}(p)(w) \geq d ] \]

\[ p \]

\[ ||\text{mandatorily}|| \]

\[ \lambda M \lambda p \lambda w : M \text{ expresses deontic modality} \]

\[ \land \] \[ M \text{ is positive} . \exists d [ d = \text{max}(S_M) \land M(d)(p)(w) ] \]

b. **Impossibility** (i.e. \( \neg \Box \equiv \Box \neg \)\(^{12} \)

\[ \lambda p \lambda w . \exists d [ d = \text{max}(S_M) \land \text{NECESSITY}(\neg p)(w) \geq d ] \]

\[ p \]

\[ ||\text{mandatorily}|| \]

\[ \neg \lambda d \lambda p \lambda w . \text{NECESSITY}(\neg p)(w) < d \]

\[ \lambda M \lambda p \lambda w : M \text{ expresses deontic modality} \]

\[ = \lambda d \lambda p \lambda w . \text{NECESSITY}(\neg p)(w) \geq d \]

\[ \land \] \[ M \text{ is positive} . \exists d [ d = \text{max}(S_M) \land M(d)(p)(w) ] \]

\[ ||\text{not}|| \]

\[ ||\text{may}|| \]

\[ \lambda X . \neg X \]

\[ \lambda d \lambda p \lambda w . \text{NECESSITY}(\neg p)(w) < d \]

Crucially, as (43b) shows, *may* on its own would not satisfy the definedness condition on *mandatorily*, as it is negative before combining with negation.

Note that my analysis predicts that elements such as *mandatorily* should not be able to act as modal operators on their own (against the assumptions of Geurts & Huitink 2006 and Zeijlstra 2008). As a matter of fact, the prediction seems to be carried out, as shown in (44a) versus (44b-d). Even though *mandatorily* can occur on its own in generic statements, it seems to be impossible in episodic statements. This observation carries over to other adverbs that modify deontic necessity and have been analyzed as modal concord elements, such as *necessarily* and *obligatorily*.

(44) Why did John sign this form?

a. * He mandatorily / necessarily / obligatorily signed it.

b. He had to sign it.

c. He mandatorily / necessarily / obligatorily had to sign it.

d. It was mandatory / necessary / obligatory (for him) to sign it.

This can be taken to indicate that *mandatorily* in (45) combines with a covert generic operator that acts as a universal modal of sorts.

---

12 It is an open question whether the constituency assumed in (43) makes the right predictions in other areas. Compare Anand & Brasoveanu (2009) for an alternative that makes use of a dual operator.
(45) In these days, visitors mandatorily signed this form.

Epistemic modals perhaps and maybe contrast with mandatorily, necessarily and obligatorily, in that they can occur on their own with a modal operator meaning. This might suggest that in “modal concord” with perhaps and maybe we are dealing with a different phenomenon from the phenomenon discussed in this paper\textsuperscript{13}.

3 Extending the Empirical Coverage

Grosz (to appear) shows that the German particles ruhig, JA and bloß also qualify as modal matching elements\textsuperscript{14}. On the one hand, they impose requirements on the modal force of an utterance (as Grosz to appear shows, all three must combine with modals that make reference to an authority, e.g. somebody’s commands, wishes or goals\textsuperscript{15}); on the other hand, they instantiate all four possible matching relations that involve (im-)possibility and (non-)necessity: (bloß has the same distribution as JA and is thus omitted from the examples.)

(44) a. Possibility (i.e. $\Box$) and “ruhig”
Der Hans darf {ruhig / *JA}. den Kühlschrank ausräumen.
the Hans may RUHIG JA the fridge empty
‘Hans may {ruhig / *JA} empty the fridge.’
$\approx$ In view of what I want / In view of my rules, it is completely possible that Hans empties the fridge (i.e. there is not the least objection).

b. Impossibility (i.e. $\neg\Box \equiv \Box\neg$) and “JA”
Der Hans darf den Kühlschrank {JA/*ruhig} nicht ausräumen.
the Hans may the fridge JA ruhig not empty
‘Hans {JA / *ruhig} may not empty the fridge.’
$\approx$ In view of what I want / In view of my rules, it is absolutely necessary that Hans do not empty the fridge (i.e. there are no mitigating circumstances).

c. Necessity (i.e. $\Box$) and “JA”
Der Hans soll {JA / *ruhig}. aufessen!
the H. shall JA ruhig eat up
‘Hans shall {JA / *ruhig} eat up!’
$\approx$ In view of what I want / In view of my rules, it is absolutely necessary that Hans eat up (i.e. there are no mitigating circumstances).

d. Non-Necessity (i.e. $\neg\Box \equiv \Box\neg$) and “ruhig”
% Du brauchst er {ruhig / *JA} nicht auf(zu)essen!
you need PRT ruhig / *JA not (to.)eat up!
‘You {ruhig / *JA} need not eat up!’
$\approx$ In view of what I want / In view of my rules, it is completely possible that you do not eat up (i.e. there is not the least objection).

\textsuperscript{13} More generally speaking, the research of Anand & Brasoveanu (2009) suggests that modal concord is a much more heterogeneous phenomenon than one might initially think.

\textsuperscript{14} See Schwager (to appear) and Portner (2010) for a different approach to the particle ruhig.

\textsuperscript{15} More precisely, these particles require a circumstantial modal base and a non-dynamic, non-epistemic ordering source, in terms of Kratzer (1981, 1991); this corresponds to Portner’s (2007) priority type.
As indicated by the paraphrases, these particles also maximize the degree of necessity or possibility in the respective constructions. We can thus conclude that JA, bloß and ruhig have exactly the same properties as other modal matching elements (like mandatorily). Their meanings can be rendered as follows, in (45) and (46).

\[
||JA / bloß|| = \lambda M \lambda p \lambda w : M \text{ expresses non-dynamic root modality } \land M \text{ is positive .} \\
\exists d \{d = \max(S_M) \land M(d)(p)(w)\}
\]

where \(\max(S_M)\) is the maximum of the (upper or totally closed) scale of \(M\)

\(\text{is positive means that necessity}(p)(w)\) exceeds the degree that \(M\) combines with

\[
||ruhig|| = \lambda M \lambda p \lambda w : M \text{ expresses non-dynamic root modality } \land M \text{ is negative .} \\
\exists d \{d = \min(S_M) \land M(d)(p)(w)\}
\]

where \(\min(S_M)\) is the minimum of the (lower or totally closed) scale of \(M\)

\(\text{is negative means that necessity}(p)(w)\) is lower than the degree that \(M\) combines with

4 Conclusion

In section 1, I have shown that we find modal-concord-like constructions in which a necessity modal combines with a negated possibility modal. I argued that such constructions cannot be easily explained under an approach such as Geurts & Huitink (2006) or Zeijlstra (2008). In section 2, I proposed an alternative analysis, which treats modal matching as degree modification over degrees of modality, based on the strengthening effect perceived in modal matching. Finally, in section 3, I showed that the German particles ruhig, JA and bloß can be analyzed as modal matching elements and that my analysis of modal matching can be straightforwardly extended to these particles. I conclude that the present analysis uniformly accounts for modal concord, modal-concord-like matching with negated modals and the distribution of German modal particles.

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