Perturbative QCD for the LHC

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IPPP, Durham University

Highlights in Computational Quantum Field Theory
5th Vienna Central European Seminar

“Particle Physics and Quantum Field Theory”

Vienna, 28–30 November 2008
Present Status of QCD

✓ Thanks to LEP, HERA and the TEVATRON
QCD now firmly established theory of strong interactions
✓ We have gained a lot of confidence in comparing theoretical predictions with experimental data
✓ No major areas of discrepancies
✓ Now prepared to enter a new era of precision physics for QCD
The running coupling in perturbative QCD

\[ \frac{d\alpha_s}{d \ln \mu^2} = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \beta_2 \alpha_s^4 - \beta_3 \alpha_s^5 - \ldots \]

Four-loop coeff.:

van Ritbergen, Vermaseren, Larin; Czakon

\[ \beta(\alpha_s) \quad n_f = 4, \overline{\text{MS}} \]

\[ \alpha_s(\mu^2) \]

3...5 flavours

\[ \alpha_s(M_Z^2) = 0.115 \]
Hard processes in perturbative QCD

Example: inclusive deep-inelastic scattering (DIS)

Kinematic variables

\[ Q^2 = -q^2 \]
\[ x = \frac{Q^2}{(2P \cdot q)} \]

Lowest order: \( x = \xi \)

Structure functions \( F_a \) [up to \( \mathcal{O}(1/Q^2) \)]

\[ F_a^p(x, Q^2) = \sum_i [c_{a,i}(\alpha_s(\mu^2), \mu^2/Q^2) \otimes f_i^p(\mu^2)](x) \]

Coefficient functions \( c_{a,i} \), renormalization/factorization scale \( \mu \)
Kinematics: parton momenta $\xi_- < \xi < 1$ probed

**HERA → LHC:**
$Q^2$ evolution across up to three orders of magnitude
Hard processes in perturbative QCD

Parton distributions $f_i$: evolution equations

$$
\frac{d}{d \ln \mu^2} f_i(\xi, \mu^2) = \sum_k \left[ P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2) \right](\xi)
$$

Initial conditions incalculable in pert. QCD.

Splitting functions $P$, Coefficient functions $c_a$

$$
P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots
$$

$$
c_a = \alpha_s^{n_a} \left[ c_a^{(0)} + \alpha_s c_a^{(1)} + \alpha_s^2 c_a^{(2)} + \ldots \right]
$$

**NLO**: standard approximation

**NNLO**: new emerging standard

Moch, Vermaseren, Vogt
Hard processes in perturbative QCD

\[
\sigma(Q^2) = \int \sum_{i,j} \left[ d\hat{\sigma}_{ij}(\alpha_s(\mu_R), \mu_R^2/Q^2, \mu_F^2/Q^2) \otimes f_i^p(\mu_F) \otimes f_j^p(\mu_F) \right]
\]

- partonic cross sections \( d\hat{\sigma}_{ij} \)
- running coupling \( \alpha_s(\mu_R) \)
- parton distributions \( f_i(x, \mu_F) \)

- renormalization/factorization scale \( \mu_R, \mu_F \)
- + parton shower + hadronisation model + underlying event + ...
The challenge

✓ Everything at the LHC (signals, backgrounds, luminosity measurement) involves QCD
✓ Strong coupling is not small: $\alpha_s(M_Z) \sim 0.12$ and running is important
  ⇒ events have high multiplicity of hard partons
  ⇒ each hard parton fragments into a cluster of collimated particles jet
  ⇒ higher order perturbative corrections can be large
  ⇒ theoretical uncertainties can be large
✓ Processes can involve multiple energy scales: e.g. $p_T^W$ and $M_W$
  ⇒ may need resummation of large logarithms
✓ Parton/hadron transition introduces further issues, but for suitable (infrared safe) observables these effects can be minimised
  ⇒ importance of infrared safe jet definition
  ⇒ accurate modelling of underlying event, hadronisation, ...
What is covered in this talk

Will focus on status of fixed order parton-level predictions
✓ Systematic to higher order/higher multiplicity in perturbation theory
✓ Appropriate for hard well separated final states
✓ Lead to a systematic reduction in renormalisation/factorisation scale uncertainties
✓ Many recent theoretical developments and new calculations/numerical programmes available

caveat Parton-level, relies on matching to experimental observables e.g. merging with parton showers and event generators, etc

CKKW, MLM, MCNLO, POWHEG

see talk by Seymour

✗ No time for many important topics;
✗ parton distributions
✗ soft gluon resummation
✗ studies of jet definitions; fast $k_T$ algorithm, infrared safe cone algorithms,...
Twin Goals:

1. Identification and study of New Physics
2. Precision measurements (e.g. $\alpha_s$, PDF’s) leading to improved theoretical predictions

- LO: backgrounds to new physics searches
- NLO: precision measurements of fundamental quantities ($\alpha_s, m_t, M_W$, new physics parameters)
- NNLO: determination of auxiliary observables, PDF’s
State of the Art - at a glance

<table>
<thead>
<tr>
<th>Relative Order</th>
<th>$2 \rightarrow 1$</th>
<th>$2 \rightarrow 2$</th>
<th>$2 \rightarrow 3$</th>
<th>$2 \rightarrow 4$</th>
<th>$2 \rightarrow 5$</th>
<th>$2 \rightarrow 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>LO</td>
<td>LO</td>
<td>LO</td>
<td>LO</td>
<td>LO</td>
<td>LO</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>NLO</td>
<td>NLO</td>
<td>NLO</td>
<td>NLO</td>
<td>NLO</td>
<td>NLO</td>
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<tr>
<td>$\alpha_s^2$</td>
<td></td>
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<tr>
<td>$\alpha_s^3$</td>
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<td></td>
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</tr>
<tr>
<td>$\alpha_s^4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_s^5$</td>
<td></td>
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</tr>
</tbody>
</table>

LO  Well under control, even for multiparticle final states
NLO Well understood for $2 \rightarrow 1$ and $2 \rightarrow 2$
NLO Many new $2 \rightarrow 3$ calculations
NLO Very first $2 \rightarrow 4$ LHC cross section this year
NNLO Recent breakthroughs for inclusive and exclusive $2 \rightarrow 1$
NNLO Recent landmark calculation of NNLO splitting functions

Moch, Vermaseren, Vogt

NNLO Still waiting for $2 \rightarrow 2$
Leading order

Many available programs for automatic evaluation of LO cross sections

✓ Feynman diagrams: matrix elements automatically generated up to $2 \to 6$
  MADGRAPH, COMPHEP, GRACE, ...

✓ Off-shell recursion relations:
  Berends, Giele; Caravaglios, Moretti
  matrix elements automatically generated up to $2 \to 8$ or more
  HELAC, AMEGIC++/COMIX, ALPHA, ...

✓ (Twistor inspired) On-shell recursion relations:
  Cachazo, Svrcek, Witten; Britto, Cachazo, Feng, Witten, AMEGIC++/COMIX; Dinsdale, Ternick, Weinzierl

✓ plus automatic integration over phase space
  HELAC/PHEGAS, MADGRAPH/MADEVENT, SHERPA/AMEGIC++, ALPHA/ALPGEN, ...

✓ very good for estimating importance of various processes in different models - properly populate phase space with multiple hard objects

✓ able to interface with parton showers CKKW in SHERPA, MLM in ALPGEN, ...
### Comparison of algorithms

- On-shell recursion relations (CSW, BCF) yield compact analytic results.
- Numerical implementations show that off-shell recursion (BG) is faster.

<table>
<thead>
<tr>
<th>Final state</th>
<th>BG</th>
<th>BCF</th>
<th>CSW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CO</td>
<td>CD</td>
<td>CO</td>
</tr>
<tr>
<td>2g</td>
<td>0.24</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>3g</td>
<td>0.45</td>
<td>0.48</td>
<td>0.42</td>
</tr>
<tr>
<td>4g</td>
<td>1.20</td>
<td>1.04</td>
<td>0.84</td>
</tr>
<tr>
<td>5g</td>
<td>3.78</td>
<td>2.69</td>
<td>2.59</td>
</tr>
<tr>
<td>6g</td>
<td>14.20</td>
<td>7.19</td>
<td>11.90</td>
</tr>
<tr>
<td>7g</td>
<td>58.50</td>
<td>23.70</td>
<td>73.60</td>
</tr>
<tr>
<td>8g</td>
<td>276</td>
<td>82.10</td>
<td>597</td>
</tr>
<tr>
<td>9g</td>
<td>1450</td>
<td>270</td>
<td>5900</td>
</tr>
<tr>
<td>10g</td>
<td>7960</td>
<td>864</td>
<td>64000</td>
</tr>
</tbody>
</table>

Duhr, Hoche, Maltoni
Example at LO

Multi-jet production at the LHC using HELAC/PHEGAS

Draggiotis, Kleiss, Papadopoloulos

<table>
<thead>
<tr>
<th># of jets</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td># of dist.processes</td>
<td>10</td>
<td>14</td>
<td>28</td>
<td>36</td>
<td>64</td>
<td>78</td>
<td>130</td>
</tr>
<tr>
<td>total # of processes</td>
<td>126</td>
<td>206</td>
<td>621</td>
<td>861</td>
<td>1862</td>
<td>2326</td>
<td>4342</td>
</tr>
<tr>
<td>(\sigma(nb))</td>
<td>-</td>
<td>91.41</td>
<td>6.54</td>
<td>0.458</td>
<td>0.030</td>
<td>0.0022</td>
<td>0.00021</td>
</tr>
<tr>
<td>% Gluonic</td>
<td>-</td>
<td>45.7</td>
<td>39.2</td>
<td>35.7</td>
<td>35.1</td>
<td>33.8</td>
<td>26.6</td>
</tr>
</tbody>
</table>

✓ For each final state, there are many distinct contributing processes e.g. \(gg \rightarrow gg, gg \rightarrow q\bar{q}, q\bar{q} \rightarrow gg, gg \rightarrow gg, q\bar{q} \rightarrow Q\bar{Q}, qQ \rightarrow qQ\) etc

✓ Assigning different quark flavours gives even more

✓ Bookkeeping, phase space generation and evaluation done automatically

✓ ALPGEN and SHERPA are also very fast for multiparticle SM processes

✓ MADGRAPH slower, but adapted for other models, effective H, MSSM, 2HDM, ...
Limitations of LO

Very large uncertainty for multiparticle final states

scale uncertainty on $\alpha_s^2$

✓ New channels open up at higher orders $qq$ + large gluon PDF
✓ Increased phase space
✓ Large $\pi^2$ coefficients in $s$-channel $\Rightarrow$ large NLO corrections 30% - 100%

parton luminosity uncertainty
$W + \text{Jets at CDF Run II with 320 pb}^{-1}$

CDF Run II Preliminary

$\frac{d\sigma}{dE} \text{[pb/GeV]}$

0 50 100 150 200 250 300 350

$\text{Jet Transverse Energy [GeV]}$

$\int dL = 320 \text{ pb}^{-1}$

$W \rightarrow e\nu \text{ events}$

$W_\text{kin}: E_T^e \geq 20 \text{[GeV]}$, $|\eta| \leq 1.1$

$W_{had}: E_T^W \geq 30 \text{[GeV]}$

$\eta$: JetClu $R=0.4$, $|\eta| < 2.0$

$\nu_\text{hadron level; no UE correction}$

ALPGEN+PYTHIA

Total $\sigma$ normalized to Data

Cross sections for the leading jet in $W + \geq 1$ jet events, second jet in $W + \geq 2$ jets events, etc

✓ ALPGEN+PYTHIA merged LO+PS prediction normalised to the inclusive cross section for each jet multiplicity

✓ Excellent qualitative agreement
Anatomy of a NLO calculation

✓ one-loop $2 \rightarrow 3$ process
looks like 3 jets in final state

✓ tree-level $2 \rightarrow 4$ process
looks like 3 or 4 jets in final state

✓ plus method for combining the infrared divergent parts - dipole subtraction

Catani, Seymour; Dittmaier, Trocsanyi, Weinzierl, Phaf

✓ automated dipole subtraction

Gleisberg, Krauss (SHERPA); Hasegawa, Moch, Uwer; Frederix, Gehrmann,
Greiner (MadDipole); Seymour, Tevlin – see talk by Hasegawa

Bottleneck: one-loop matrix elements
Availability of NLO calculations

✓ 2 → 2 processes
✓ parton level integrators available for all 2 → 2 Standard Model and MSSM processes for some time
✓ extensively used at LEP, TEVATRON and HERA
EVENT, JETRAD, MCFM, DISENT, DIPHOX, HQQB, NLOJET++, VBFNLO etc
✓ can be matched with parton shower MC@NLO, POWHEG – Frixione, Webber; Nason, Oleari, Ridolfi; Krämer, Soper see talk by Seymour

✓ 2 → 3 processes
✓ many 2 → 3 processes now available at NLO
  e.g. backgrounds pp → 3 jets, V + 2 jets, γγ + jet, V + b¯b, VV + jet, t¯tt + jet
  as well as signals pp → t¯ttH, b¯bbH, H + 2 jets, HHH, t¯tt+jet

http://www.cedar.ac.uk/hepcode

⇒ First 2 → 4 LHC cross section recently computed
Inclusive Jet Production using the Kt Algorithm

Single jet inclusive differential cross section in different rapidity slices

- Described by NLO QCD
- Excellent quantitative agreement → CDF Run I $\alpha_s$ measurement
### LHC priority NLO wish list, Les Houches 2005/7

<table>
<thead>
<tr>
<th>process</th>
<th>background</th>
<th>status</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pp \to VV + 1\text{ jet}$</td>
<td>$WBF \ H \to VV$</td>
<td>$WWj \ (07)$</td>
</tr>
<tr>
<td>$pp \to t\bar{t} + b\bar{b}$</td>
<td>$t\bar{t}H$</td>
<td>$q\bar{q} \to t\bar{t}b\bar{b} \ (08)$</td>
</tr>
<tr>
<td>$pp \to t\bar{t} + 2\text{ jets}$</td>
<td>$t\bar{t}H$</td>
<td>$t\bar{t}j \ (07)$</td>
</tr>
<tr>
<td>$pp \to VV + b\bar{b}$</td>
<td>$WBF \ H \to VV, t\bar{t}H, \text{ NP}$</td>
<td>$WBF \ pp \to VVjj \ (07)$</td>
</tr>
<tr>
<td>$pp \to VV + 2\text{ jets}$</td>
<td>$WBF \ H \to VV$</td>
<td></td>
</tr>
<tr>
<td>$pp \to V + 3\text{ jets}$</td>
<td>$\text{NP}$</td>
<td></td>
</tr>
<tr>
<td>$pp \to VVV$</td>
<td>$\text{SUSY trilepton}$</td>
<td>$ZZZ \ (07), WWZ \ (07), WW\ W \ (08), ZZZW \ (08)$</td>
</tr>
<tr>
<td>$pp \to b\bar{b}b\bar{b}$</td>
<td>$\text{Higgs and NP}$</td>
<td></td>
</tr>
</tbody>
</table>

- **✓** $pp \to H + 2\text{ jets}$ via gluon fusion
  - Campbell, Ellis, Zanderighi, hep-ph/0608194

- **✓** $pp \to H + 2\text{ jets}$ via WBF, electroweak and QCD corrections
  - Ciccolini, Denner, Dittmaier, arXiv/0710.4749

- **✓** $pp \to H + 3\text{ jets}$ via WBF,
  - Figy, Hankele, Zeppenfeld, arXiv/0710.5621

- **✓** ............

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Perturbative QCD for the LHC – p.20
**Vector boson pair plus jet**

**QCD corrections to** $pp \to W^+W^- j + X$

Dittmaier, Kallweit, Uwer, arXiv/0710.1577; Campbell, Ellis, Zanderighi, arXiv/0710.1832

✓ Background to Higgs in both WBF, GF channels - $H \to W^+W^-$ with one jet missed, or Higgs recoiling against jet

✓ For inclusive cuts, NLO increases cross section by about 25%

✓ Factorisation scale uncertainty small, renormalisation scale uncertainty reduced by $\sim 50\%$

✓ Shapes of NLO inclusive distributions very similar to LO

✓ For WBF cuts, with one or both jets forward, $WWj$ is one of dominant backgrounds NLO increased by $\sim 70\%$ cf LO

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**Cuts I**

![Graph](image-url)

- LO $\mu_R=\mu, \mu_F=80$ GeV
- LO $\mu_F=\mu, \mu_R=80$ GeV
- NLO $\mu_R=\mu, \mu_F=80$ GeV
- NLO $\mu_F=\mu, \mu_R=80$ GeV

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Perturbative QCD for the LHC – p.21
Top pair plus jet

QCD corrections to $pp \to t\bar{t}j + X$

- Background to Higgs in WBF, $t\bar{t}H$ channels
- Measurement of $t$ properties

Residual scale dependence reduced

NLO corrections essentially eliminate forward-backward charge asymmetry at Tevatron
QCD corrections to $pp \rightarrow ZZZ + X$

Lazopoulos, Melnikov, Petriello, hep-ph/0703273

- Background to various SUSY tri-lepton signatures, gauge boson coupling measurements,

- Large, 50% corrections not seen by LO scale variation! 15% shift from pdfs, 35% shift from $\pi^2$ terms
Top pair plus Z Production

QCD corrections to $pp \rightarrow t\bar{t}Z + X$

Lazopoulos, Melnikov, Petriello, arXiv/0709.4044,
Lazopoulos, McElmurry, Melnikov, Petriello, arXiv/0804.2220

- Background to various SUSY tri-lepton signatures, gauge boson coupling measurements,
- Fully numerical calculation - using sector decomposition and contour deformation
- For reasonable choices of $\mu$, corrections as large as 75%
Top pair plus bottom pair Production

QCD corrections to $q\bar{q} \rightarrow t\bar{t}b\bar{b} + X$

Bredenstein, Denner, Dittmaier, Pozzorini, arXiv/0807.1248

✓ Background to the Higgs signal in $t\bar{t}H$ production where the Higgs decays into a bottom pair

✓ First successful demonstration of Feynman diagrammatic evaluation of $2 \rightarrow 4$ process at LHC

✓ Dominant $gg \rightarrow t\bar{t}b\bar{b} + X$ process underway
The one-loop problem

Any one-loop integral can be written as

\[ M = \sum d(D) \text{boxes}(D) + \sum c(D) \text{triangles}(D) + \sum b(D) \text{bubbles}(D) + \sum a(D) \text{tadpoles}(D) \]

✓ most of the scalar loop integrals boxes etc are known analytically around \( D = 4 \)

✓ only problem is to compute the \( D \)-dimensional coefficients \( a(D) \) etc.

The only problem is complexity - the number of terms generated is too large to deal with, even with computer algebra systems, and there can be very large cancellations.
Lots of ideas and strategies

✓ Improved tensor reduction: Denner, Dittmaier; Binoth, Guillet, Heinrich, Pilon, Reiter, Schubert, ... – see talk by Reiter

✓ Numerical evaluation of integral recursion relations Giele, Ellis, Zanderighi

✓ 4-d Unitarity and cut constructibility Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng; ...

✓ D-dimensional unitarity Anastasiou, Britto, Cachazo, Feng, Kunszt, Mastrolia

✓ Numerical loop integration: accuracy only has to match real emission contribution Nagy, Soper, hep-ph/0610028

Sector decomposition plus contour deformation automated by Anastasiou, Beerli, Daleo, hep-ph/0703282

✓ Algebraic reduction of the integrand

Ossola, Papadopoulos, Pittau, hep-ph/0609007; Ellis, Giele, Kunszt, arXiv/0708.2398

✓ ........


hep-ph/0704.1271
One-loop six gluon amplitude

- Analytic computation
  Bedford, Berger, Bern, Bidder, Bjerrum-Bohr, Brandhuber, Britto, Buchbinder, Cachazo, Dixon, Dunbar, Feng, Forde, Kosower, Mastrolia, Perkins, Spence, Travaglini, Xiao, Yang, Zhu

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>$\mathcal{N} = 4$</th>
<th>$\mathcal{N} = 1$</th>
<th>$\mathcal{N} = 0$ (cut)</th>
<th>$\mathcal{N} = 0$ (rat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>− − + + + +</td>
<td>BDDK (94)</td>
<td>BDDK (94)</td>
<td>BDDK (94)</td>
<td>BDK (94)</td>
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<tr>
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<td>BDDK (94)</td>
<td>BBST (04)</td>
<td>BBDFK (06), XYZ (06)</td>
</tr>
<tr>
<td>− + + − + +</td>
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<td>BDDK (94)</td>
<td>BBST (04)</td>
<td>BBDFK (06), XYZ (06)</td>
</tr>
<tr>
<td>− − + − + +</td>
<td>BDDK (94)</td>
<td>BDDK (94)</td>
<td>BBDI (05), BFM (06)</td>
<td>BBDFK (06), XYZ (06)</td>
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<tr>
<td>− − − + + +</td>
<td>BDDK (94)</td>
<td>BDDK (94)</td>
<td>BFM (06)</td>
<td>XYZ (06)</td>
</tr>
<tr>
<td>− − − + + −</td>
<td>BDDK (94)</td>
<td>BBDP (05), BBCF (05)</td>
<td>BFM (06)</td>
<td>XYZ (06)</td>
</tr>
<tr>
<td>− + − + − +</td>
<td>BDDK (94)</td>
<td>BBDP (05), BBCF (05)</td>
<td>BFM (06)</td>
<td>XYZ (06)</td>
</tr>
</tbody>
</table>

- Numerical evaluation via recursion Ellis, Giele, Zanderighi (06)
- Numerical evaluation based on unitarity Ellis, Giele, Kunszt (07)
For theories with massless internal particles, it might be easier to compute

\[ M = \sum d(4)\text{boxes}(D) + \sum c(4)\text{triangles}(D) + \sum b(4)\text{bubbles}(D) + R \]

where the coefficients are now 4-dimensional and \( R \) is a rational (non-logarithmic) term.

✓ Coefficients of loop integrals obtained with generalised unitarity
✓ Rational parts obtained using on-shell recursion relation.
✓ Numerical implementation in BlackHat

Berger, Bern, Dixon, Forde, Kosower

Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre
The multi-leg one-loop problem - 2

Alternatively

\[ M = \sum e'(\text{pentagons}(D + 2)) + \sum d'(\text{boxes}(D)) + d''(D - 4)(D - 2)\text{boxes}(D + 4) + \sum c'(\text{triangles}(D)) + c'(D - 4)\text{triangles}(D + 2) + \sum b'(\text{bubbles}(D)) + b'(D - 4)\text{bubbles}(D + 2) \]

where the coefficients don’t depend on the dimension.

Giele, Kunszt, Melnikov

✓ To probe the higher dimension terms, one can employ unitarity in higher (integer) dimension

Ellis, Giele, Kunszt, Melnikov

✓ Numerical implementation in ROCKET

Giele, Zanderighi

Perturbative QCD for the LHC – p.30
### Numerical one-loop evaluation: BlackHat v Rocket

<table>
<thead>
<tr>
<th>BlackHat</th>
<th>Rocket</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ based on $D = 4$ unitarity and on-shell recursion</td>
<td>✓ based on OPP and $D_s$ dimension unitarity</td>
</tr>
<tr>
<td>✓ up to 8 gluon amplitudes numerically</td>
<td>✓ off-shell recursion for tree-input</td>
</tr>
<tr>
<td>✓ leading colour $V q\bar{q}ggg$ numerically</td>
<td>✓ up to 20 gluon amplitudes numerically</td>
</tr>
<tr>
<td></td>
<td>✓ all vector boson plus five parton processes numerically</td>
</tr>
<tr>
<td></td>
<td>Ellis, Giele, Zanderighi arXiv:0810.2762</td>
</tr>
</tbody>
</table>
Why go beyond NLO?

In many cases, the uncertainty from the pdf’s and from the choice of renormalisation scale still give NLO uncertainties that are as big or bigger than the experimental errors.

\[
\alpha_s(M_Z) = 0.1178^{+6\%}_{-4\%}(\text{scale})^{+5\%}_{-5\%}(\text{pdf})
\]
Why go beyond NLO? - continued

When is NNLO needed?

✓ When corrections are large - e.g. H production
✓ For benchmark measurements where experimental errors are small

What is known so far?

✓ Inclusive cross sections for $W$, $Z$ and $H$ production
  van Neerven, Harlander, Kilgore, Anastasiou, Melnikov, Ravindram, Smith
✓ Semi-inclusive $2 \rightarrow 1$ distributions - $W$, $Z$ and $H$ rapidity distributions
  Anastasiou, Dixon, Melnikov, Petriello
✓ Fully differential $pp \rightarrow H, W, Z + X$
  Anastasiou, Melnikov, Petriello
✓ DGLAP splitting kernels
  Moch, Vermaseren, Vogt
✓ NNLO parton distributions
  Martin, Stirling, Thorne, Watt
Anatomy of a NNLO calculation

✓ two-loop $2 \rightarrow 2$ matrix element

- looks like 2 jets in final state

✓ one-loop $2 \rightarrow 3$

- looks like 2 or 3 jets in final state

✓ tree-level $2 \rightarrow 4$

- looks like 2, 3 or 4 jets in final state

✓ plus method for combining the infrared divergent parts
The many (thousands) of tensor integrals appearing in two-loop graphs can be written in terms of a few Master Integrals $\text{MI}_j$

\[
2 = \sum_j a_j \text{MI}_j
\]

where the $a_j$ are polynomials in kinematical variables and the space-time dimension $D$.

The $\text{MI}_j$ can be expanded in $\epsilon = (4 - D)/2$ so that

\[
\sum \text{diagrams} = \sum_{i=1}^{4} \frac{X_i}{\epsilon^i} + X_0
\]
Reduction of Tensor Integrals

✓ Integration by Parts

\[ \int d^D k_1 \int d^D k_2 \frac{\partial}{\partial k_1^\mu} \left[ \frac{v^\mu}{A_1^{\nu_1} \cdots A_n^{\nu_n}} \right] = 0 \]

where \( v \) is any momentum in the problem, \( k_i, p_i \).

Chetrykin, Kataev, Tkachov

✓ Packages for solving the IbP identities

✓ AIR

Anastasiou, Lazopoulos, hep-ph/0404258

✓ IDSolver

Czakon

✓ FIRE

A. Smirnov, arXiv:0807.3243
Methods for calculating master integrals

✓ Mellin-Barnes contour integrals
  Davydychev; Smirnov; Tausk

✓ Differential equations in external scales and match to boundary conditions with fewer scales
  Remiddi, Gehrmann

✓ Nested sums from Schwinger parameterisation together with Hopf algebra techniques to relate to standard sums
  Moch, Uwer, Weinzierl

✓ Numerical method based on iterated sector decomposition - used to check many of above results
  Binoth, Heinrich
Drell Yan production

Most accurate prediction yet
✓ NNLO splitting functions
✓ NNLO PDF fits
✓ NNLO Drell-Yan cross section
⇒ High precision
Total error of $4\% - 5.5\%$

Martin et al

Aim to able to use as Standard Candle for luminosity measurements.
Gauge boson production at the LHC

Gold-plated process

At LHC NNLO perturbative accuracy better than 1%
⇒ use to determine parton-parton luminosities at the LHC

Anastasiou, Dixon, Melnikov, Petriello
Higgs boson production at the LHC

Total cross section

Fully differential

NNLO needed for reliable predictions

Harlander, Kilgore; Anastasiou, Melnikov, Petriello; ...
Higgs boson production at the LHC

✓ First study of fully inclusive $pp \rightarrow H \rightarrow WW \rightarrow \ell\nu\ell\nu$ with $m_H \sim 165$ GeV
  Anastasiou, Dissertori, Stöckli, arXiv/0707.2373

✓ Apply experimental cuts to reduce backgrounds from $t\bar{t}$, non-resonant $W^+W^-$ production

✓ Cuts affect LO/NLO/NNLO cross sections differently
  shouldn’t use inclusive K-factor

\[ pp \rightarrow H + X \rightarrow WW + X \rightarrow e^+\nu e^-\nu + X \]
Other NNLO calculations on horizon

- $pp \rightarrow jet + X$
  - needed to constraint PDF’s and fix strong coupling
  - matrix elements known for some time
    - Anastasiou et al, Bern et al
  - antenna subtraction terms worked out
    - Daleo, Gehrmann, Maitre

- $pp \rightarrow t\bar{t}$
  - necessary for precise $m_t$ determination
  - matrix elements partially known
    - Czakon, Mitov, Moch; Bonciani, Ferroglia, Gehrmann, Studerus, Maitre

- $pp \rightarrow VV$
  - signal: to study the gauge structure of the Standard Model
  - background: for Higgs boson production and decay in the intermediate mass range
  - large NLO corrections
    - Chachamis, Czakon, Eiras
Summary

QCD  A lot still to do, but progress being made towards main targets

LO  largely solved (plus BSM models)
✓  high multiplicity merged with parton shower, ALPGEN, SHERPA, . . .
✗  large theoretical uncertainty

NLO  QCD corrections generally large 30% – 100% - much larger than scale variation suggests
✓  Cuts tend to spoil use of inclusive K-factor
✓  Serious effort on Les Houches NLO wish list, several new NLO calculations $WWj, ttj, VVV, t\bar{t}Z$
✗  2 $\rightarrow$ 4 barrier starting to be breached $q\bar{q} \rightarrow t\bar{t}b\bar{b}$
✓  Numerical methods becoming powerful

NNLO  Inclusive and exclusive results for $H, W, Z$ production
✓  DGLAP splitting kernels $\Rightarrow$ NNLO PDF fits
✗  2 $\rightarrow$ 2 calculations coming onto horizon