Baryon generation in non-equilibrium electroweak phase transition

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Motivation

Chern-Simons and Winding Number in a Tachyonic Electroweak Transition

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Baryon asymmetry, from the theory of nucleosynthesis:

$$n_B/n_\gamma \simeq 6.5 \cdot 10^{-10}$$

Sakharov conditions (1967)

- Baryon number is non-conserved
- C and CP are non-conserved
- Non-equilibrium processes

Baryon number generation in SM

Classically divergence free currents get divergences after quantization

Adler-Bell-Jackiw (1969), Fujikawa (1979) $\Psi \to e^{i(a+b\gamma_5)\theta(x)}\Psi$

in the Standard Model:

- left handed: $SU(2)_L \times U(1)_Y$
- right handed: $U(1)_Y$

Baryon current: $J_\mu^B = \frac{1}{3}\overline{u}\gamma_\mu u + \frac{1}{3}\overline{d}\gamma_\mu d$ from anomaly:

$$\partial_\mu J_\mu^B = \frac{N_F}{32}\pi^2 \left(-g_2^2 F_{\mu\nu}^a \tilde{F}^{a\mu\nu} + g_1^2 f_{\mu\nu}\tilde{f}^{\mu\nu}\right)$$

where $F^{a\mu\nu}$ is the $SU(2)_L$ field strength and $f^{\mu\nu}$ is the $U(1)_Y$ field strength
Leptons: same contribution $\to B - L$ is conserved

$$\partial_\mu J_\mu^B = \frac{N_F}{32\pi^2}\left(-g_2^2 \partial_\mu K^\mu + g_1^2 \partial_\mu k_\mu\right)$$

where K_μ is the Chern-Simons current. $\Delta B = 3\Delta N_{CS}$

Chern-Simons Number and Winding Number

$$N_{CS} = -\frac{g_2^2}{16\pi^2}\int d^3x 2\epsilon^{ijk}\mathrm{Tr}\left(\partial_i A_j A_k + i\frac{2}{3}g_2 A_i A_j A_k\right) \notin \mathbf{N}$$

Gauge transformation: $A_i \to U A_i U^{-1} + \frac{i}{g_2}\partial_i U U^{-1}$

$$\delta N_{CS} = \frac{1}{24\pi^2}\int d^x \mathrm{Tr}\left[(\partial_i U)U^{-1}(\partial_j U)U^{-1}(\partial_k U)U^{-1}\right]\epsilon^{ijk}$$

Vacuum configurations with standard maps:

$$U^{(1)}(x) = \frac{x_0 + i x_i \sigma_i}{r} \qquad r = \sqrt{x_0^2 + x_i x_i} \qquad \delta N_{CS}\left(U^{(1)}\right) = 1$$

Classical Vacua of the broken phase:

$$G_{vac}^{(n)} = \left\{U = \left(U^{(1)}\right)^n, A_i = \frac{i}{g}\partial_i U U^{-1}, \Phi = U\cdot(0,v), N_{CS} = n\right\}$$

Sphaleron

Configurations betwwen two different vacua: Sphaleron

Klinkhammer, Manton (1984); Forgács, Horváth (1984)

$$A_i = \frac{i}{g} f(gvr) U^\infty \partial_i U^\infty \qquad U^\infty = U^{(1)}(x_0 = 0, x_i)$$

$$\Phi = \frac{i}{\sqrt{2}} h(gvr) U^\infty \cdot (0, v) \qquad N_{CS}(\text{Sphaleron}) = \frac{1}{2}$$

Dublett Higgs:

$$\Phi = \begin{pmatrix} \varphi_d^* & \varphi_u \\ -\varphi_u^* & \varphi_d \end{pmatrix} = \frac{\rho}{\sqrt{2}} U, \quad \rho^2 = 2(\varphi_u^* \varphi_u + \varphi_d^* \varphi_d), \quad U(x) \in \text{SU(2)}$$

If $\rho \neq 0$, the Higgs winding number is defined:

$$N_W = \int d^3 x n_w \qquad n_W = \frac{1}{24\pi^2} \epsilon^{ijk} \text{Tr} \left[(\partial_i U) U^{-1} (\partial_j U) U^{-1} (\partial_k U) U^{-1} \right]$$

In vacuum configurations $N_W = N_{CS}$ integer. $N_W - N_{CS}$ is gauge invariant

Baryon generation in SM

Cosmology: Inflation $\rightarrow$ initial asymmetry is washed away

after reheating the Universe cools $\rightarrow$ electroweak phase transition

If $m_H > 67$ GeV, electroweak phase transition = crossover
$\rightarrow$ not non-equilibrium enough

Csikor, Fodor, Heitger (1999)

First choice: extension of the SM

second choice: Inflation is not a GUT scale process
ends with the electroweak phase transition

Krauss, Trodden; Garcia-Bellido et al. (1999)

preheating in this case= tachyonic instability $\rightarrow$ large occupation numbers, classical approximation is valid

small momentum modes have large effective T
$\rightarrow$ Chern-Simons number changes frequently

$T_r < T_c \rightarrow$ generated baryon number is not washed away

Hybrid Inflation

Higgs: Φ symmetry breaking potential
Inflaton: Ψ mass term only
biquadtratic coupling

Linde (1993)

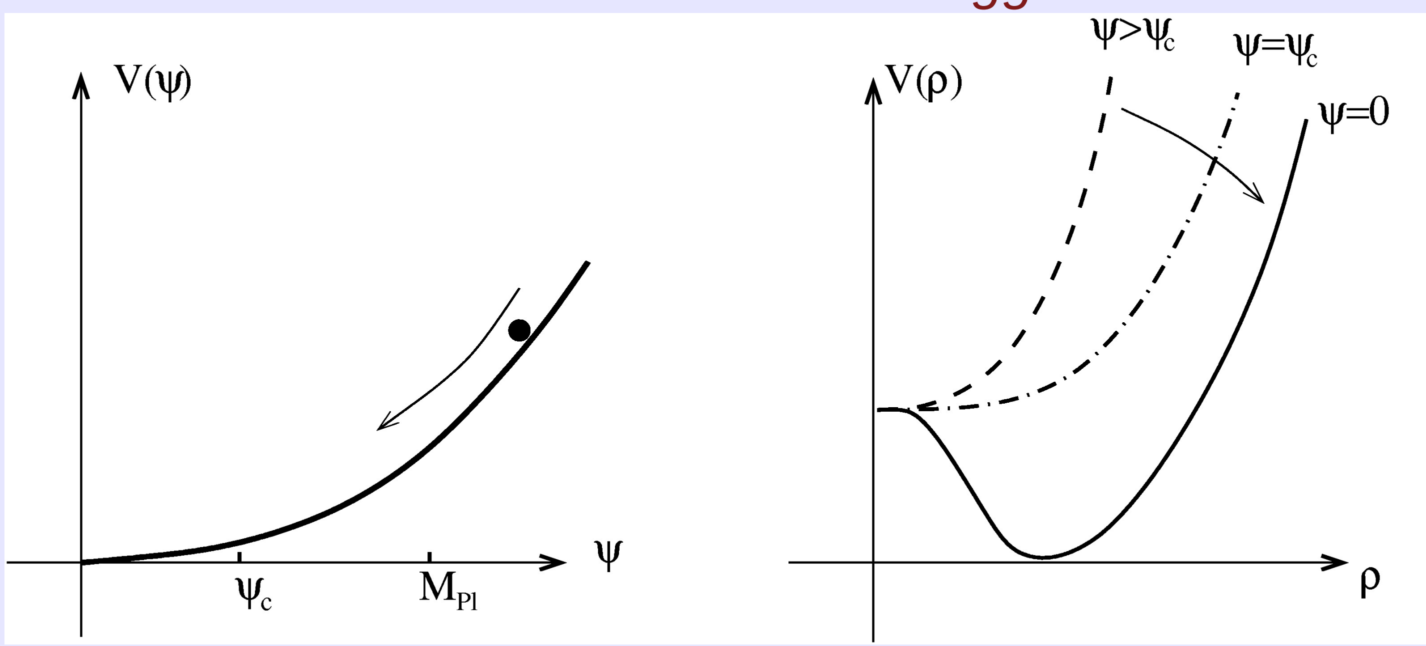
V_0 ensures zero cosmological constant after transition

$$V = V_0 + \frac{1}{2}m_\Psi^2\Psi^2 + \frac{1}{2}(\mu^2 + g^2\Psi^2)|\Phi|^2 + \frac{\lambda}{24}|\Phi|^4$$

After slow roll inflaton crosses its critical value $\Psi_c = |\mu|/g$
$\rightarrow$ tachyonic instability

In electroweak preheating
quantum corrections
$\rightarrow$ inverted hybrid inflation
inflaton rolls away from the origin

Inflaton - Higgs



Questions to investigate

- What are the parameters of the model, what are the predictions?
  new particle: inflaton two zero spin scalar: $m_1 \approx 130\mathrm{GeV} \quad m_2 \approx 400\mathrm{GeV}$

  van Tent, Smit, Tranberg (04)

- How much CP violation is needed for the experimental vaule of the n_B/n_γ ratio?

  Tranberg, Smit(03)

- Is the N_{CS} generated in local processes?
  (sphaleron transition?)

  How is the zero temperatue sphaleron picture modified?
  (Non-equilibrium situation)

  To study this question one doesn't need the CP-violating term in the action.
  (which just shifts the average)

Methods of numerical investigations

Reheating: High occupation numbers in the low-k region
Classical approximation can be applied
compared with 2PI: Classical approximation is valid

Arrizabalaga, Smit, Tranberg (04)

Method: Solving classical EOM on a cubic space-time lattice.

Initial Conditions: mimicking quantum ground state
Each mode with momentum k should have energy: $\epsilon_k = \frac{1}{2}\omega_k$

Gauge fields can have zero energy: excited via the source term in their EOM

High momentum Higgs fields should also have zero energy:
No renormalization necessary, $V_{pot} \gg \epsilon_{noise}$

$\rightarrow$ Only fill scalar modes with $k^2 < m^2$

Problem with Gauss constraint and $Q_{total} = 0$
Construct initial scalar fields with Monte Carlo sampling
Assign E_L to satisfy Gauss constraint.

Modelling the mass switch done by the inflaton field:

Quench: At $t = 0$ $\qquad m^2_{Higgs} \rightarrow -m^2_{Higgs}$

Simulations in 3D

Effective CP-violating term coming from integrating out (heavy) fermions

$$\frac{3\delta_{CP}}{16\pi^2 M^2}\Phi^+\Phi \mathrm{Tr}(F^{\mu\nu}\tilde{F}_{\mu\nu})$$

Ensemble average over a number of runs:
measure the resulting

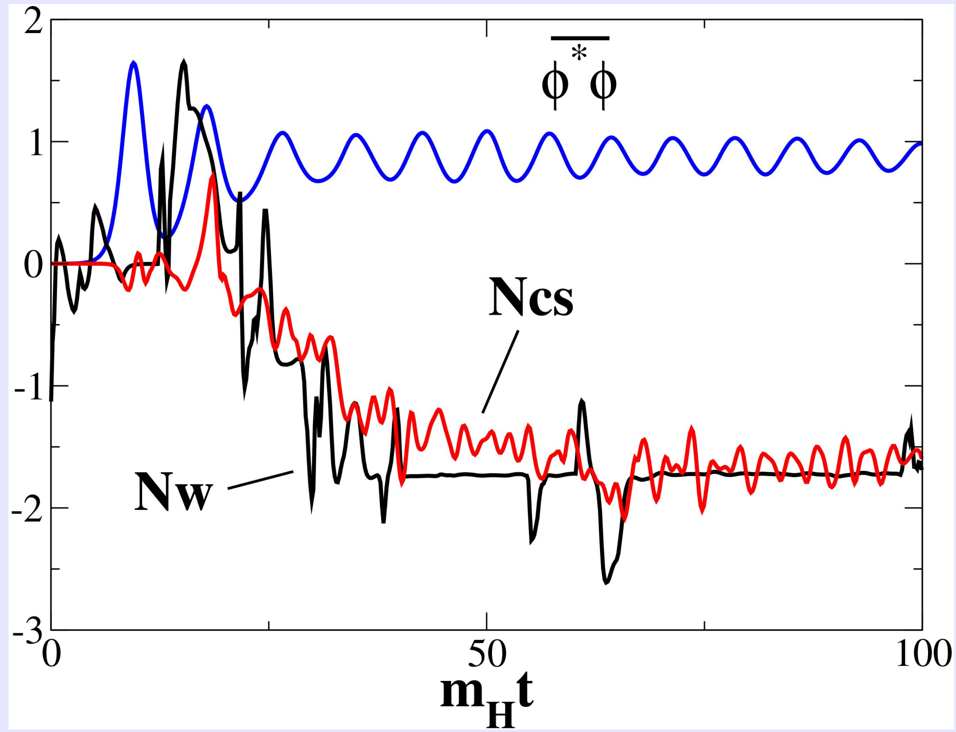
$$<\Delta N_{CS}>=\frac{1}{3}\Delta N_B$$

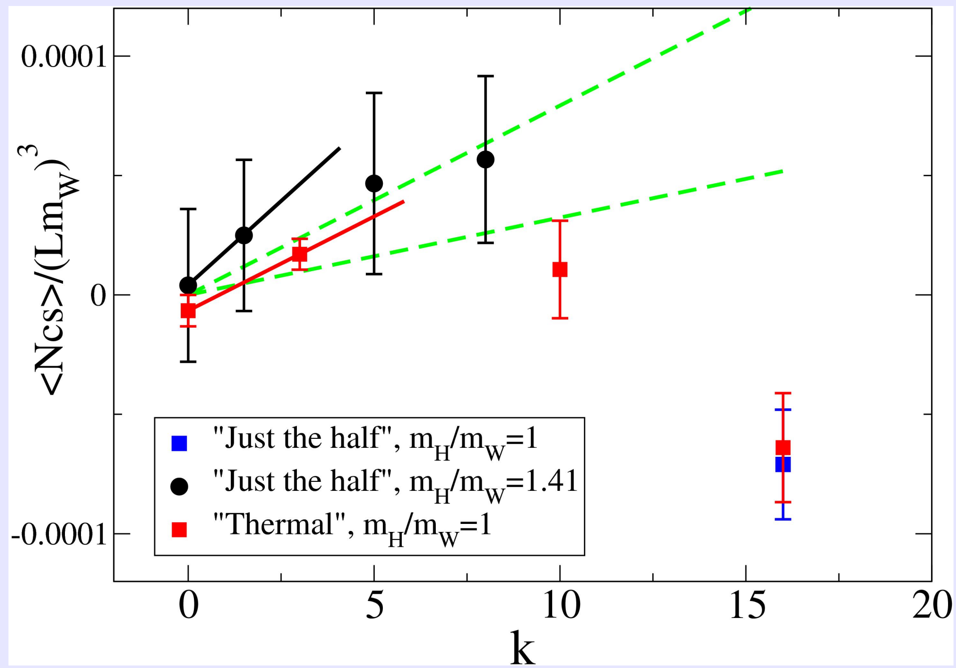
in function of δ_{CP}

Using $M = m_W$, required δ_{CP} for observed baryon asymmetry

$$\delta_{CP}\approx 0.7\times 10^{-5}$$

Tranberg, Smit (03)





Half-knot in 1 dimension

Integrated to the whole space winding number should be integer
locally there is a big density if $|\Phi|$ is small
for typical configurations $\approx 1/2$ in 1 dimension:

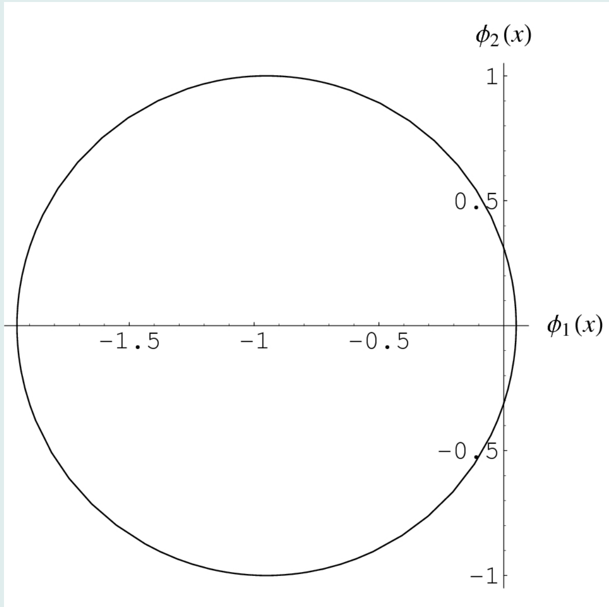
$$\Phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) = \frac{\rho}{\sqrt{2}}\,\Omega, \quad \Omega \in U(1).$$

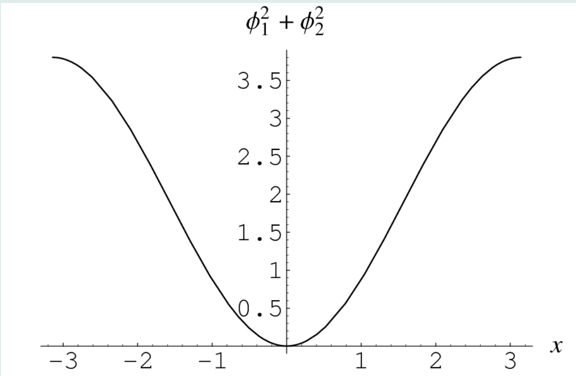
winding density: ($x \equiv x^1$): $n_W = -\frac{i}{2\pi}\,\Omega^*\partial_x\Omega = \frac{1}{2\pi\rho^2}\,(\phi_1\partial_x\phi_2 - \phi_2\partial_x\phi_1)$
e.g. : $\phi_1(x) = cos(x) - 0.95\phi_2 = sin(x)$

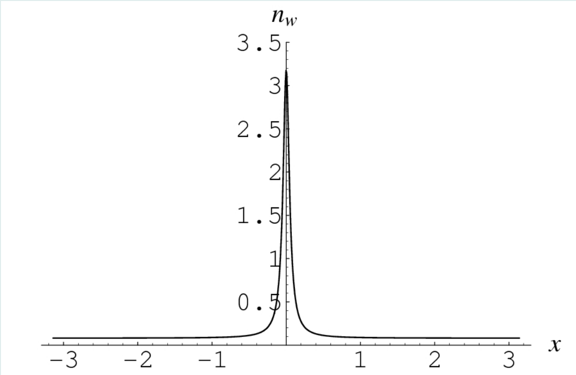
formalized "half-knot": with linear approximation

$$\phi_\alpha = c_\alpha + d_\alpha x, \quad \alpha = 1, 2.$$

circle $\rightarrow$ straight line
contribution to winding number:
$N_W^{\text{peak}} \equiv \int_{-\infty}^{\infty} dx\, n_W =$
$= \frac{1}{2}\,\text{sgn}(c_1 d_2 - c_2 d_1) = \pm\frac{1}{2}$







Half-knot in 3 dimensions

Parametrization with real fields:

$$\Phi = \frac{1}{\sqrt{2}} \left(\phi_4 1 + i\phi_a \tau^a \right),$$

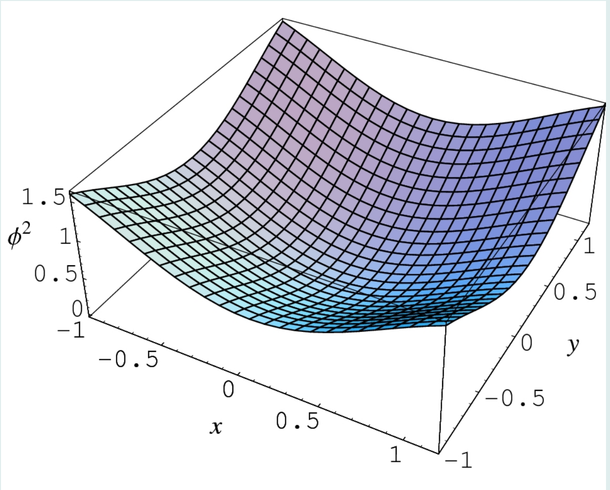
Configuration with waves:

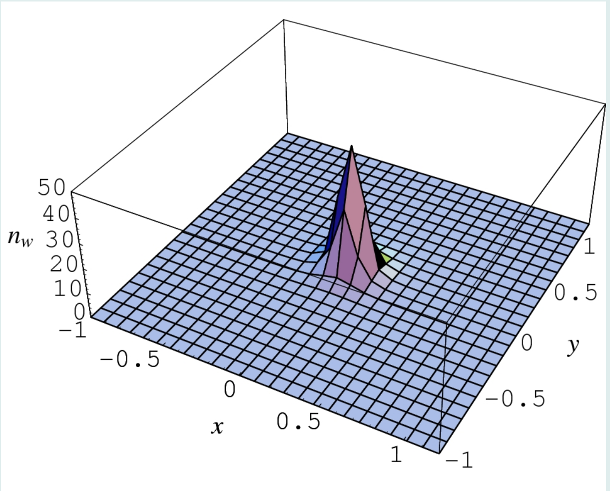
$$\phi_\alpha(x) = \sin(\mathbf{x} \cdot \mathbf{k}_\alpha - \epsilon_\alpha), \quad \alpha = 1, \ldots, 4.$$

The Higgs is small arounf the origin if $\epsilon_\alpha \ll 1$ formalized with linear approximation:

$$\phi_\alpha(\mathbf{x}) = c_\alpha + d_{\alpha k} x^k \qquad n_{\mathrm{W}} = \frac{1}{2\pi^2 \rho^4} \det M$$

where M is a 4×4 matrix of the $d_{\alpha 1}$, $d_{\alpha 2}$, $d_{\alpha 3}$, c_α vectors





$$N_W = 0.43$$

$$N_W = \int d^3x n_W = \frac{1}{2} \operatorname{sgn} \det M = \pm \frac{1}{2}$$

The sign might change if $|\Phi| = 0$ in a point (“goes to the other side”)

Half-knots in tachyonic instability

Parameters:

$$S = -\int d^4x \left[\frac{1}{2g^2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \mathrm{Tr} \left[(D_\mu \Phi)^\dagger D^\mu \Phi \right] + \lambda \left(\frac{1}{2} \mathrm{Tr} \left[\Phi^\dagger \Phi \right] - \frac{v^2}{2} \right)^2 \right]$$

$$g^2 = 4/9, \qquad \lambda/g^2 = 1/4 \rightarrow m_H/m_W = \sqrt{2} \qquad a m_H = 0.35 \qquad N^3 = 60^3$$

Number of zeros of the Higgs: $\sim k_{max}^3$, modes with smaller k grow faster:$\rightarrow$ Number of zeros decreases

early half-knots gauge fields are excited after formation, N_{CS} approaches N_W.
late half-knots appeares where the Higgs-field gets small

CP-violation can act on the late half-knot

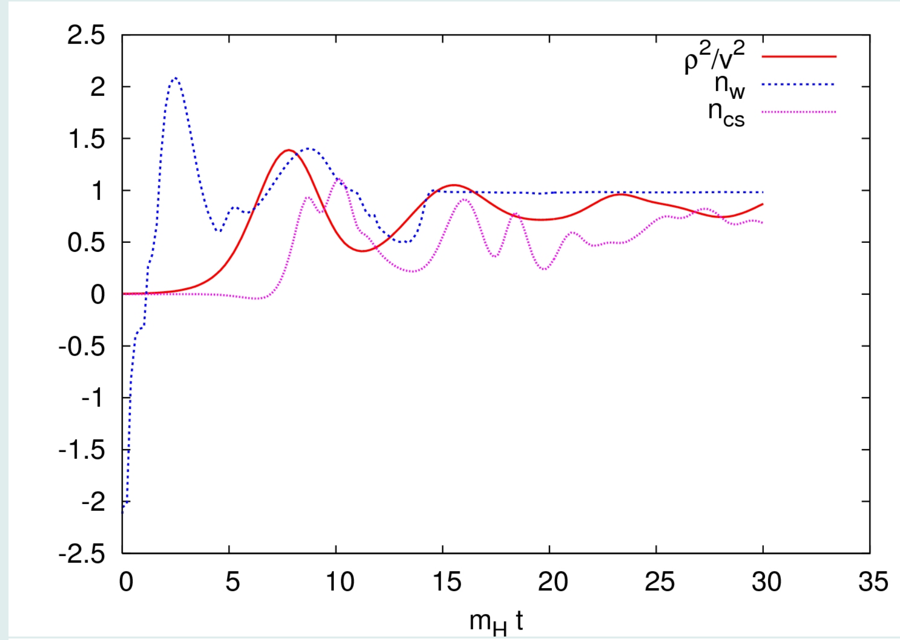
Typical trajectories

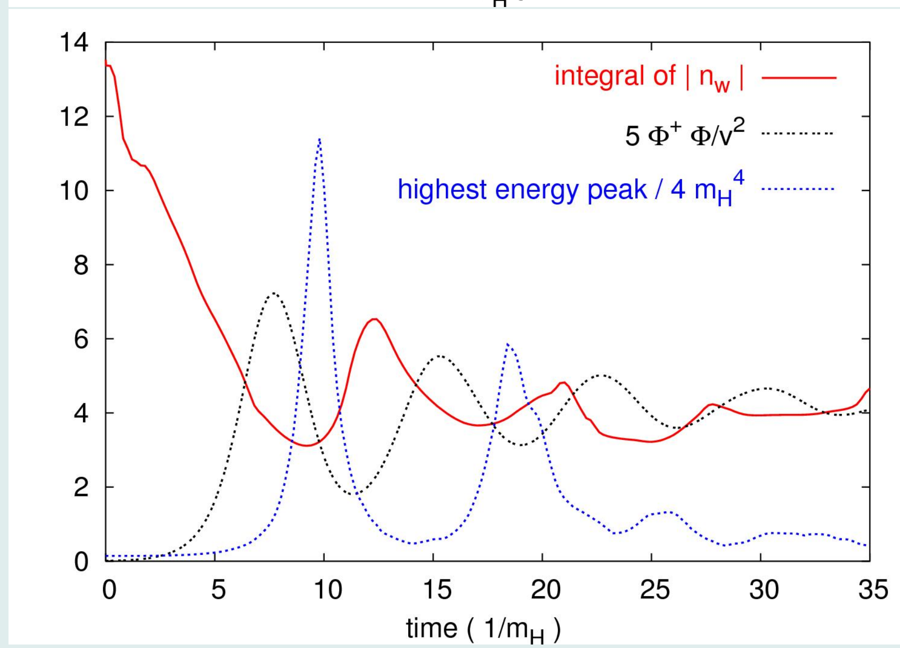
discretisation errors $\rightarrow$ winding number is not integer initially
at $m_H t \approx 50$: $N_{CS} \approx N_W$

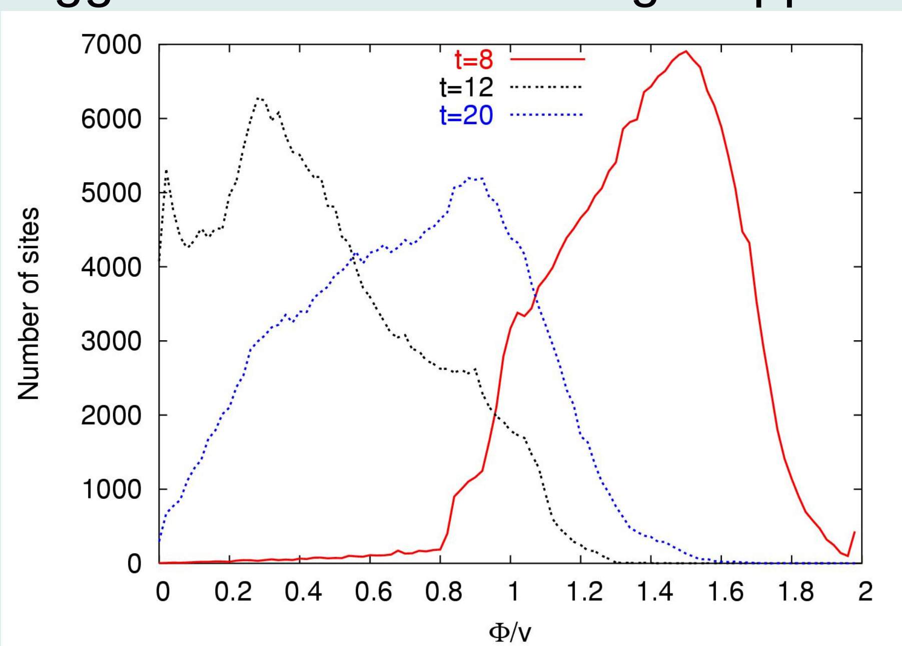
$\int |n_W| d^3x \sim$ number of configurations

peaks in energy density:
$t = 10, 18, 26$
$\rightarrow$ local process

Generations: At each roll-back of the Higgs field new “blobs” might appear







Early half-knot

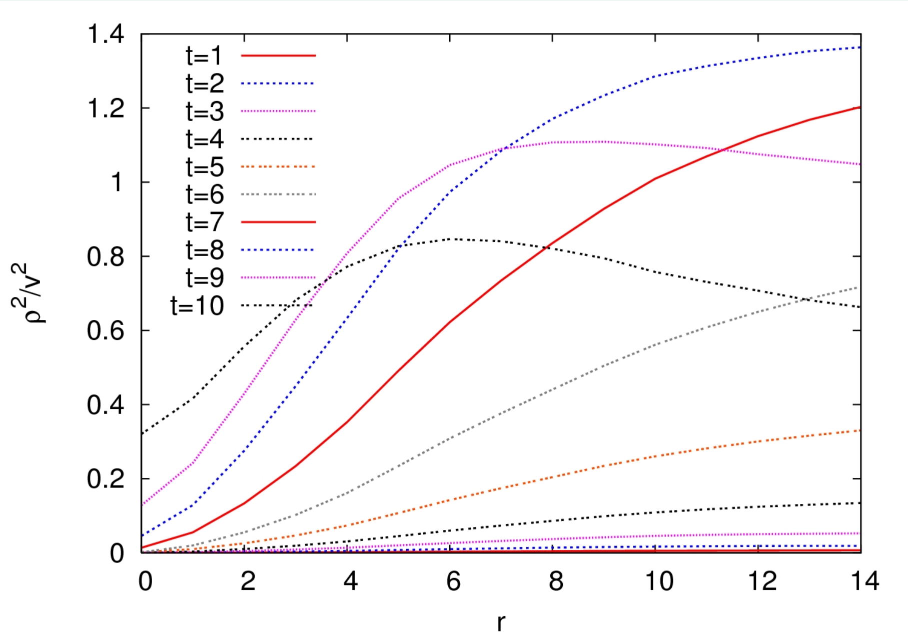
Quantities integrated in a sphere

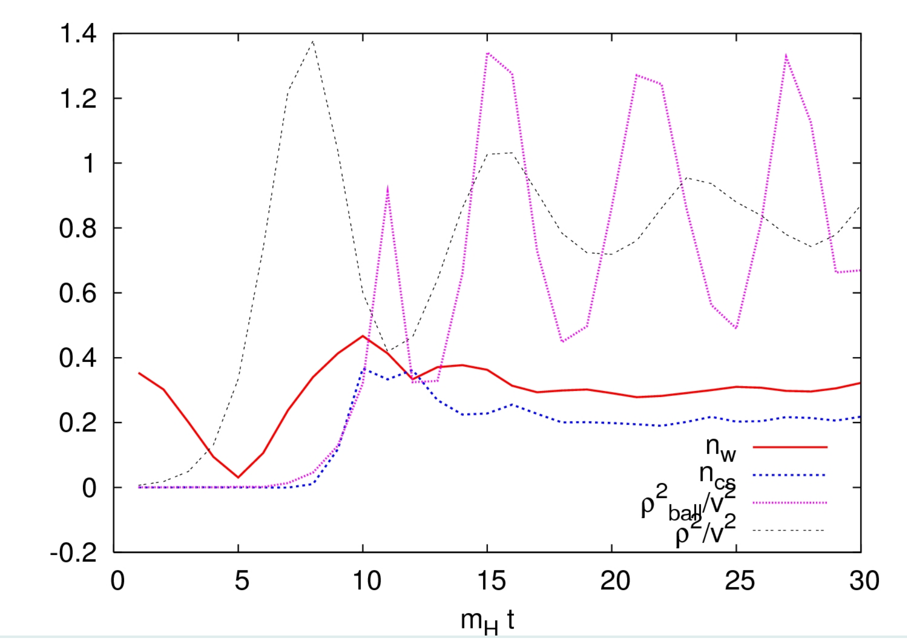
$$N_W^{\rm ball} = \int_{\rm ball} d^3x \, n_W$$

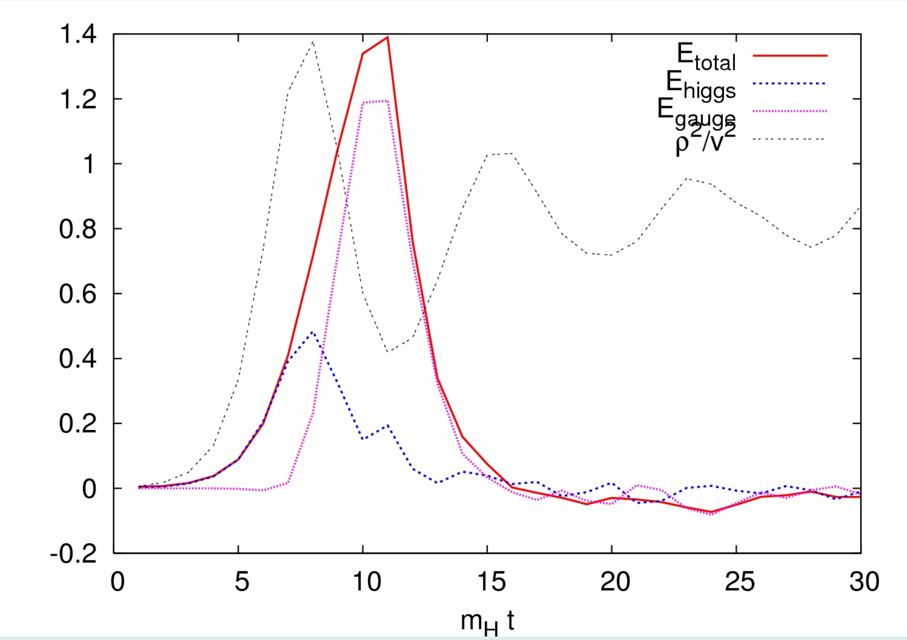
Center is located by the peaks in winding density

ρ^2 starts to grow later, shows little damping $\longrightarrow$ oscillon?

Energy in the sphere $\sim$ Sphaleron energy



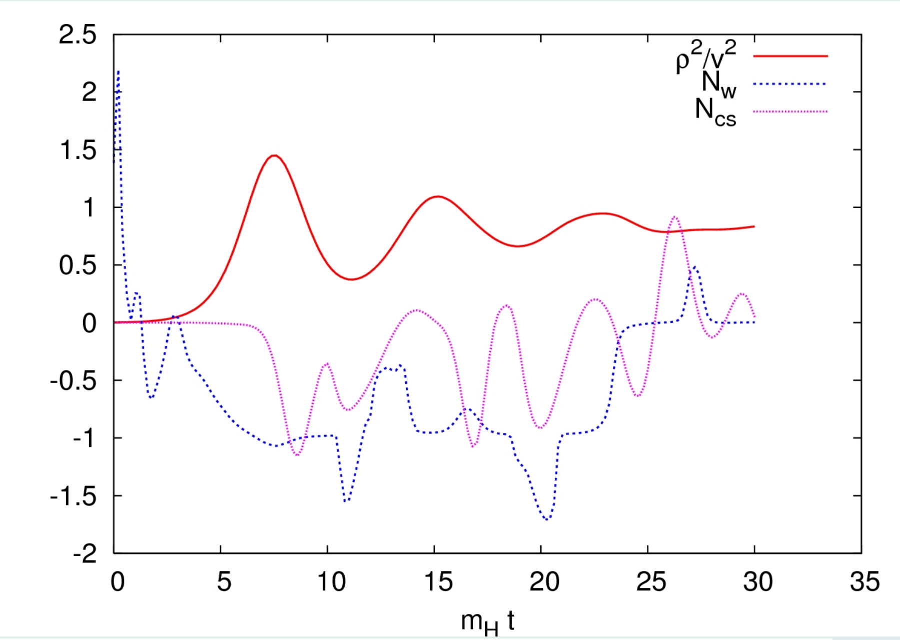




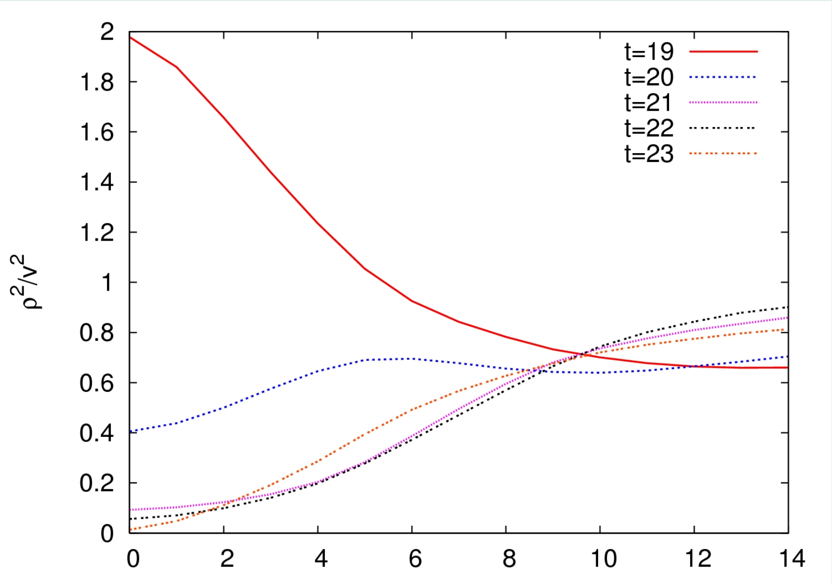
Late transition

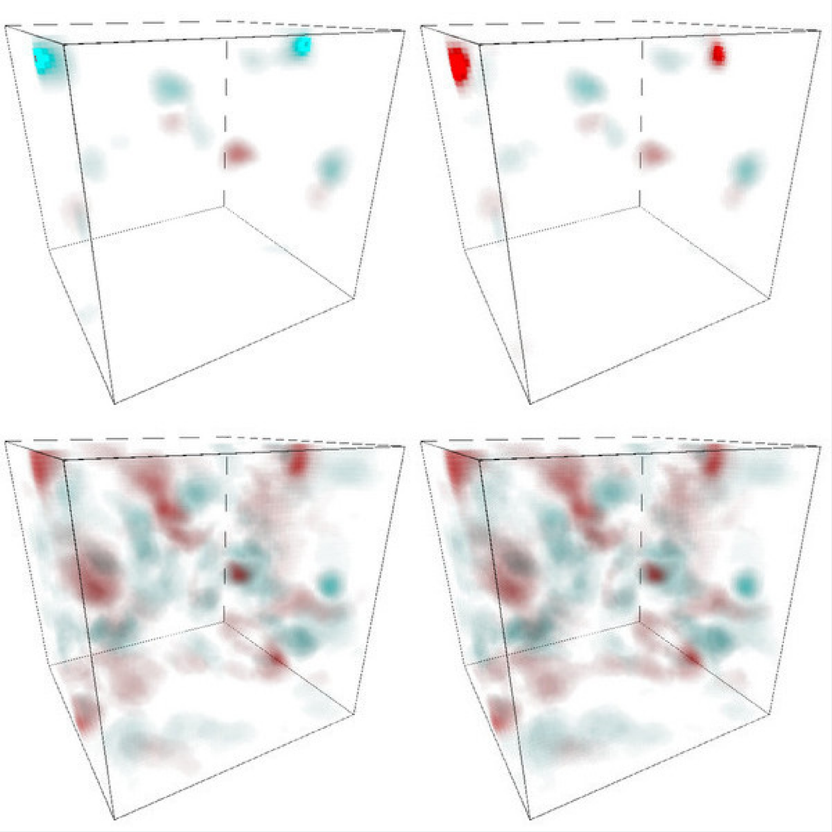
If the Higgs is small in the center of a half-knot $\rightarrow N_W$ might change sign

between $t = 23$ and $t = 24$ the half-knot changes sign
while $N_W: \quad -1 \rightarrow 0$



ρ^2 profile:



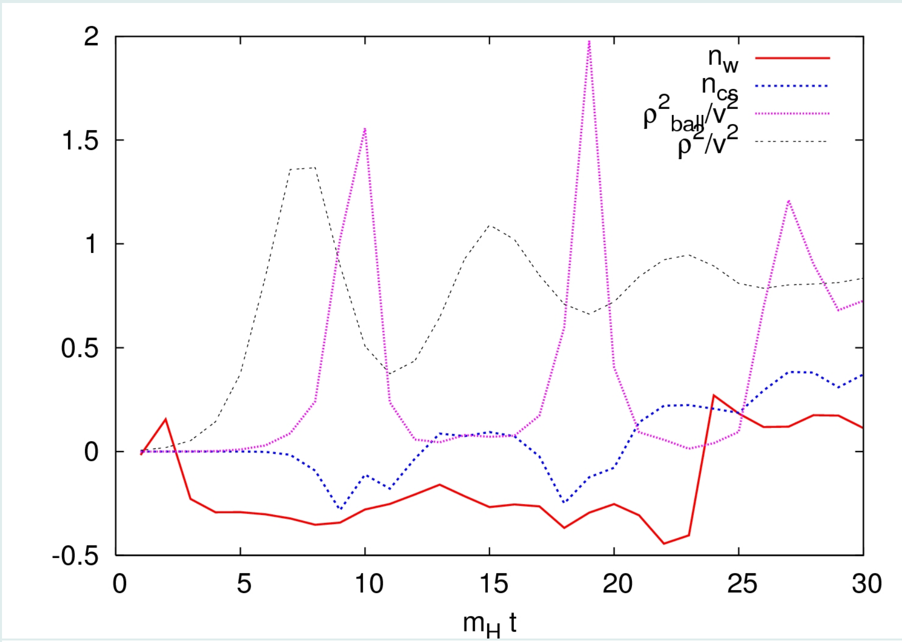


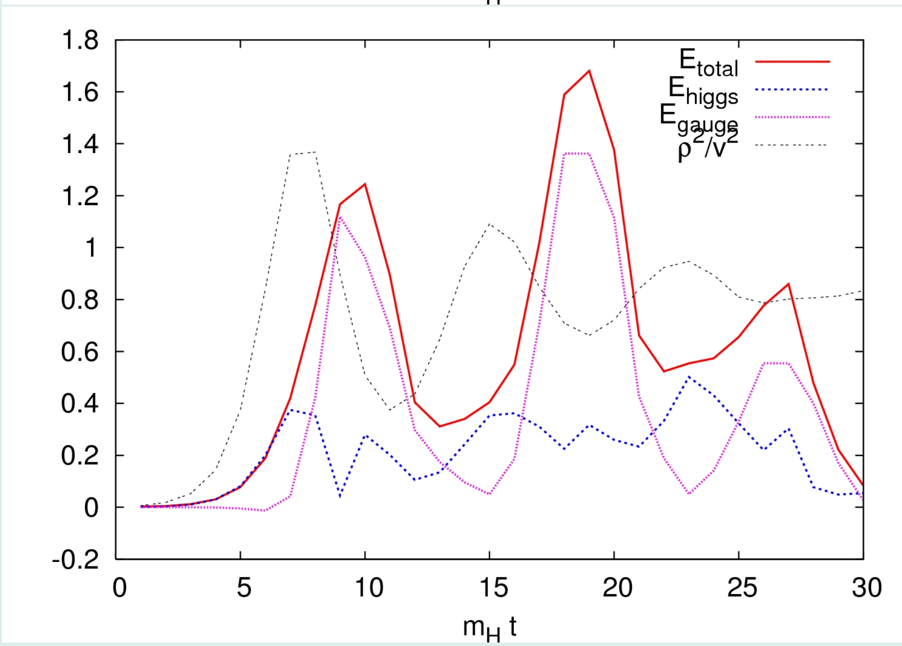
Late transition

quantities integrated in a sphere

at $t = 23$ sign changes
N_{CS} varies slowly

Sphaleron transition:
N_{CS} grows $O(1)$
N_W jumps $O(1)$
Higgs field has a zero in the center
energy around the defect $O(E_{sph})$





Distribution of the winding number

Independent defects, in a big enough volume: Poisson distribution

$$\text{probability of } n \text{ defects} \qquad P_n = \frac{\overline{n}^n}{n!} e^{-\overline{n}}$$

+1 winding number with probability: a ($a = 1/2$ with no CP-violation)

$$N_w = k - (n - k) \qquad k \text{ is the number of positive defects}$$

Probability of arriving at a fixed N_w:

$$P_{N_w} = \sum_{n=|N_w|}^{\infty} \binom{n}{k} a^k (1-a)^{n-k} P_n \qquad k = \frac{N_w + n}{2}$$

Summing gives Bessel function of the first kind:

$$P_{N_w} = I_{N_w}\left(2\overline{n}\sqrt{a(1-a)}\right)(a(1-a))^{-N_w/2} a^{N_w} \exp(-\overline{n})$$

$$\text{No CP-violation} \rightarrow P_N = I_N(\overline{n}) \exp(-\overline{n})$$

Distribution of the winding number

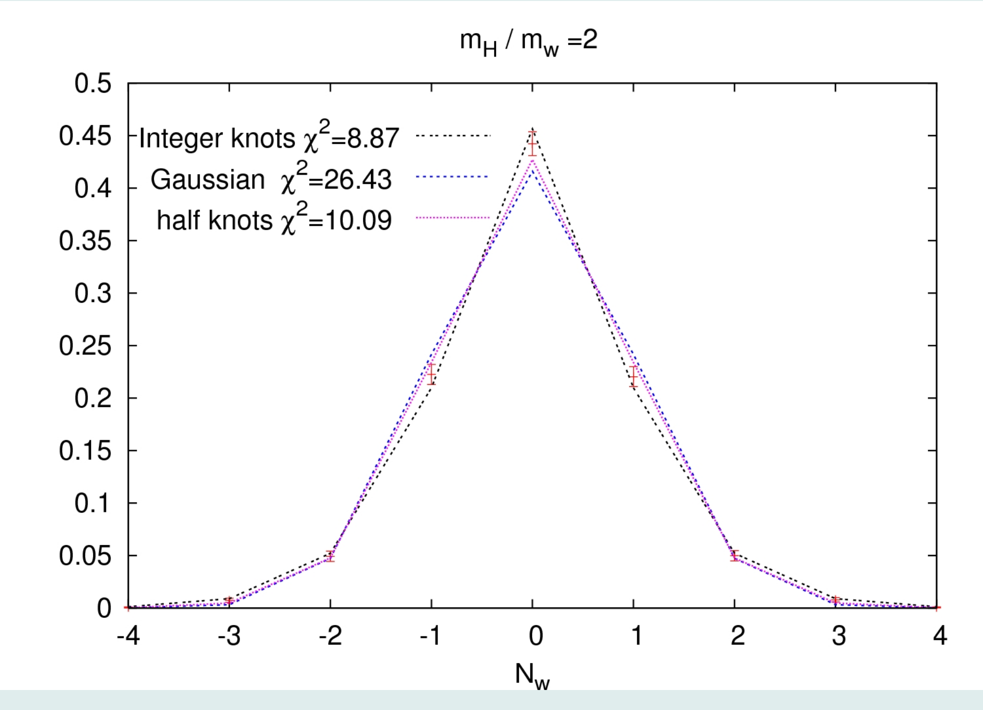
With half-knot: each defect has $N_W = \frac{1}{2}$
odd number of defects: $\pm\frac{1}{2}$ with equal probability

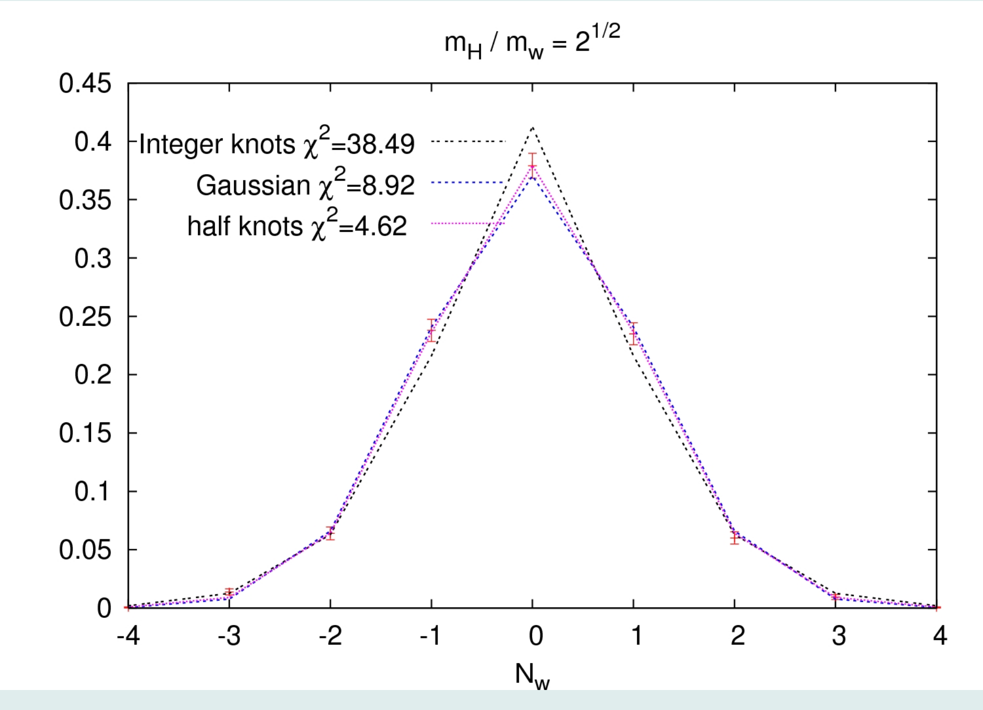
$$P_N^{(1/2)}(r) = e^{-\overline{n}} \left[I_{2N}(\overline{n}) + \frac{1}{2} I_{2N+1}(\overline{n}) + \frac{1}{2} I_{2N-1}(\overline{n}) \right]$$

If we restrict to even number of defects:

$$P_N'^{(1/2)}(r) = I_{2N}(r) / \cosh(r)$$

Two distribution gives similar results:





Conclusions

- In $SU(2)$ tachyonic preheating the Chern-Simons number change can be described with local objects: Half-knots
- These local objects have typically $\pm\frac{1}{2}$ winding number
- Sphaleron-like transition between opposite sign half-knots
- The distribution of N_W supports the idea of half-knots
- The defects are produced in “generations”