Nucleon Deformation from Lattice QCD

Antonios Tsapalis

National Technical University of Athens

School of Applied Mathematics and Physical Sciences

&

Hellenic Naval Academy

5th Vienna Central European Seminar, Nov. 28-30, 2008

Outline

- Introduction to Lattice QCD
- Hadronic States from the Lattice
- Limitations and Systematics
- Nucleon Deformation and N→Δ(1232) Electromagnetic Transition Form Factors

  C. Alexandrou, G. Koutsou, H. Neff, J. Negele, W. Schroers, A. Tsapalis
  PRD 77, 085012 (2008)

- Δ(1232) Deformation and Δ Electromagnetic Form Factors

  C. Alexandrou, T. Korzec, G. Koutsou, Th. Leontiou, C. Lorce´, J. Negele, V. Pascalutsa, A. Tsapalis, M. Vanderhaeghen ***arXiv:0810.3976***

- Conclusions – Prospects

Quarks and Gluons

$$S[\psi_f, \overline{\psi}_f, A_\mu] = \int d^4x [\frac{1}{2} Tr F_{\mu\nu} F^{\mu\nu} + \sum_f \overline{\psi}_f (i\gamma_\mu D^\mu - m_f)\psi_f]$$

$$F_{\mu\nu} = [D_\mu, D_\nu] \qquad D_\mu = \partial_\mu + igA_\mu$$

invariant under:

P, C, T,

local color SU(3) rotations

global flavour SU(N_f) rotations (approximate)

$g(q^2) \to 0$ $q^2 \to$ large (asymptotic freedom)

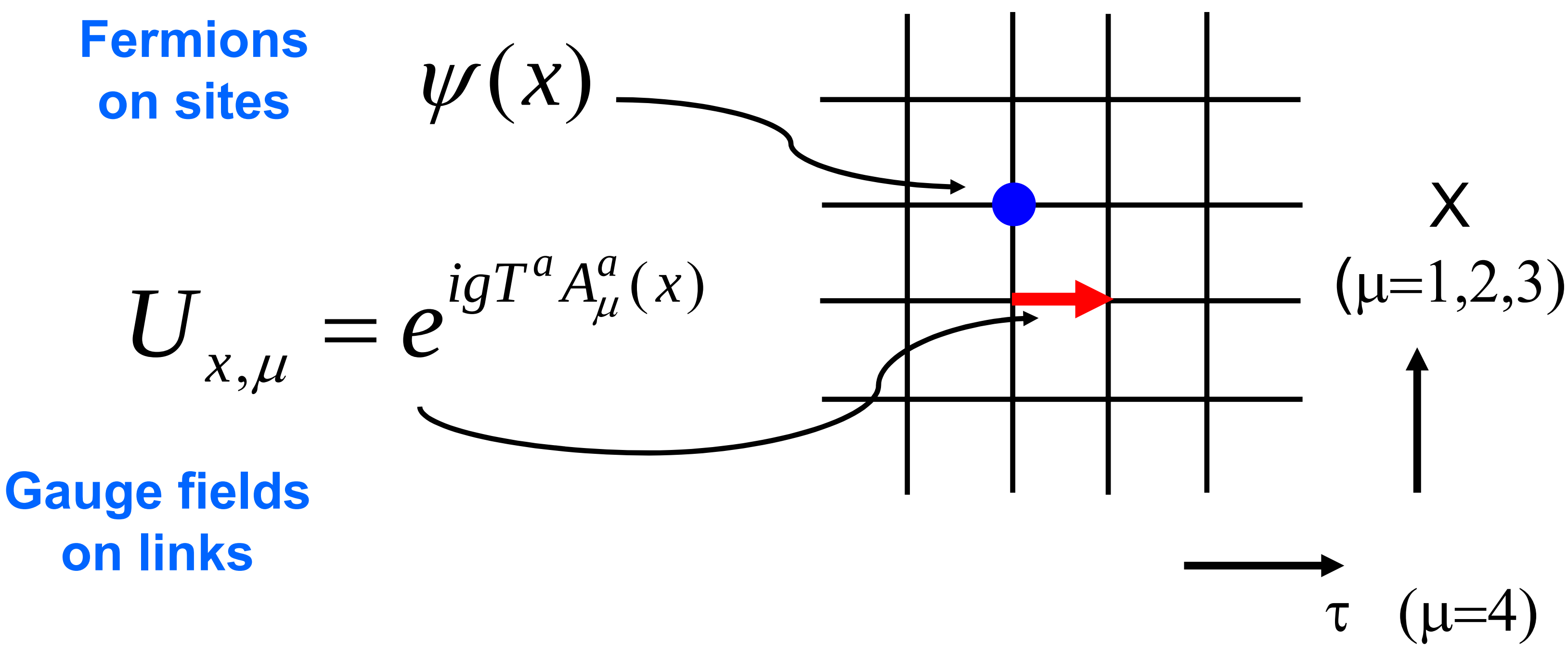
Perturbative techniques suffice for the description of high energy scattering processes

Lattice QCD

- Rotate to Euclidean time: t –> -i τ

$$|\Psi(\tau)> = e^{-H\tau}|\Psi(0)>$$

- Discretize space-time



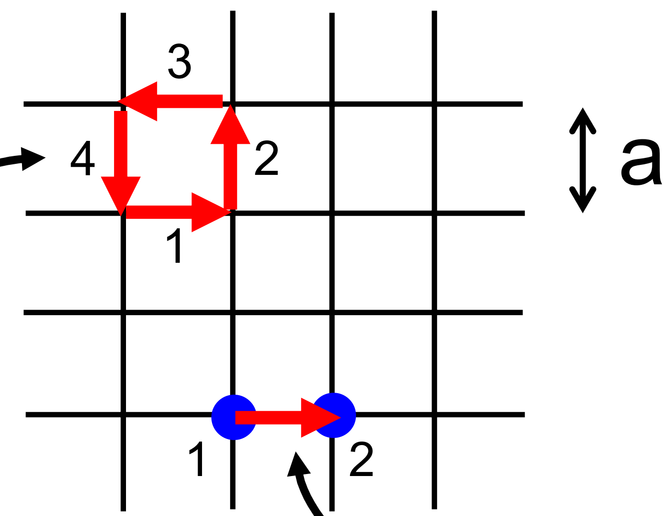
Wilson formulation (1974)

$$\frac{1}{2} Tr F_{\mu\nu} F^{\mu\nu}$$

$$\downarrow$$

$$1 - \frac{1}{6}\{Tr[U_1 U_2 U_3^+ U_4^+] + h.c.\}$$

Plaquette gauge action



$$\overline{\psi}(i\gamma_\mu D^\mu - m)\psi$$

$$\downarrow$$

$$(m+4)\overline{\psi}_1\psi_1 - \frac{1}{2}\{\overline{\psi}_1[(1-\gamma_\mu)U_\mu + (1+\gamma_\mu)U_\mu^+]\}\psi_2$$

Wilson-Dirac operator D_W

Lattice QCD Action

$$S_{gauge} = \frac{6}{g^2} \sum_{\mu,\nu} \sum_{x} a^4 \left\{ 1 - \frac{1}{6} Tr[P_{\mu\nu}(x) + P_{\mu\nu}^{+}(x)] \right\}$$

$$S_{quark} = \sum_{x,y} a^4 \overline{\psi}_x D_W(x,y) \psi_y$$

$$S_{LQCD} = S_{gauge} + S_{quark}$$

Invariant under local SU(3) transformations g_x on sites

$$U_{x,\mu} \rightarrow g_x U_{x,\mu} g_{x+\hat{\mu}}^{+} \quad \psi_x \rightarrow g_x \psi_x \ , \quad \overline{\psi}_x \rightarrow \overline{\psi}_x g_x^{+}$$

Generate an ***ensemble*** of gauge fields $\{U\}$ distributed with the Boltzmann weight

$$Z = \int D\psi D\overline{\psi} DU e^{-\beta S_{gauge}[U] + \sum_{x,y} \overline{\psi}_x D_W \psi_y}$$

$$= \int DU e^{-\beta S_{gauge}[U]} \det[D_W(U)]$$

$$\beta = \frac{6}{g^2}$$

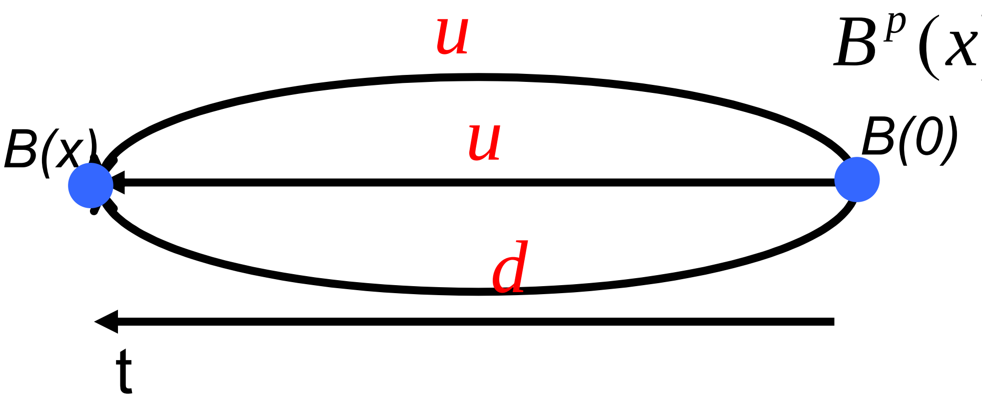
Calculate any n-point function of QCD

$$\langle \hat{O} \rangle = \frac{1}{Z} \int DU\, D\psi\, D\overline{\psi}\; O[U, \psi_x, \overline{\psi}_y] e^{-\beta S_{gauge}[U] + \sum_{x,y} \overline{\psi}_x D_W \psi_y}$$

$$= \frac{1}{Z} \int DU\; O[U, D_{W\,x,y}^{-1}] e^{-\beta S_{gauge}[U]} \det[D_W(U)]$$

stochastic solution (not simulation) of QCD

Hadron masses in Lattice QCD



$$B^{p}(x)=\varepsilon^{abc}[u^{a}(x)C\gamma_{5}d^{b}(x)]u^{c}(x)$$

- construct interpolating field for hadron state
- generate a baryon at t=0
- annihilate the baryon at time t
- measure the 2-pt function
- extract the energy from the exponential decay of the state in Euclidean time

Hadron 2-pt functions are products of quark propagators

nucleon interpolating field : $B^p(x) = \varepsilon^{abc}[u^a(x) C\gamma_5 d^b(x)] u^c(x)$

$$\langle \Omega | T(B_x \overline{B}_y) | \Omega \rangle = \varepsilon^{abc} \varepsilon^{a'b'c'}$$

$$\left\langle D^{aa'}{}_W{}^{-1}(x,y) \otimes D^{bb'}{}_W{}^{-1}(x,y) \otimes D^{cc'}{}_W{}^{-1}(x,y) \right\rangle$$

expensive part: calculate quark propagator on each configuration

$$\langle \Omega | T(\psi_x \overline{\psi}_y) | \Omega \rangle = D_W{}^{-1}(x,y)$$

Invert the ($3 \times 4 \times L^3 \times T$) x ($3 \times 4 \times L^3 \times T$) Wilson-Dirac matrix with standard linear system solvers (conjugate gradient etc..)

$$\int d\vec{x} e^{-i\vec{p}\cdot\vec{x}} \left\langle \Omega \middle| B(\vec{x},\tau)\overline{B}(0) \middle| \Omega \right\rangle =$$

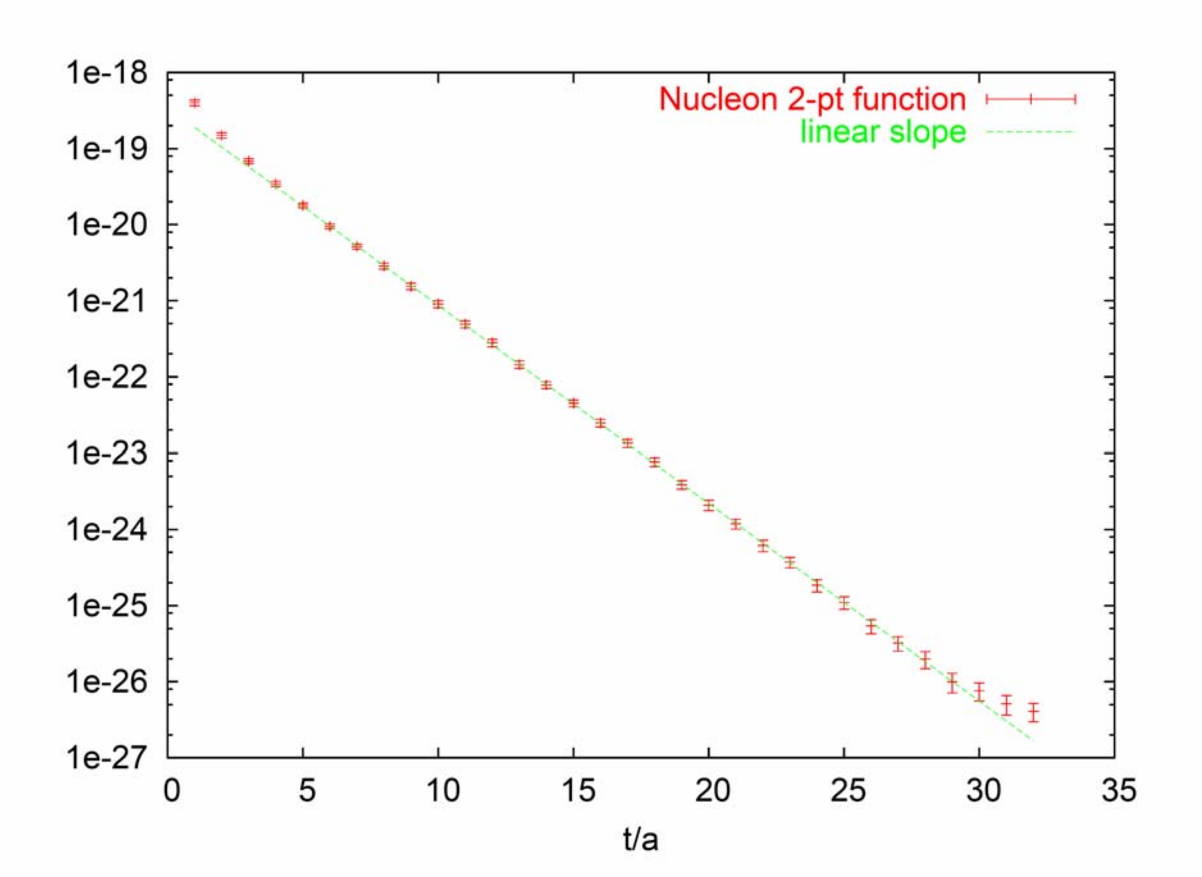
$$\int d\vec{x} e^{-i\vec{p}\cdot\vec{x}} \left\langle \Omega \middle| B(\vec{x},\tau) \,||\, \sum_{\vec{q}} N(\vec{q})\rangle\langle N(\vec{q})\,| + |\, N'(\vec{q})\rangle\langle N'(\vec{q}) + \ldots \,||\, \overline{B}(0) \middle| \Omega \right\rangle =$$

$$\sum_{\vec{q}} \int d\vec{x} e^{-i\vec{p}\cdot\vec{x}} e^{-E_N\tau} e^{i\vec{q}\cdot\vec{x}} \langle \Omega \,|\, B \,|\, N(\vec{q})\rangle\langle N(\vec{q}) \,|\, B \,|\, \Omega\rangle + \text{ excited states } =$$

$$Z_N^2 e^{-E_N\tau} + Z_{N'}^2 e^{-E_{N'}\tau} + \ldots = Z_N^2 e^{-E_N\tau}(1 + C_1 e^{-\Delta E\tau} + \ldots.)$$

use $\langle N(\vec{q}) \,|\, \overline{B}(\vec{x},\tau) \,|\, \Omega\rangle = Z_N e^{-i\vec{q}\cdot\vec{x}} e^{-E_N\tau}$

maximize the overlap Z_N to the state using
"smeared" interpolating fields



$M_N a = 0.595 \pm 0.007$

Fixing one hadron mass to its physical value determines the lattice spacing a – for the rest of the hadrons we get a ***prediction*** for the mass

Limitations

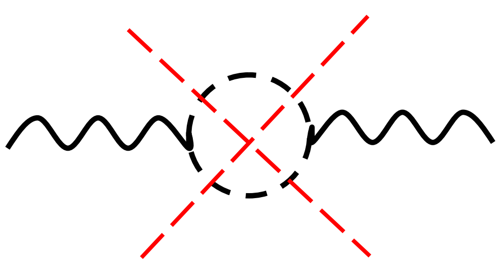
physical answers emerge at the limit

Size of ensemble {U}	➔ infinite	O(100-1000)
Lattice volume La	➔ infinite	La ~ 2-3 fm
Lattice spacing a	➔ 0	a ~ 0.1 fm
		(cutoff p/a ~ 6 GeV)

- det(D_W) very expensive to include in Z

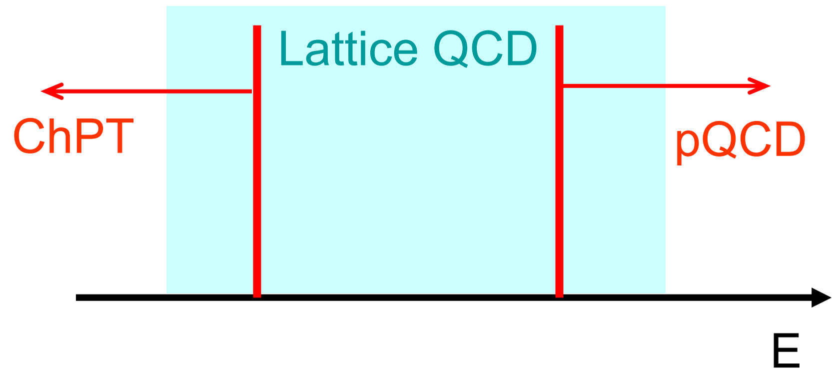
set det(D_W) = 1 **quenched approximation**

ignore quark loops



• D_W breaks chiral symmetry -- additive quark mass renormalization

heavy quarks ; m_π > 350 MeV need to extrapolate to small m_π



improve D_W with $O(a^2)$ term: **clover fermions**

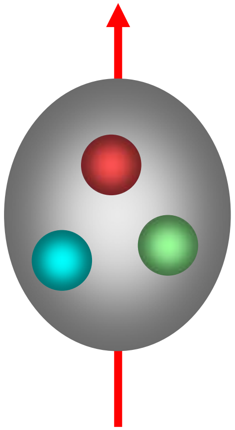
…or implement **twisted mass fermions**

Around 1998 lattice fermion operators were discovered which maintain *exactly* chiral symmetry

Overlap and Domain Wall operators

Very CPU expensive – 30-50 times more than D_W

Nucleon Deformation



$$S = \frac{1}{2}$$

Spectroscopic quadrupole moment ***vanishes***

$$< \frac{1}{2} | Q | \frac{1}{2} >= 0$$

i.e. one photon measurements cannot reveal the shape

Intrinsic quadrupole moment w.r.t. body-fixed frame exists

$$Q_0 = \int d\vec{r} \rho(\vec{r})(3z^2 - r^2)$$

$Q_0 > 0$ prolate

$Q_0 < 0$ oblate

modelling required !

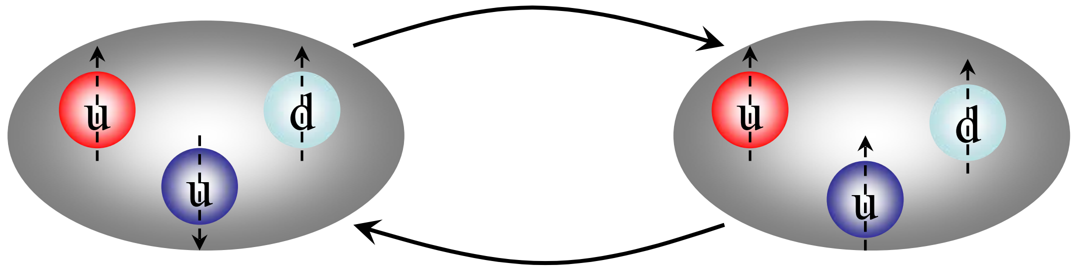
$N \rightarrow \Delta(1232)$

γ^* M1 , E2 , C2

N(qqq)

$I = \frac{1}{2}$ $J = \frac{1}{2}$

938 MeV



Δ(qqq)

$I = \frac{3}{2}$ $J = \frac{3}{2}$

1232 MeV

M_{1+}, E_{1+}, S_{1+} π^o

Spherical $\Rightarrow$ M1

Deformed $\Rightarrow$ M1 , E2 , C2

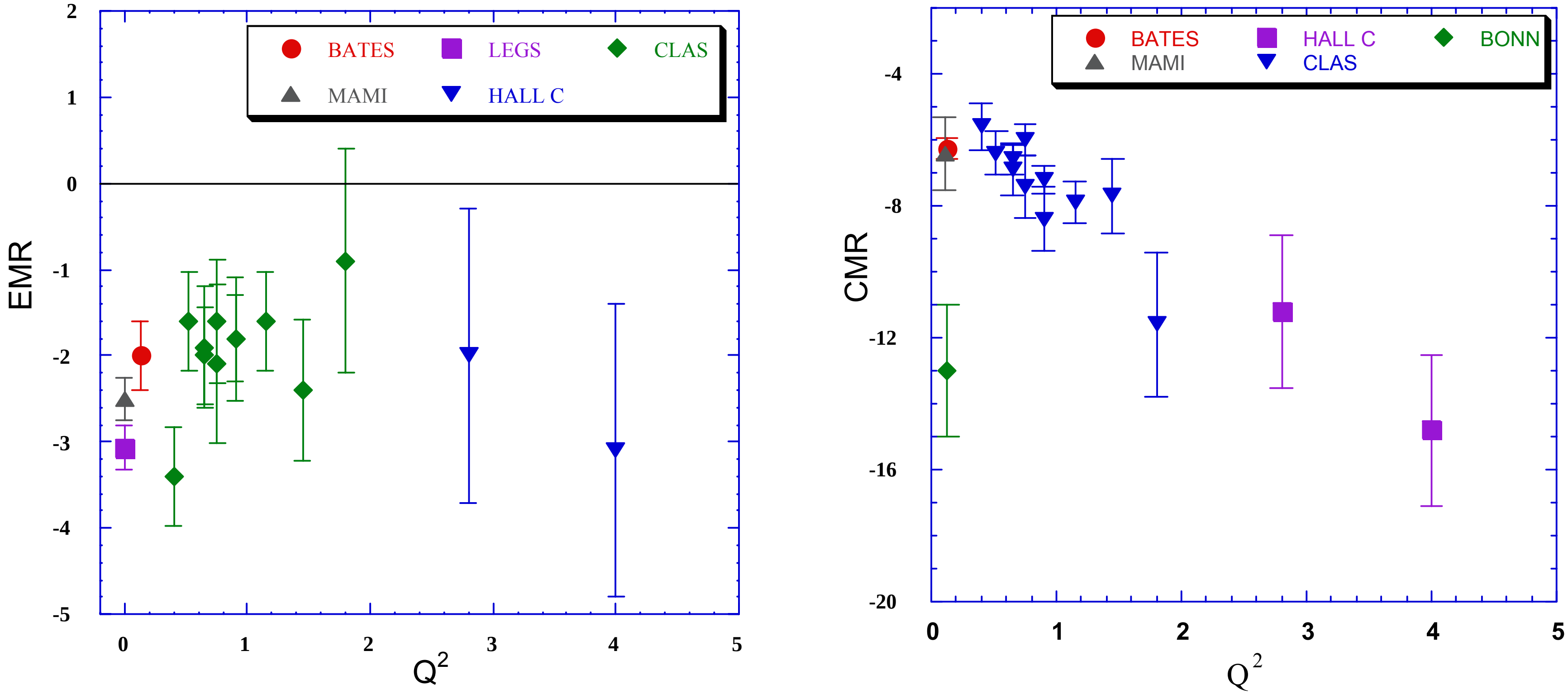
Deformation signal

$$\text{EMR} = \text{Re}\left(\frac{E_{1+}^{3/2}}{M_{1+}^{3/2}}\right)$$

$$\text{CMR} = \text{Re}\left(\frac{S_{1+}^{3/2}}{M_{1+}^{3/2}}\right)$$

EMR & CMR
Experimental Status

uncertainties in modelling final state interactions



Thanks to N. Sparveris (Athens, IASA & JLab)

The Transition Matrix Element

$$< \Delta(p',s') \,|\, J^{\mu} \,|\, N(p,s) >= i\sqrt{\frac{2}{3}}\left(\frac{m_{\Delta}m_{N}}{E_{\Delta}E_{N}}\right)^{1/2} \bar{u}_{\tau}(p',s')O^{\tau\mu}u(p,s)$$

H.F.Jones and M.C.Scadron, Ann. Phys. (N.Y.) 81,1 (1973)

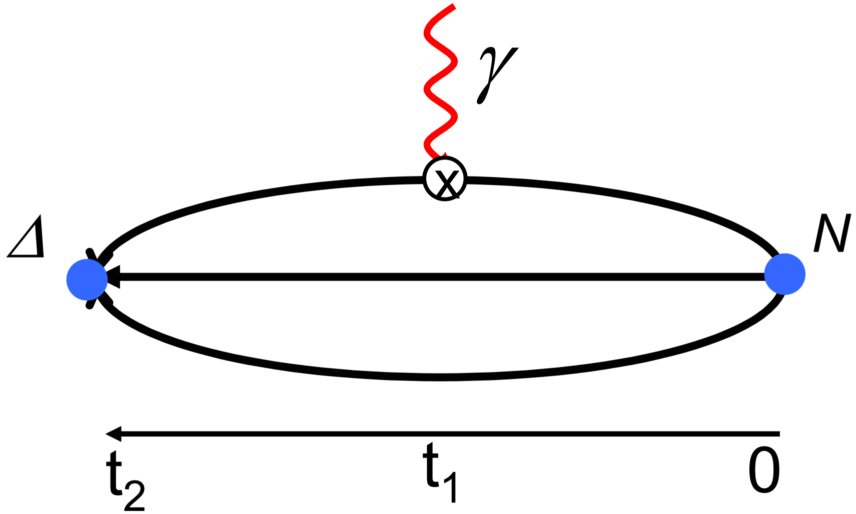
$$O^{\tau\mu} = G_{M1}(q^2)K_{M1}^{\tau\mu} + G_{E2}(q^2)K_{E2}^{\tau\mu} + G_{C2}(q^2)K_{C2}^{\tau\mu}$$

magnetic dipole | electric quadrupole | scalar quadrupole

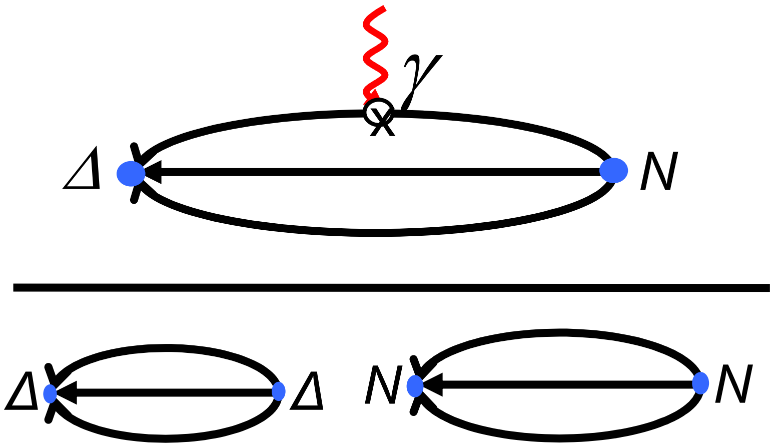
static Δ frame

$$\mathrm{EMR} = -\frac{G_{E2}(q^2)}{G_{M1}(q^2)} \qquad \mathrm{CMR} = -\frac{|\vec{q}|}{2M_{\Delta}}\frac{G_{C2}(q^2)}{G_{M1}(q^2)}$$

Lattice QCD Calculation



- generate a nucleon at t=0
- inject a photon with momentum **q** at $t=t_1$
- annihilate a Delta at time $t=t_2$
- measure the 3-pt function
- extract the form factors from suitable ratios of 3-pt and 2-pt functions



$$\sim \frac{\langle G_{\sigma}^{\Delta j^{\mu} N}(t_{2,}t_{1};\vec{p}\,';\vec{p};\Gamma)\rangle}{\sqrt{\langle G_{ii}^{\Delta\Delta}(t_{2};\vec{p}\,';\Gamma_{4})\rangle\langle G^{NN}(2t_{2}-2t_{1};\vec{p};\Gamma_{4})\rangle}}$$

$$\langle G_{\tau}^{\Delta j^{\mu} N}(t_{2,}t_{1};\vec{p}\,';\vec{p};\Gamma)\rangle =$$

$$\int d\vec{x}_{2} e^{-i\vec{p}\,'\cdot\vec{x}_{2}} \int d\vec{x}_{1} e^{-i(\vec{p}\,'-\vec{p})\cdot\vec{x}_{1}} \langle \Omega | T(\chi_{\tau a}^{\Delta}(x_{2})\, j^{\mu}(x_{1})\, \chi_{b}^{N}(0)) | \Omega\rangle \Gamma_{ba} \rightarrow$$

$$Z_{\Delta} Z_{N} e^{-E_{\Delta}'(t_{2}-t_{1})} e^{-E_{N} t_{1}} \times [K_{M1}^{\tau\mu} G_{M1}(q^{2}) + K_{E2}^{\tau\mu} G_{E2}(q^{2}) + K_{C2}^{\tau\mu} G_{C2}(q^{2})]$$

Known kinematical functions of p, p´

Select μ , τ , Γ indices to isolate different form factors

$$\Gamma_{i} = \frac{1}{2}\begin{pmatrix} \sigma_{i} & 0 \\ 0 & 0 \end{pmatrix}, \quad \Gamma_{4} = \frac{1}{2}\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

Lattice parameters

Wilson N_F = 0 β=6.0

32^3x64 a=0.09 fm

m_π = 0.56 GeV

m_π = 0.49 GeV

m_π = 0.41 GeV

Wilson N_F = 2 β=5.6 a=0.08 fm

24^3x40 m_π = 0.69 GeV (TXL)

24^3x40 m_π = 0.51 GeV (TXL)

24^3x32 m_π = 0.38 GeV (DESY)

Hybrid scheme

MILC N_F = 2 + 1 staggered (KS) sea

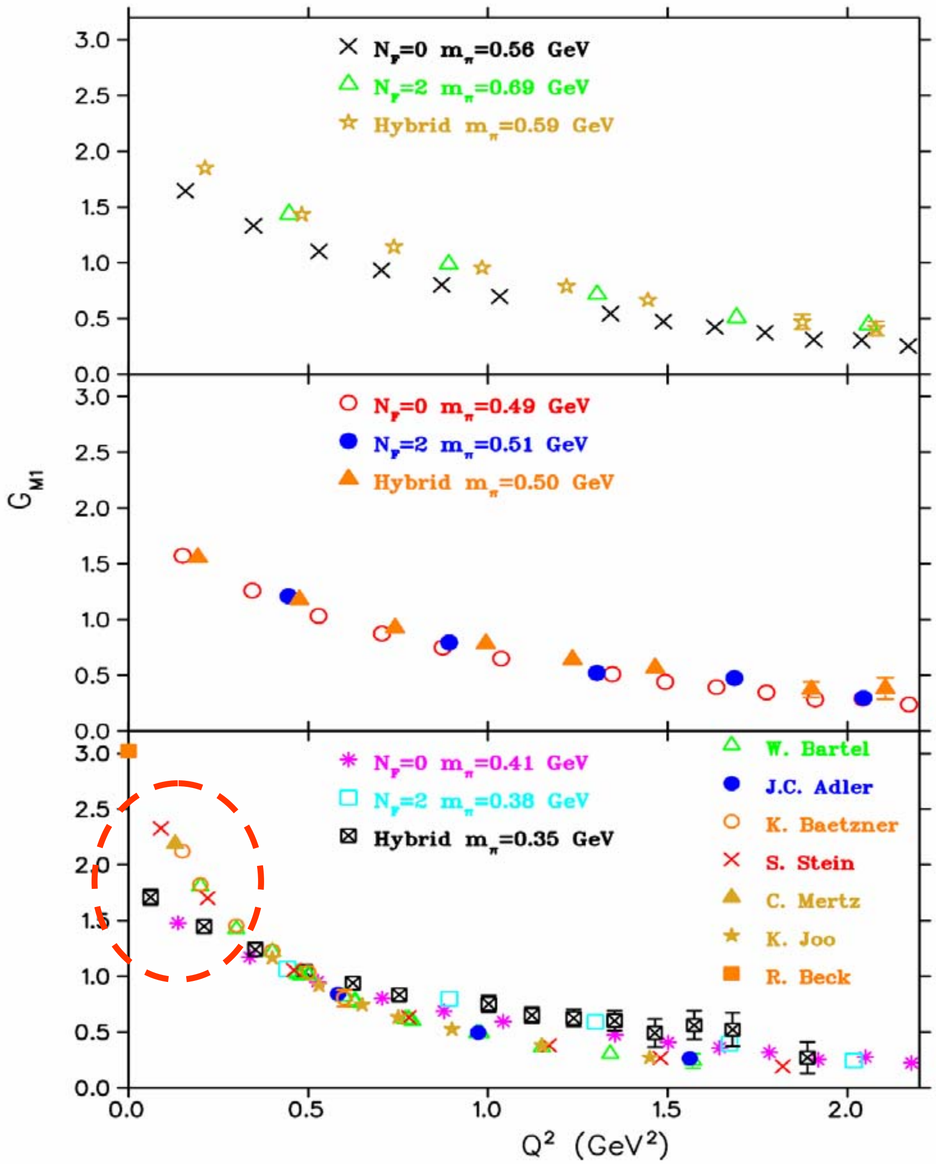
Domain Wall valence (L_5=16)

a=0.125 fm

am_s	am_u	m_π	L^3xT
0.05	0.03	0.59 GeV	20^3x64
0.05	0.02	0.50 GeV	20^3x64
0.05	0.01	0.36 GeV	28^3x64

strange quark mass fixed on physical value

G_{M1}



quenched QCD results in

C. Alexandrou, Ph. de Forcrand, H. Neff, J. Negele, W. Schroers, A. Tsapalis
PRL 94, 021601 (2005)

dynamical QCD results in

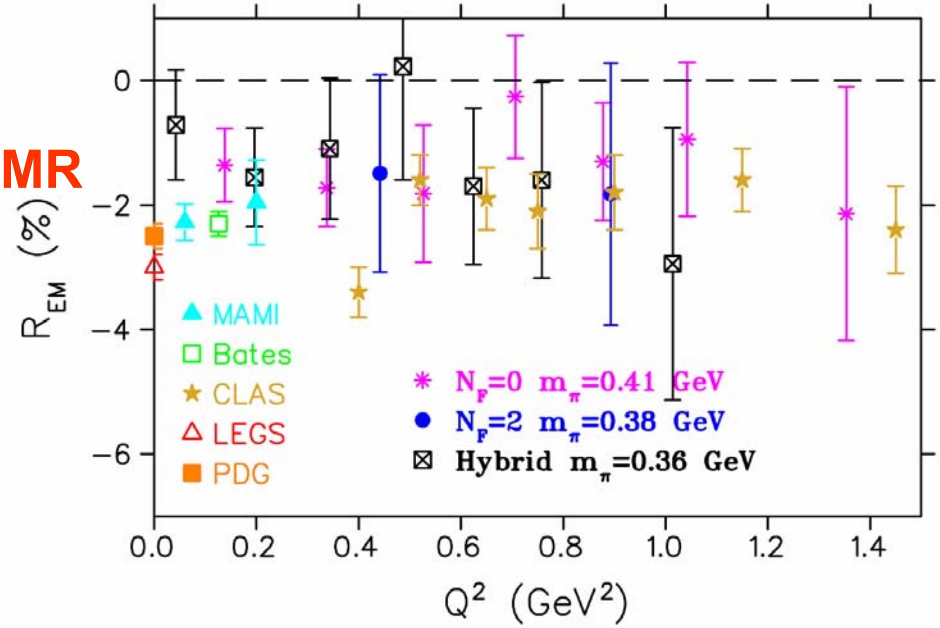
C. Alexandrou, G. Koutsou, H. Neff, J. Negele, W. Schroers, A. Tsapalis
PRD 77, 085012 (2008)

dipole fits well – requires larger mass 1.3 GeV vs 0.78_{exp}

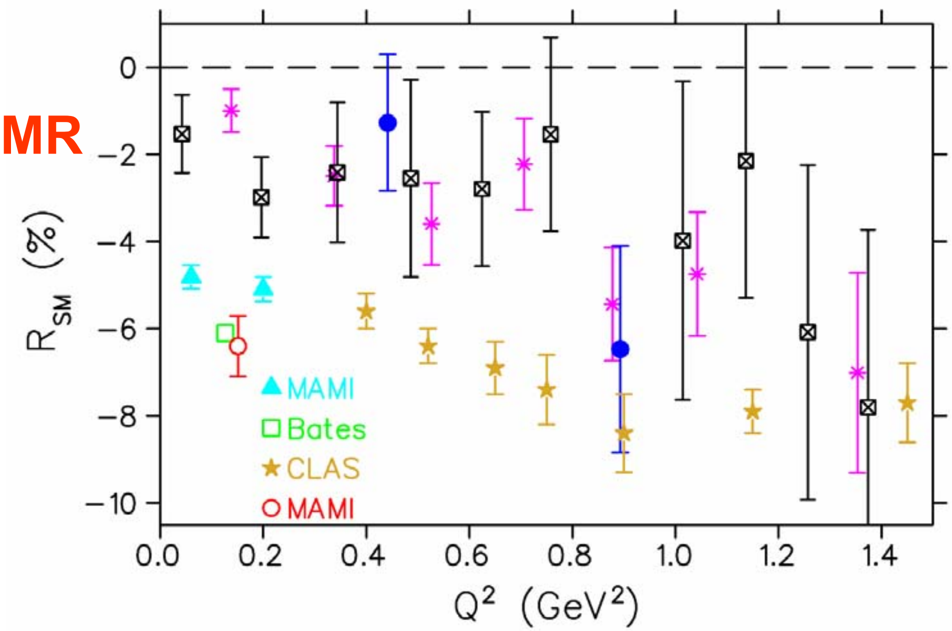
slower falloff at small Q^2
missing pion cloud

no unquenching effects visible

EMR



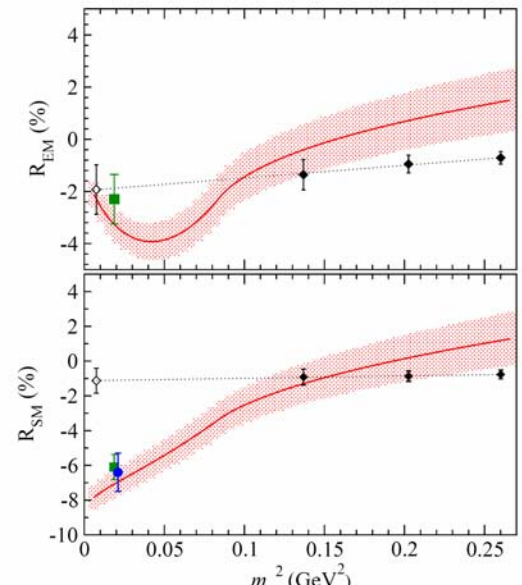
CMR



qualitative agreement with experiment

quadrupole amplitudes in N→ Δ verified from QCD

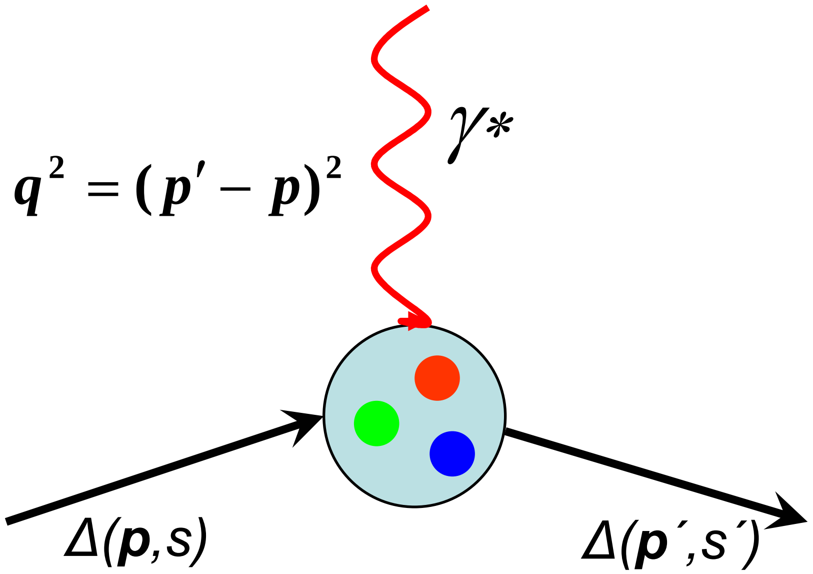
deformed N & Δ states



Low Q^2 data in agreement with chiral EFT predicted behavior

V. Pascalutsa & M. Vanderhaeghen, PRL 95(2005) 232001

Δ(1232) Deformation



J=3/2

elastic matrix element can reveal quadrupole amplitudes

$$\left\langle \frac{3}{2} \,|\, Q \,|\, \frac{3}{2} \right\rangle \neq 0$$

$$< \Delta(p', s') \,|\, J_\mu \,|\, \Delta(p, s) > =$$

$$\bar{u}^\sigma(p,s) \left[g_{\sigma\tau} \left(a_1(q^2)\gamma_\mu + \frac{a_2(q^2)}{2m_\Delta}(p_\mu + p'_\mu) \right) + \frac{q_\sigma q_\tau}{4m_\Delta^2} \left(c_1(q^2)\gamma_\mu + \frac{c_2(q^2)}{2m_\Delta}(p_\mu + p'_\mu) \right) \right] u_\tau(p,s)$$

four independent q^2 dependent form factors: a_1, a_2, c_1, c_2

reparameterize in terms of spin/parity based physical form factors

$$\left.\begin{array}{l} a_1(q^2) \\ a_2(q^2) \\ c_1(q^2) \\ c_2(q^2) \end{array}\right\} \Rightarrow \left\{\begin{array}{ll} G_{E0}(q^2) & \text{Electric charge form factor} \\ G_{M1}(q^2) & \text{Magnetic dipole form factor} \\ G_{E2}(q^2) & \text{Electric quadrupole form factor} \\ G_{M3}(q^2) & \text{Magnetic octupole form factor} \end{array}\right.$$

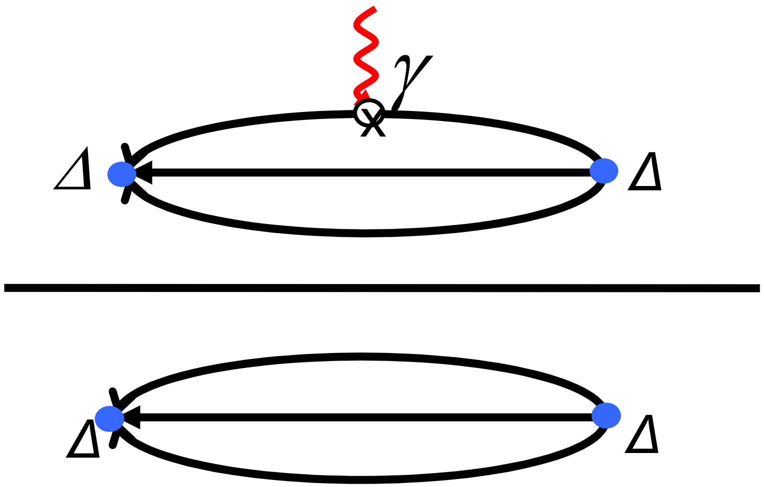
useful for the extraction of static properies & distributions

rms radius $\left\langle r^2 \right\rangle = -6 \left.\frac{dG_{E0}(Q^2)}{dQ^2}\right|_{Q^2=0}$

magnetic moment $\mu_\Delta = \frac{e}{2m_\Delta} G_{M1}(0)$

quadrupole moment (deformation) $\sim G_{E2}(0)$

magnetic octupole moment $O = \frac{e}{2m_\Delta^3} G_{M3}(0)$



$$\sim \frac{\langle G^{\Delta j^\mu \Delta}_{\sigma\tau}(t_2, t_1; \vec{p}\,'; \vec{p}; \Gamma)\rangle}{\sqrt{\langle G^{\Delta\Delta}_{ii}(t_2; \vec{p}\,'; \Gamma_4)\rangle \langle G^{\Delta\Delta}_{ii}(2t_2 - 2t_1; \vec{p}; \Gamma_4)\rangle}}$$

$$\langle G^{\Delta j^\mu \Delta}_{\sigma\tau}(t_2, t_1; \vec{p}\,'; \vec{p}; \Gamma)\rangle =$$

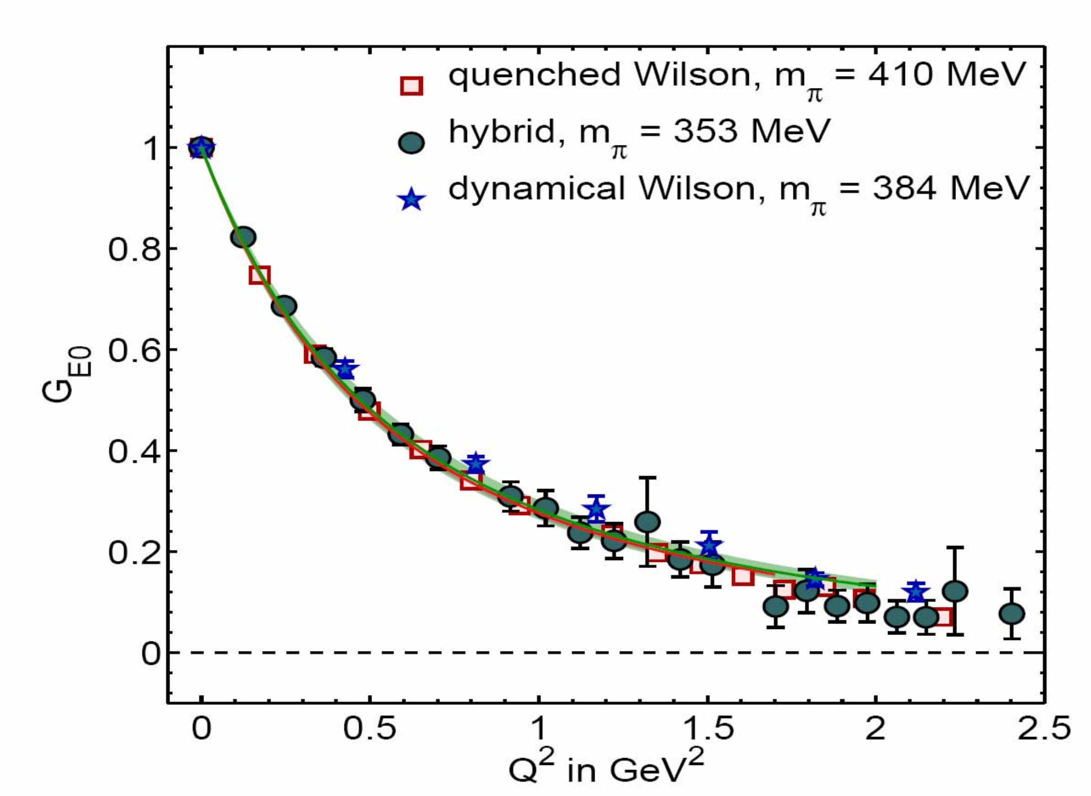
$$\int d\vec{x}_2 e^{-i\vec{p}\,'\cdot\vec{x}_2} \int d\vec{x}_1 e^{-i(\vec{p}\,'-\vec{p})\cdot\vec{x}_1} \langle\Omega | T(\chi^\Delta_{\sigma a}(x_2) j^\mu(x_1) \chi^\Delta_{\tau b}(0)) | \Omega\rangle \Gamma_{ba} \to$$

$$Z^2_\Delta e^{-E'_\Delta(t_2-t_1)} e^{-E_\Delta t_1} \times [K^{\sigma\mu\tau}_{E0} G_{E0}(q^2) + K^{\sigma\mu\tau}_{M1} G_{M1}(q^2) + K^{\sigma\mu\tau}_{E2} G_{E2}(q^2) + K^{\sigma\mu\tau}_{M3} G_{M3}(q^2)]$$

Known kinematical functions of p, p´

Select σ, μ , τ indices to isolate different form factors

Electric charge form factor & rms radius



dipole fits well the data

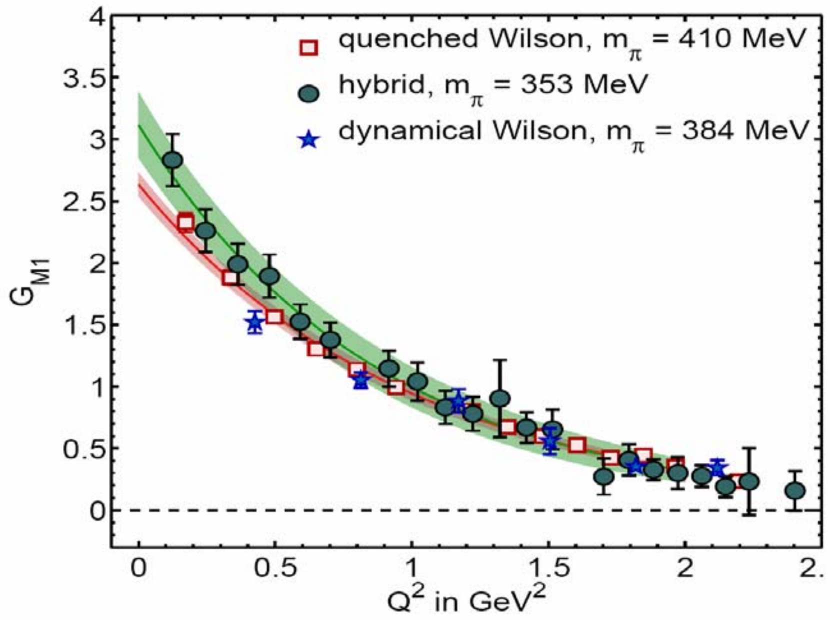
$$G_{E0}(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\Lambda_{E0}^2}\right)^2}$$

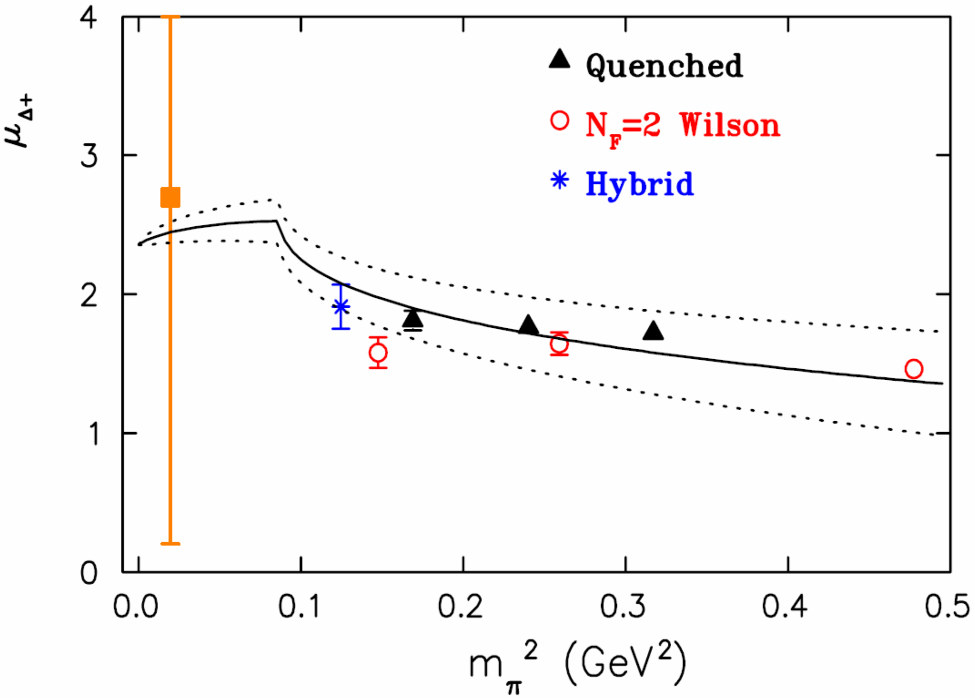
rms radius from slope at Q^2=0

$$\langle r^2 \rangle = -6 \frac{dG_{E0}(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

	$\sqrt{\langle r^2 \rangle}$ (fm)
quenched Wilson	0.652(9)
N_F=2 Wilson	0.611(17)
N_F=2+1 Hybrid	0.641(22)

Magnetic dipole form factor and magnetic moment





dipole and exponential fit equally well the data

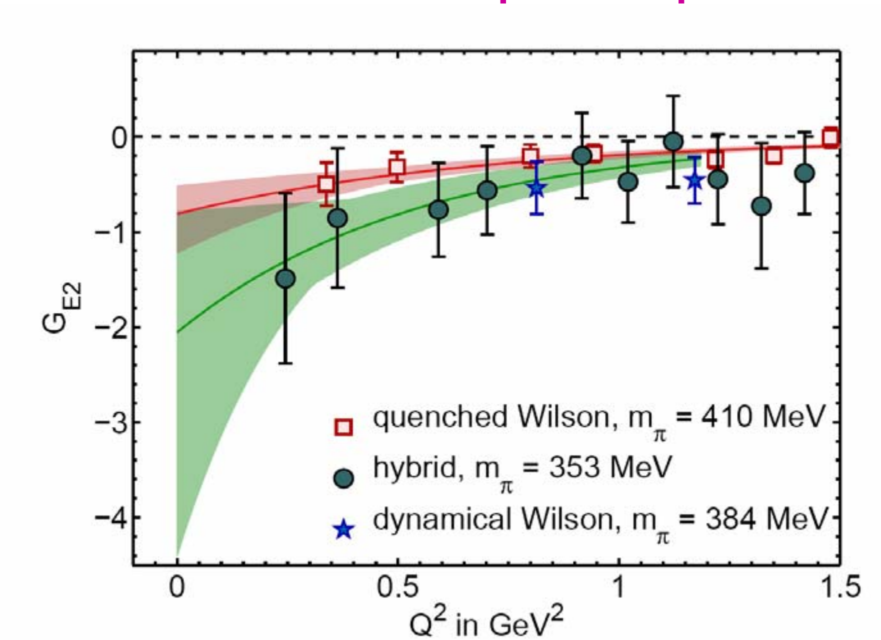
$$G_{M1}(Q^2) = G_{M1} \exp\left(-\frac{Q^2}{\Lambda_{M1}^2}\right)$$

chiral extrapolation

V. Pascalutsa & M. Vanderhaeghen, PRL 95(2005) 232001

	$\mu_{\Delta^+}\ (e/2m_N)$
quenched Wilson	1.811(69)
N_F=2 Wilson	1.58(11) ← finite volume effect ?
N_F=2+1 Hybrid	1.91(16)

Electric quadrupole form factor & deformation



exponential fits with Q^2 to extract $G_{E2}(0)$

consistently negative $G_{E2}(0)$

Quadrupole moment $Q^{\Delta}_{3/2}$ is connected to transverse

quark charge densities in the infinite momentum frame

.. which in turn are connected to the FFs at Q^2=0

$$Q^{\Delta}_{3/2} = \frac{e}{2m_\Delta^2}\left\{2\left[G_{M1}(0) - 3e_\Delta\right] + \left[G_{E2}(0) + 3e_\Delta\right]\right\}$$

Consistently ***positive*** $Q^{\Delta}_{3/2}$ for spin +3/2 Δ^+ state in the IMF

elongated along spin axis (**prolate**)

Conclusions

- Lattice QCD is the established technique for the study of hadron properties from first principles with controlled approximations.
- Hadron Structure can be reliably revealed through Form Factors, Structure functions and GPDs calculated on the Lattice.
- Nucleon deformation is studied through the $\gamma^* N \rightarrow \Delta$ transition form factors for Q^2 up to 1.5 GeV^2. EMR & CMR ratios in agreement to experiment and chiral EFT predictions for pion masses down to 350 MeV.
- Δ(1232) e.m. form factors are directly calculated on quenched and dynamical lattices. Magnetic moments, rms radii and quadrupole deformation are extracted with precision that supercedes experimental measurements.
- Similar calculations of axial & pseudoscalar form factors complete the picture of the complicated dynamics in hadron states.