The median winter snowline in the Alps

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Abstract
The relative duration of snow cover in a season is a number between zero and unity; here it represents the probability to encounter, at a given station, snow of at least 5 cm depth. We use routine station data of snow depth for the winters 1961–2000 to explore the pattern of relative snow duration in the Alps. A horizontal isoline is drawn across all stations that exhibit 50 \% snow duration; we consider this isoline the median snowline. We further introduce the mountain temperature as linear expansion of the Central European temperature with respect to station coordinates; it separates the large-scale European temperature from the local-scale vertical lapse rate and serves as substitute for the station temperature. The mountain temperature allows to condense the snow data of all stations and years into one analytical curve, the state function of snow duration. This curve yields every desired snowline; the median snowline coincides with the altitude of maximum sensitivity of snow duration to European temperature. The median snowline in winter is located at an average altitude of 641 m and slightly slopes downward towards the eastern Alps. The average altitude varies considerably from winter to winter under the influence of European temperature fluctuations; it shifts upward by about 123 m per °C climate warming.

Zusammenfassung

1 Introduction
The idea of the snowline is not new. HANN (1883) has defined the snowline (or snow limit) as the lowest altitude of the perennial snow cover, equivalent to the lower boundary of the snow-covered area at the end of summer; it yields a climatological annual average. This concept has been adopted by KÖRNER (2003) based on TROLL (1961) who understands the snowline as thermal boundary above which the ground remains snow covered all-the-year. However, seasonal fluctuations are of similar impact upon the snowline, also in accord with HANN (1883); for example, at Mt. Säntis (NE-Switzerland) he locates the snowline at 740 m in the second decade of December and at 1930 m in June. And for the Inn valley close to Innsbruck HANN (1908) finds, averaged over the north and south faces, a value of 667 m for the snowline in winter and 2575 m in summer. A modified approach is to consider the geographical distribution of snow cover. For example, the Dictionary of Earth Science (PARKER, 1997) defines the snowline as the boundary of an area with more than 50 \% snow cover. As to recent applications, a study in the Indian Himalayas (KAUR et al., 2010) uses satellite measurements of monthly snow cover; the snowline is located at the elevation that separates the area with snow cover from the area free of snow. These references are not exhaustive; yet they suffice to show that the snowline is not a generally accepted notion but remains somewhat vague. We have considered it worthwhile to rigorously define what we feel should be the natural meaning of the snowline and to demonstrate its climatological significance, here restricted to the winter season in the Alps.

Our ‘natural’ approach is to take the line that separates complete from zero snow cover. We place its average position where there is a 50 \% probability to encounter snow at any time, identical to 50 \% probability for no
snow. We will get this information from observed snow depths by connecting neighbored climate stations with the same relative snow duration; relative snow duration is the percentage of days in a given season with snow depth above a specified threshold. This understanding applies principally to every value of relative snow duration between zero and unity. Here we want to focus upon the 50 %-snowline, called the median snowline.

It is the first purpose of this study to find, from observed data, the altitude of the median winter snowline in the Alps, including its time fluctuations from year to year. The second purpose is to understand the mechanism that generates the field of snowlines. We shall show that the controlling agent is the mean winter temperature of continental Europe. We will particularly demonstrate that the median snowline is unique in that it represents the altitude of maximum sensitivity of snow cover duration with respect to European temperature.

The snowline concept is the novelty of this paper. Its main goal is to develop a theoretical understanding with which the qualitative description of a snowline can be formalized and quantitatively derived from snow depth measurements. It will be based upon the following earlier publications: HANTEL et al. (2000), referred to as 'Paper I'; WIELKE et al. (2004) together with WIELKE et al. (2005), ‘Paper II’; and HANTEL and HIRTL-WIELKE (2007), ‘Paper III’. In Paper I the dependence of relative snow duration upon European temperature has been investigated for Austrian climate stations; the same has been done for Swiss climate stations in Paper II. The theoretical model has been rigorously developed in Paper III.

The present study is organized as follows. We start with the data description and, in a preliminary step, study the mean altitude of 50 %-snow duration in the Alps from one individual winter to the next; only snow data are used in this step. Then we review our model (i.e., Papers I, II, III); it combines local-scale snow information with continent-scale temperature information and condenses both into one formula, called the ‘state function of snow duration’; it represents the entire data volume of the entire observation period. From the state function all relevant parameters can be analytically derived. They yield the innovative quantity presented here: altitude and horizontal pattern of the median snowline, complete with its temperature sensitivity and time trend.

2 Basic data and quality checks

As snow data base for winter (DJF) we take the ‘All Alps’ snow depth data set 1961–2000 from the 268 climate stations used in Paper III. Measured quantity is the snow depth, observed daily at each station (Fig. 1). Following Paper I, a day with snow depth above or below threshold 5 cm is counted as $\nu = 1$ or $\nu = 0$. The DJF average of $\nu$ yields the relative snow duration $\overline{\nu} = n$ of this station winter. $n$ is close to 1 at high stations in cold winters (‘always snow’) and close to 0 at low stations in mild winters (‘never snow’). The frequency distribution of $n$ (not shown) is bimodal with a minimum of less than 40 station winters around $n = 0.5$ and maxima of more than 80 winters close to $n = 0, n = 1$.

If less than 15 observations at a station are reported in a given winter the respective station winter is a priori dropped. This excludes 11 stations. The year of a specific winter is referenced according to its January; for example, winter 2000 is December 1999, January 2000, February 2000. The first winter (1961) of the record comprises only January and February, the December of the year 2000 has not been used.

Further, station winters with exactly $n = 0$ and those with exactly $n = 1$, referred to as ‘saturated’, are also a priori dropped. The reason is that saturated snow data do not carry relevant information since they have observation variance zero; we do not accept them as measurements (for further discussion of this point see section 4). This excludes 7 stations which report only saturated data. Each of the remaining stations contains at least one unsaturated $n$-value (most of them much more).

Observing the two a priori requirements just described yields the basic station data set. It consists of 250 stations (black and blue rhomboids in Fig. 1) yielding 10000 principally usable station winters. Many of these stations still contain individual winters that are saturated and thus have also to be dropped. This procedure ends with 5705 unsaturated $n$-values, one for each station winter (corresponding to 22.8 winters per station).

As temperature data base we take the monthly grid-ded CRU temperatures (BROHAN et al., 2006) with a horizontal resolution of 0.5 degrees. These are averaged

![Figure 1: Location of Alpine climate stations providing the basic snow duration data set. Blue rhomboids: stations that do not pass the correlation criterion; black rhomboids: stations used for the final evaluation. Average of CRU temperature over red box in inset (5.5—17.5°E, 43.5°—49.5°N) is representative for European temperature. Thick light blue rhomboid: Position of $x = 0, y = 0$ in definition of mountain temperature.](image-url)
The median winter snowline follows from the laboratory model of Hantel and Hirtl-Wielke (2007) and in this sense is mandatory.

Figure 2: Relative duration \( n \) of snow cover at ‘All Alps’ climate stations in the winters of 1978 (blue) and 1998 (red), plotted versus station altitude \( z \). Fit with logistic model \( P(z) = \Phi(x) \) with \( \Phi = \) error function and \( x = \sqrt{2\pi r_0(z - z_0)} \). Parameters in 1978 (131 data points): \( r_{1978}^0 = 1.76(\pm 0.46) \times 10^{-3} \text{m}^{-1} \); \( z_{1978} = 529(\pm 30) \text{m} \). Parameters in 1998 (132 data points): \( r_{1998}^0 = 1.33(\pm 3.66) \times 10^{-3} \text{m}^{-1} \); \( z_{1998} = 866(\pm 109) \text{m} \). Note that 7 blue and 16 red crosses which were used for estimating the fit profiles could not be drawn because they are outside the plot.

The mean altitude of these is \( 728\pm 412 \text{m} \) (63 stations are located below 500 m a.s.l., 108 are between 500 m and 1500 m a.s.l. and 36 are above 1500 m a.s.l.). The surviving 5370 \( n \)-values for 5370 station winters represent the snow data base of this study (black symbols in Fig. 1, corresponding to 25.9 winters per station).

3 Mean altitude of the median snowline

The snow duration comes as function of time \( \theta \) and of the space coordinates \( x, y, z \) of the climate station:

\[
    n = n(\theta, x, y, z).
\]

We plot \( n \) versus altitude \( z \) in a given year, irrespective of \( x, y \). The fit curve \( P(z) \) for \( n \) must not be linear since \( n \) is the mean of the binary stochastic variable \( \nu \). For variables of this type a logistic curve is the proper fitting function (Hosmer and Lemeshow, 2000). Out of the class of logistic curves (Mazumdar, 1999) we take here the error function.

Such curve is fitted to the blue symbols in Fig. 2, valid for 1978. The fit curve cuts the median value \( n = 0.5 \) at an altitude \( H^{1978} = 529 \text{m} \). We consider \( H \) as the averaged altitude of the median snowline; it is equal to the reference parameter \( z_0 \) of the interpolating error function (see caption of Fig. 2).

The winter of 1978 was relatively cold (European temperature \( T^{1978} = -0.08 \text{C} \)). The milder winter 1998 (\( T^{1998} = 2.52 \text{C} \)) generates lower relative snow durations (red symbols in Fig. 2) which yield the average median snowline altitude \( H^{1998} = 866 \text{m} \) (see caption of Fig. 2). Beniston et al. (2003), using Swiss data from 18 observing sites, consider also snow cover duration versus altitude profiles but restricted to a linear fit; they find, opposite to our result of Fig. 2, a smaller vertical slope in cold than in mild winters.

The result \( H^{1998} < H^{1978} \) is as expected: The snowline tends to be higher when the temperature is higher.

Station temperatures are also available at all stations of Fig. 1. However, we do not use them in this study with the two exceptions of Fig. 5 and Fig. 7. It is only in these two figures that we compare the results found with the CRU-data to those found with station temperatures.

As quality check for the snow data we adopted a criterion already used in Papers I-III: We require that the linear correlation coefficient between \( n \) and \( T \) during the observation period should be negative at each station. 43 stations (comprising 335 station winters or 5.9% of the total \( n \)-values) that violated this condition (blue symbols in Fig. 1) were discarded, leaving 207 useable stations. The mean altitude of these is \( 728\pm 412 \text{m} \) (63 stations are located below 500 m a.s.l., 108 are between 500 m and 1500 m a.s.l. and 36 are above 1500 m a.s.l.). The surviving 5370 \( n \)-values for 5370 station winters represent the snow data base of this study (black symbols in Fig. 1, corresponding to 25.9 winters per station).

1 All standard deviation estimates in this study are given as one sigma.

2 Choice of the error function has been convenient in our programming but is not mandatory here; one could take other logistic functions in Fig. 2 as well. On the other hand, in section 4.2 below, the error function follows from the laboratory model of Hantel and Hirtl-Wielke (2007) and in this sense is mandatory.
The median winter snowline $H$ for each year determined according to Fig. 2 for Alpine climate stations plotted versus time. Light grey dots: Data points excluded because fit has yielded a negative $H$-value (see text).

Fig. 3 shows the time series of $H$ over the 40-year observation period. In some years the fit yields a negative $H$. This is formally possible since the logistic function is not restricted to altitudes $z>0$; we consider the 3 corresponding $H$-values (light grey rhomboids in Fig. 3) as outliers.

The upward increase of $n$ in the individual profiles 1978 and 1998 in Fig. 2 is evidently due to the familiar upward decrease of temperature; this is a local effect. Conversely, the gross difference between the entire profiles 1978 and 1998 is a large-scale effect caused by the European temperature. This interpretation is supported by relating the time series of $H$ in Fig. 3 to the time series of $T$ for the same period. The corresponding scatter diagram is shown in Fig. 4 for the two European temperatures defined above. The difference between the impact of either $T_{CRU\text{large}}$ or $T_{CRU\text{small}}$ in Fig. 4 is marginal, the correlation is significant in both cases.

Fig. 4 suggests that the interannual altitude fluctuations of the median snowline (Fig. 3) are controlled by the European temperature. This European effect of Fig. 4 (from now on represented by $T = T_{CRU\text{small}}$) needs to be separated from the familiar vertical lapse rate effect of Fig. 2. It is for this purpose that we have developed the model that will be reviewed in the next section.

4 Review of the snow duration model

The snow duration model as we use it here has grown in steps from Paper I for Austrian climate stations [HANTEL et al. (2000)], over Paper II for Swiss stations [WIELKE et al. (2004) together with WIELKE et al. (2005)], to Paper III for All-Alps stations [HANTEL and HIRTLE-WIELKE (2007)]. The concepts required will be reviewed in this section.

4.1 The probabilistic model – the local mode

Basic hypothesis of our probabilistic model is that the seasonal snow cover duration $n$ is in approximate thermodynamic equilibrium with seasonal mean temperature $t$. We describe this mechanism through a normally distributed stochastic variable from an ensemble with mean $t$ and standard deviation $\epsilon$. The probability $P$ that $t$ is less than a reference $t_0$ is [see standard statistics texts, e.g. TAYLOR (1997)]:

$$P(t) = \Phi \left( \frac{t_0 - t}{\epsilon} \right).$$

(4.1)

$\Phi$ is the error function [BRONSTEIN et al., 1999] defined as:

$$\Phi(\chi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\chi} e^{-\vartheta^2/2}d\vartheta.$$

(4.2)

The more $t$ is below $t_0$, the more positive is the argument of $\Phi$ in (4.1) and the closer to unity is $P$.

In the ideal case (an experiment with distilled water in the laboratory, see Fig. 3 of Paper III) we interpret $t$ as temperature ($t_0$ = freezing temperature). When equilibrium has been reached $P$ is identical to the probability to find water in the frozen phase. We eliminate $\epsilon$ in Eq. (4.1) in favor of the negative parameter $s_0$ by putting $-\epsilon^{-1} = \sqrt{2\pi}s_0$. The probability for ice is now:

$$P(t) = \Phi(\chi) \quad \text{with} \quad \chi = \sqrt{2\pi}s_0(t-t_0).$$

(4.3)

A preliminary form of this model was first applied (in the so-called ‘local mode’) to snow duration data in winter and spring at individual Austrian climate stations (Paper I, Figs. 4, 6, 7) and at Swiss climate stations.
(Paper II, Fig. 1); the parameters $s_0$, $t_0$ were determined through fitting the theoretical function to the data$^3$. For $t$ we first chose the European temperature $T$. The results were encouraging. Papers I, II showed that at individual climate stations the interannual $T$ -variations cause a strong $n$ -variation, quite well described by the model (4.3).

4.2 Revisiting the error model of Paper I

Before proceeding some remarks are necessary concerning the error model applied for estimating the parameters $s_0$ and $t_0$ in (4.3). The nonlinear fit used here as well as in Paper I follows standard statistical interpolation recipes [e.g., TAYLOR (1997)] and consists of minimizing the cost function:

$$J(s_0, t_0) = \sum_{i=1}^{I} \left[ \frac{n_i - n^i}{\sigma(n_i)} \right]^2$$

(4.4)

The index $i$ for the data points runs from 1 to $I$, with $I$ the number of station winters. $n_i$ is the measured snow duration and $n^i = P(t_i)$ is the nonlinear model value with $t_i$ the measured temperature.

$\sigma(n_i)$ is the standard deviation of the measured $n_i$. We may recall that $n$ is calculated from the daily $\nu$ (with $\nu = 0$ for snow depth below threshold, $\nu = 1$ for snow depth above threshold). Now it can be shown that the stochastic quantity $\nu$ is Bernoulli-distributed (DEGROOT, 1986). It has a parabolic variance distribution; the variance of $n$ at the limits of the interval is exactly zero. For this reason saturated values $n_i = 0$, $n_i = 1$ cannot be accepted as observations; the corresponding weight $1/\sigma(n_i)^2$ would make the respective term infinite in the cost function. It follows that saturated $n$’s do not belong to a snow duration data sample and have a priori to be dropped.

These specifications of the nonlinear fit yield the parameters $s_0$ and $t_0$ that make $J$ a minimum. There is a further specification in the error model of Paper I that concerns the ‘rectified’ fit and the ‘extended’ fit. In Papers I-III the extended fit was used which tends to overestimate the parameter $s_0$. We shall not use these fits here but exclusively apply the nonlinear fit represented by $J(s_0, t_0)$, together with the parabolic profile for $\sigma(n_i)$.

4.3 The probabilistic model – many stations

When many climate stations are involved the role of the temperature requires further analysis. We shall from here on reserve the letter $t$ for station temperature (seasonal average of local daily station observations) and $T$ for European temperature (seasonal average of area average of monthly gridded CRU temperatures). Instead of plotting $n$ in the local mode as function of $T$ one could as well plot $n$, still in the local mode, as function of $t$ (this was done in Fig. 1 of Paper III). The reason is that $t$ and $T$ are quite well correlated at individual stations (see, e.g., Fig. 6 of Paper III). The plot $n(t)$, now for the present All-Alps set, is shown in Fig. 5, together

![Figure 5: Winter snow duration $n$ at 210 Alpine climate stations plotted versus station temperature. Thick grey curve $P(t)$ interpolates $n$-values. Each dot represents one out of 5381 station winters. Grey shading captures 68% of data points (corresponding to one standard deviation in $t$-direction); shading does not show accuracy of fitted state curve. For discussion of parameters see text.](image1)

![Figure 6: Winter snow duration $n$ at 207 Alpine climate stations plotted versus mountain temperature. Thick grey curve $N(\tau)$ represents state function of $n$. Each dot represents one out of 5370 station winters. Grey shading captures 68% of data points (corresponding to one standard deviation in $\tau$-direction); shading does not show accuracy of fitted state curve. Selected parameters are shown in the inset; for discussion see text.](image2)

$^3$In Paper I we used the hyperbolic tangent function for interpolation, without a physical argument. In Paper III we introduced the physical mechanism described here; it leads to the Gaussian error function for interpolation. The differences between hyperbolic tangent and error function are numerically small.
with the interpolating function $P(t)$ defined in (4.3)$^4$.

Fig. 5 includes estimates for $s_0$, $t_0$. The parameter $s_0$ is the maximum temperature sensitivity of the snow duration. The value is to be interpreted as follows: When $t$ decreases (increases) by 1 degree $n$ increases (decreases) by 16%. For example, a station with $n = 0.5$ (45 snow cover days per winter) would, under a hypothesized warming of 1°C, experience a decrease of the winter snow duration down to $n = 0.34$; this would correspond to a reduction by 14 snow days leaving meager 31 snow cover days per winter for this station.

4.4 The concept of mountain temperature

The plot $P(t)$ in Fig. 5 reveals the temperature dependence of $n$; however, the $z$-information of Fig. 2 is lost. It follows that snowlines cannot be gained from Fig. 5.

Now the station temperature $t$ is influenced by two mechanisms: The large-scale climate process condensed in the European temperature $T$; and the local effects, notably the vertical lapse rate of temperature but also the horizontal temperature structure. Thus we have attempted to replace $t$ through a combination of $T$ and station coordinates $x$, $y$, $z$; they are condensed in the mountain temperature defined as$^5$:

$$
\tau = T + ax + by + cz \quad (4.5)
$$

$a$, $b$, $c$ are the constants of this linear expansion; (4.5) can equivalently be interpreted as multilinear regression analysis of $t$. We have plotted the station temperatures $t$ and $\tau$ – the latter fitted from (4.5) with $t$ as dependent variable – against each other with good results in a first step (70% explained variance, details not shown).

4.5 The global mode and the state function of snow duration

We now change perspective. We will not get $\tau$ from the observed $t$-field, but from the $n$-field. We replace $t$ through $\tau$ in (4.3) and understand $P(\tau)$ as probability of snowcover duration, with the same functional relationship as in (4.1); the parameters $s_0$, $\tau_0$ (augmented by the coefficients $a$, $b$, $c$) are to be fitted to the observed snow data. In this way our equilibrium hypothesis above is implemented in the ‘global mode’ and yields what may be called the state function of the snow duration:

$$
N(\tau) = \Phi(\chi) \quad \text{with} \quad \chi = \sqrt{2\pi} s_0 (\tau - \tau_0) \quad (4.6)
$$

$\Phi$ is as defined in (4.2). $N(\tau)$ is specified by the parameter vector $(s_0, \tau_0, a, b, c)$. Time $\theta$ is implicit in the data vector $(n, T, x, y, z)$ through the time dependence of the large-scale climate temperature $T(\theta)$. The parameter vector is estimated from the data vector through our fitting routine discussed above. Local temperature $t$ is not involved.

4.6 Temperature sensitivity of the state function

Fig. 6 shows the state function for the Alpine data set. The profile $N(\tau)$ represents the observed $n$-data of all station winters over the entire period 1961-2000. The complete parameter vector is listed in Table 1 together with a couple of derived quantities. All error estimates have been obtained through a bootstrap routine (EFRON and TIBSHIRANI, 1998) with 2000 runs each. $s_0$ is the extreme slope of the curve $N(\tau)$ for $\tau = \tau_0$; the corresponding function value is $N(\tau_0) = 0.5$.

Fig. 6 yields practically the same interpolating curve as does Fig. 5 which suggests that $t$ is reasonably represented by $\tau$. This is independently shown in Fig. 7. The difference between the means $\tau$, $\bar{T}$ in Fig. 7 corresponds to the difference between the fit constants $\tau_0$, $t_0$ in Figs. 5, 6. The added value of the mountain temperature (4.5) is that large-scale and local-scale temperature effects become separated. The parameters $a$, $b$, $c$ that define $\tau$ follow from the nonlinear fit of the observed $n$-data; no $t$-information is used for $\tau$. Given this independence between the data sources of $t$ and $\tau$ the NRMSE-value seen in Fig. 7 must be considered quite good (the most ideal value would be NRMSE $= 0$).

The fitted curve $N(\tau)$ from Fig. 6 is reproduced in Fig. 8 together with a statistical summary of the station winter data. Both Figs. 6, 8 suggest that the mountain

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$^4$There are 5381 station winters in Fig. 5 instead of the 5370 available values. The reason is that the correlation criterion applied in Fig. 5 excludes not 43 but only 40 climate stations which adds 11 station winters to the number of data points.

$^5$In Papers I-III the parameter $\tau$ was called ‘Alpine temperature’; here we have switched to ‘mountain temperature’ for greater generality.
Table 1: Parameters of state function and derived quantities for winter snow duration. Snow data from Alpine climate stations 1961–2000. $T$ = European temperature, $\theta$ = time. The term ‘temperature gradient’ refers to the gradient of the mountain temperature $\tau$.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum sensitivity of state curve</td>
<td>$s_0$</td>
<td>$-0.17(\pm0.01)\degree C^{-1}$</td>
</tr>
<tr>
<td>Reference parameter for $\tau$</td>
<td>$\tau_0$</td>
<td>$-5.01(\pm0.69)\degree C$</td>
</tr>
<tr>
<td>West-to-east temperature gradient</td>
<td>$a$</td>
<td>$-0.45(\pm0.06)\degree C/km$</td>
</tr>
<tr>
<td>South-to-north temperature gradient</td>
<td>$b$</td>
<td>$0.42(\pm0.24)\degree C/\text{lat}$</td>
</tr>
<tr>
<td>Altitudinal temperature gradient</td>
<td>$c$</td>
<td>$-8.10(\pm1.12)\degree C/km^{-1}$</td>
</tr>
</tbody>
</table>

The sensitivity with respect to $\tau$: $\frac{\partial N}{\partial \tau} = \frac{dN}{dt} \frac{\partial \tau}{\partial \tau} = \frac{dN}{dt}$.

The partial derivative is understood for fixed station vector $(x, y, z)$. Note that (4.8) applies to all $\tau$, not just to $\tau_0$. Thus formula (4.8) represents the entire sensitivity profile of Alpine climatological snow cover. The sensitivity is maximum for $\tau = \tau_0$, adopted at the altitude of the median snowline; above and below the sensitivity decreases and becomes zero at very low and very high altitudes.

A point much discussed in Papers I and II is to what extent $s_0$ depends on the specifications of the error model. Our earlier evaluations yield, with the nonlinear fit, a value for $s_0$ between $-0.13\degree C^{-1}$ [only Switzerland data; see Fig. 3 of WIELKE et al. (2005)] and $-0.20\degree C^{-1}$ [only Austrian data; see Fig. 8 of HANTEL et al. (2000)]. With the present All-Alps data we find $s_0 = -0.17\degree C^{-1}$ (Figs. 6, 8 and Table 1). We consider this an acceptable coincidence, given the different data bases and the shorter observation period in papers I, II (1961-1990) as compared to the present study. These considerations show further that the All-Alps sensitivity -0.33 per $\degree C$ warming, obtained with the extended fit and published in Paper III, must now be considered an overestimate. The maximum sensitivity $s_0 = -0.17\degree C^{-1}$ found here is more realistic.

5 Characteristics of the snowline

Complete snowline information can be drawn from $N(\tau)$ by straightforward reasoning; the mountain temperature concept allows to derive the characteristics of the snowline from the state function. The altitude of the median snowline is:

$$H(0.5, T, x, y) = \frac{\tau_0 - T - ax - by}{c}. \quad (5.1)$$
It follows from solving Eq. (4.5) for \( z = H \) with the condition \( \tau = \tau_0 \); this yields the value \( n = N(\chi = 0) = 0.5 \) of the state function which is the definition of the median snowline. It is at this altitude that the temperature sensitivity adopts the extreme value \( \tau_0 \). \( H = 641 \text{ m} \) in Tab. 1 has been entered for \( x = 0, y = 0 \) (located in Tyrol, \( \lambda = 10.5^\circ \text{E}, \phi = 46.8^\circ \text{N} \), thick blue rhomboid in Fig. 1). Note that the standard deviation of \( H \) in Tab. 1 is gained from the bootstrap results for \( \tau_0 \) and \( c \) and does not take into account the interannual fluctuations of \( T \); the estimate \( \pm 26 \text{ m} \) is smaller than might be concluded from the fluctuations seen in Fig. 3.

The altitude of an arbitrary snowline specified by \( n \) is:

\[
H(n, T, x, y) = \frac{\tau(n) - T - ax - by}{c} = \frac{1}{c} \frac{\partial H}{\partial T} = \frac{1}{c}.
\]

The specification of \( \tau(n) \) can be implemented through the inverse \( N^{-1} \) of the state function as \( \tau(n) = N^{-1}(n) \). For example, for \( n = 0.5 \) the fitted \( N \) from Figs. 6, 8 yields \( \tau = \tau_0 = -5.0^\circ \text{C} \). The partial derivative in (5.2) is understood for fixed snowline \( n \) and fixed station coordinates \( x, y \). Eq. (5.1) is a special case of the general formula (5.2). Both reflect the downward move of the median snowline in cold years and the upward move in warm years (note that \( c < 0 \)).

The temperature sensitivity of the snowline altitude, \( \partial H / \partial T \) according to (5.2), is constant across the entire domain and thus the same on all snowlines. The corresponding numerical value (Tab. 1) suggests that a climate warming of 1°C shifts all snowlines in the Alps corresponding to the extreme altitude trend of all snowlines. This may not do justice to the strong interannual fluctuations of \( T \).

The horizontal slope of the plane \( H(n, T, x, y) \) with the earth’s surface for fixed \( \tau(n) \), here \( \tau = \tau_0 \), fixed European temperature (mean value 1961–2000), fixed parameters \( a, b, c \) and variable horizontal coordinates \( x, y \). The parameters are taken from Table 1.

The inclination of the plane \( H(n, T, x, y) \), i.e., the horizontal slope of the snowline altitude, is gained by differentiating formula (5.2) with respect to \( x \) in eastern and to \( y \) in northern direction, with \( n \) and \( T \) kept constant. This yields:

\[
\frac{\partial H}{\partial x} = \frac{-a}{c} = -56 \text{ m} / \circ \text{longitude}; \\
\frac{\partial H}{\partial y} = \frac{-b}{c} = 52 \text{ m} / \circ \text{latitude}.
\]

This estimate corresponds to a (significant) downward slope of about 560 m from the western to the easternmost Alps (\( \Delta x \approx 10^\circ \text{longitude} \)) and a (non significant) upward slope of about 52 m from the southern to the northern Alps (\( \Delta y \approx 3^\circ \text{latitude} \)). This is made visible in Fig. 9 by the perimeter of the plane \( H(0.50, T, x, y) \) which is drawn in red color.

With regard to other possible snowlines a higher located one would generate a much smaller area than does the 50% snowline in Fig. 9. Another important difference between the snowline patterns of Fig. 9 and any other snowline is the difference in temperature sensitivity. It is a maximum for the median snowline, which implies that for high and low located stations a change of European temperature is of comparatively little impact upon the snowline.

The horizontal slope of the plane \( H(n, T, x, y) \) is the same for all snowlines. This may not do justice to the complicated orography of the Alps; the simplification of our present model does not allow for a horizontal change of the lapse rate parameter \( c \). Now it would be easier to implement more sophisticated functions than the simple linear expansion represented by our \( \tau \); for example, higher than linear expansions or thin-plate spline functions could be chosen. We have not done this in the present study; however, generalizations of this type will be a challenge for further study.

6 The 3D-pattern of the median snowline across the Alps

The median snowline in the Alps in winter is drawn in Fig. 9. The snowline is generated by cutting the horizontally inclined plane \( H(n, T, x, y) \) with the earth’s surface for fixed \( \tau(n) \), here \( \tau = \tau_0 \), fixed European temperature (mean value 1961–2000), fixed parameters \( a, b, c \) and variable horizontal coordinates \( x, y \). The parameters are taken from Table 1.

The inclination of the plane \( H(n, T, x, y) \), i.e., the horizontal slope of the snowline altitude, is gained by differentiating formula (5.2) with respect to \( x \) in eastern and to \( y \) in northern direction, with \( n \) and \( T \) kept constant. This yields:

\[
\frac{\partial H}{\partial x} = \frac{-a}{c} = -56 \text{ m} / \circ \text{longitude}; \\
\frac{\partial H}{\partial y} = \frac{-b}{c} = 52 \text{ m} / \circ \text{latitude}.
\]

This estimate corresponds to a (significant) downward slope of about 560 m from the western to the easternmost Alps (\( \Delta x \approx 10^\circ \text{longitude} \)) and a (non significant) upward slope of about 52 m from the southern to the northern Alps (\( \Delta y \approx 3^\circ \text{latitude} \)). This is made visible in Fig. 9 by the perimeter of the plane \( H(0.50, T, x, y) \) which is drawn in red color.

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The horizontal slope of the plane \( H(n, T, x, y) \) is the same for all snowlines. This may not do justice to the complicated orography of the Alps; the simplification of our present model does not allow for a horizontal change of the lapse rate parameter \( c \). Now it would be easier to implement more sophisticated functions than the simple linear expansion represented by our \( \tau \); for example, higher than linear expansions or thin-plate spline functions could be chosen. We have not done this in the present study; however, generalizations of this type will be a challenge for further study.

7 Conclusions

The practical innovation of this study is the snowline concept; it represents the entire winter snow information of the Alps in form of one simple visualization.
Specifically the up-and-down motion of the winter snow cover from year to year becomes visible; for example, one could animate the snowline variations with time by running Fig. 9 through the entire climate period. We have shown that the most important snowline is the median snowline at 0.5 because the sensitivity of the state function at its altitude to changes in European temperature is a maximum.

The present evaluations have been limited to snow depth threshold 5 cm. We have made experiments with other thresholds (not reported here) but the results are much the same, except that the median snowline is located somewhat higher; an approximate estimate is an increase of $H$ by about 30 m per cm threshold (not elaborated here in detail).

The theoretical innovation of the present model is that the entire snowline information can be drawn from the state function of the snow duration. The state function $N(\tau)$ is a monotonous function of the mountain temperature; both $N$ and $\tau$ are gained from the entire set of available daily snow duration data of 207 stations covering the Alps over 4 decades, along with the annual mean winter temperature averaged over Europe. No information on station temperature $t$ is required. Yet the field of $\tau$ is in the end well correlated with $t$. This suggests that the duration of winter snow cover is largely controlled by temperature; it justifies a posteriori our model application of the freezing/thawing process to the snow duration. Further, the separation between the small-scale local and the continental-scale European impact upon the station temperature is made visible through the mountain temperature. This makes the mountain temperature an analysis instrument for the snow duration because it reveals the dependence of the snowline upon the European temperature.

We hope that the present approach proves to be sufficiently robust so that it can be applied to, and will be fruitful for, other mountain regions of the world.

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**References**


