Soft-Collinear Effective Theory and B-Meson Decays

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Outline:

1. Motivation
2. Formalism
3. Applications
   - SCET in Inclusive $B$ Decays
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4. Summary
Factorization in QCD: Separate short- and long-distance modes

Short-distance modes:
- Heavy massive particles (e.g. electroweak gauge bosons, top quark, Higgs)
- Quantum fluctuations with large virtualities.

⇒ Dynamics encoded in short-distance coefficients (coefficient functions).
Calculable in (RG-improved) Perturbation Theory.

Long-distance modes:
- Particles with small masses/virtualities (light quarks, gluons, photons).

⇒ Dynamics in (hadronic/partonic) matrix elements of composite operators. Requires non-perturbative methods to deal with masses/virtualities of order $\Lambda \equiv \Lambda_{\text{QCD}}$. 

IR-divergences of short-distance coefficients

$\uparrow$

factorization scale $\mu$

$\downarrow$

UV-divergences of operator matrix elements
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QCD Factorization in $B$-decays

- We are interested in fundamental flavour parameters, CKM angles, quark masses, ... in the SM or its NP extensions.
- We have to analyze weak decays of $b$-quarks, but within a non-perturbative hadronic environment!

The Procedure:
- Integrate out EW gauge bosons (NP particles) at (above) the EW scale:
  - Effective Hamiltonian for
    - semi-leptonic decays (“tree”)
    - non-leptonic decays, FCNCs and meson-mixings (“penguins”, “boxes”)
- RG-running of Wilson coefficients from $\mu \sim M_W$ to $\mu \sim m_b$
- QCD factorization to separate
  - perturbative effects (i.e. virtualities scale with $m_b \gg \Lambda_{QCD}$)
  - genuine hadronic properties (i.e. universal for $B$-meson and its decay products)
QCD Factorization in $B$-decays

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- We have to analyze weak decays of $b$-quarks, but within a non-perturbative hadronic environment!

The result: Precise determination of the CKM triangle

Hadronic parameters:
- decay constants
- transition form factors
- HQET parameters
- shape functions
- light-cone distribution amplitudes
...
Nature of IR divergences:

- for instance, $b \rightarrow X_s \gamma$ ($X_s$ : s-quark jet)
  - Large recoil energy: $E_X \sim m_b/2$
  - Invariant mass of “hard-collinear” jet: $p_X^2 \sim \Lambda m_b$

- $b$-quark interactions with soft gluons described by HQET
- interactions between soft and collinear jet modes:
  - Sudakov logarithms in $b \rightarrow s$ form factor (involving $\ln^2 p_X^2/m_b^2$)
  - propagation of s-quark in soft background described by jet function
  - non-trivial dependence on (residual) $b$-quark light-cone momentum:
    - $b$-quark PDF (“shape function” for inclusive decays)
    - $B$-meson LCDA (“light-cone distribution amplitude” for exclusive decays)
Effective Field Theory Approach: QCD → SCET (→ HQET)

- Introduce separate field operators for each type of (relevant) IR-mode:
  - **collinear fields** (light quarks and gluons/photons) for each jet direction.
  - **soft fields** (light quarks and gluons/photons)
  - **quasi-static heavy-quark fields** (with soft residual momentum)

- Construct interaction terms, performing multipole-expansion of soft-collinear vertices → SCET Feynman rules.

- **Perturbative matching** to QCD at hard scale $\mu_h \sim m_b$:
  - short-distance coefficient functions (still depend on jet-energy)

- **RG running** in SCET resums large logs between hard and jet scale (incl. Sudakov logarithms)

- matching onto HQET at jet scale yields non-local operators
  - $\rightarrow$ **$b$-quark PDF** (for inclusive $B$ decays)
  - $\rightarrow$ **$B$-meson LCDA** (for exclusive $B$ decays)


Th. Feldmann (TUM)
Formalism: (see also Andre Hoang’s talk …)

Light-cone kinematics

- Specify collinear momentum direction by light-like vectors $n_+^{\mu}$ and $n_-^{\mu}$, (e.g. in rest frame or c.m.s.: $n_+^{\mu} = (1, 0, 0, 1)$ and $n_-^{\mu} = (1, 0, 0, -1)$)
- Decomposition of any Lorentz vector:

\[
p^{\mu} = (n_+ p) \frac{n_+^{\mu}}{2} + p_\perp^{\mu} + (n_- p) \frac{n_-^{\mu}}{2}
\]

- collinear particles: $(n_+ p) \gg p_\perp \gg (n_- p)$
- soft particles: $(n_+ p) \sim p_\perp \sim (n_- p)$

Physical applications require different variants of SCET

- **SCET$_I$:** (inclusive reactions (jets); intermediate step for exclusive reactions)

  (hard-)collinear particles: $p_\perp^2 \sim (n_- p)(n_+ p) \sim \Lambda (n_+ p)$

- **SCET$_II$:** (exclusive reactions)

  collinear particles: $p_\perp^2 \sim (n_- p)(n_+ p) \sim \Lambda^2$
Large and small collinear spinor components

- different derivative terms for (massless) collinear quarks scale as

\[
L_{\text{coll.}}^q = \bar{q} \left[ \left( in+D \right) \frac{\eta_-}{2} + iD_{\perp} + \left( in-D \right) \frac{\eta_+}{2} \right] q
\]

\[
\uparrow \quad \uparrow \quad \uparrow
\]

\[O(n+p) \gg O(p_{\perp}) \gg O(n-p)\]

- → large and small spinor components

\[
\xi(x) = \frac{\eta_- + \eta_+}{4} q(x), \quad \eta(x) = \frac{\eta_+ - \eta_-}{4} q(x)
\]

- solve e.o.m. for small spinor component \(\eta(x)\)

\[
\Rightarrow L^\xi = \bar{\xi} \left[ \left( in-D \right) + iD_{\perp} \frac{1}{\left( in+D \right)} iD_{\perp} \right] \frac{\eta_+}{2} \xi
\]

(still exact for interactions with collinear gluons)
Multipole Expansion (SCET_I)

- For some directions, soft fields have larger wavelengths than collinear ones:
  
  \[
  \text{soft:} \quad (n^-x, x_\perp, n^+ x) \sim \left( \Lambda^{-1}, \Lambda^{-1}, \Lambda^{-1} \right) 
  
  \text{collinear:} \quad (n^-x, x_\perp, n^+ x) \sim \left( (n+p)^{-1}, (n+p \Lambda)^{-1/2}, \Lambda^{-1} \right) 
  \]

⇒ At soft-collinear vertices, expand all soft fields around \( x_-^\mu = (n^+ x)^{\frac{n^\mu}{2}} \).

Field Redefinitions

- Leading interactions with soft gluons via
  
  \[
  i(n^- D) \rightarrow i(n^- \partial) + g(n^- A_c)(x) + g(n^- A_s)(x) 
  \]

- Perform field redefinitions for collinear quarks and gluons,
  
  \[
  \xi_c(x) \rightarrow Y_s(x_-) \xi_c(x), \quad A_c(x) \rightarrow Y_s(x_-) A_c(x) Y_s^\dagger(x_-) 
  \]

  with soft Wilson line

  \[
  Y_s(x_-) = P e^{-ig \int_0^\infty dt n_- A_s(x_- + tn_-)} \quad \text{,} \quad (i n_- \partial + g n_- A_s) Y_s = 0. 
  \]

⇒ Soft gluons decouple from collinear fields to first approximation.
Matching of external heavy-to-light currents in SCET\textsubscript{I}

Expansion in 1/$m_b$ yields:

$$\bar{\psi}(x) \Gamma Q(x) \rightarrow C_\Gamma(\mu) (\bar{\xi} W_c)(x) \Gamma (Y_s^+ h_v)(x^-) + \ldots$$

- Collinear Wilson line $W_c$ with $(i n_+ \partial + n_+ A_c) W_c = 0$, from resummation of (unsuppressed) collinear gluon radiation from heavy quark.
- Wilson coefficient $C_\Gamma$ absorbs short-distance corrections from virtual hard gluons.
- Soft and collinear divergences $\rightarrow$ Sudakov logarithms:

$$C_\Gamma(\mu) = 1 - \frac{\alpha_s C_F}{4\pi} \left( 2 \ln^2 \left( \frac{n_+ p}{\mu} \right) + \ldots \right) + \ldots$$

(sub-leading currents in the power-expansion can be identified in a similar manner)
Current renormalization in SCET

- Resum logarithms between hard scale \((n+p)\) and jet scale \(\mu \sim |\vec{p}_\perp| \sim \sqrt{\Lambda m_b}\)
  using renormalization group in SCET\(_1\)

\[
\frac{dC(\mu)}{d \ln \mu} = \left( \Gamma_{\text{cusp}}(\alpha_s) \ln \frac{(n+p)}{\mu} + \gamma(\alpha_s) \right) C(\mu)
\]

with (universal) “cusp anomalous dimension”

\[
\Gamma_{\text{cusp}} = 4 \frac{\alpha_s C_F}{4\pi} + \ldots
\]
Jet Function in inclusive reactions

\[ J(p^2, \mu) \propto \frac{1}{\pi} \text{Im} \left\{ i \int d^4x \ e^{-ipx} \langle 0 \left| T \left( W_c^\dagger \xi_c \right)(0) \left( \bar{\xi}_c W_c \right)(x) \right| 0 \rangle \right\} \]

- factorize soft and (hard-)collinear corrections to propagation of energetic quark
  (e.g. in hadronic tensor for \( b \to s\gamma \))

- one-loop result in terms of modified plus distributions:

\[
J(p^2, \mu) = \delta(p^2) + \frac{\alpha_s C_F}{4\pi} \left\{ (7 - \pi^2) \delta(p^2) - 3 \left( \frac{1}{p^2} \right)_\ast + 4 \left( \frac{\ln(p^2/\mu^2)}{p^2} \right)_\ast \right\}
\]

- two-loop result and solution of RG-equation also known [Becher/Neubert 06]
Applications
1.) $B \rightarrow X_u \ell \nu \rightarrow$ determination of $|V_{ub}|$

Factorization Theorem (leading power)

$p_X^- = E_X - |\vec{p}_X|$ spectrum in $B \rightarrow X_u \ell \nu$:

\[
\frac{d\Gamma_u}{dp_X^-} \propto \int_0^1 dy \, y^{1-2a} \, H_u(y; \mu_h) \, U(\mu_h, \mu_i) \int_0^{p_X^-} d\hat{\omega} \, J\left(ym_b(p_X^- - \hat{\omega}); \mu_i\right) \hat{S}(\hat{\omega}; \mu_i)
\]

- Hard function (QCD)
- Jet function (SCET)
- Shape fct.

- Cut $p_X^- \leq \Delta < M_D^2/M_B \simeq 0.66$ GeV, in order to suppress charm background
- RG-evolution functions $U(\mu_h, \mu_i)$ and $a = a(\mu_h, \mu_i)$

- Similar factorization theorem for $B \rightarrow X_s \gamma$ at large photon energy
- Factorization at higher orders complicated by resolved photon effects

[Paz/Lee/Neubert 09]
Issues:

Perturbative calculation of hard function(s)
- **NLO**: [Bosch/Lange/Neubert/Paz 04; Bauer/Manohar 03; Bauer/Fleming/Pirjol/Stewart 00]
- **NNLO**: [Asatrian et al. 08; Beneke et al. 08; Bell 08]

Perturbative calculation of jet function
- (massless) **NNLO**: [Becher/Neubert 05/06]
- (massive) **NLO**: [Boos/TF/Mannel/Pecjak 05; Fleming/Hoang/Mantry/Stewart 07]

Shape-function evolution
- **2-loop**: [Becher/Neubert 05]

Extracting the shape function from $B \rightarrow X_s \gamma$:
- shape-function independent relations
  - [Lange/Neubert/Paz, Lange 05; Hoang/Ligeti/Luke 05; Leibovich/Low/Rothstein 00]
  - model parametrizations and theoretical uncertainties [see below →]

Sub-leading shape-function effects
- classification
  - [Lee/Stewart 04; Bosch/Neubert/Paz 04; Beneke et al. 04; Tackmann 05]
  - shape-function independent relations (tree-level)
    - [K. Lee 08]
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    [K. Lee 08]
The $B$-meson shape function ($= b$-quark pdf in HQET)

**Definition and Properties**

\[ \hat{S}(\hat{\omega} = \bar{\Lambda} - \omega) = \langle B| h_\nu \delta(\omega - n_- \cdot D) h_\nu|B \rangle, \quad (n_- \cdot v = 1, \bar{\Lambda} = M_B - m_b) \]

- support $0 \leq \hat{\omega} < \infty$
- depends on renormalization scheme for $b$-quark mass
- radiative tail at large $\hat{\omega} \Rightarrow$ positive moments diverge

**Phenomenological constraints**

- Moments of $B \rightarrow X_c \ell \nu$ spectra (HQET parameters $\bar{\Lambda}$, $\mu_\pi^2 \rightarrow$ SF scheme)
- Photon spectrum in $B \rightarrow X_s \gamma$ (through factorization formula)
The $B$-meson shape function

$= b$-quark pdf in HQET

**Approach 1:**

- Parametrization at **low input scales**, e.g.

\[
S(\omega, \mu_0) = \frac{N}{\Lambda} \left( \frac{\hat{\omega}}{\Lambda} \right)^{b-1} \exp \left( -b \frac{\hat{\omega}}{\Lambda} \right) + \frac{\alpha_s(\mu_0)}{\pi} \times [\text{radiative tail}]
\]

- adjust to HQET parameters $\bar{\Lambda}$, $\mu^2_{\pi}$
- RG evolution to intermediate scale $\mu_i$
- compare with $B \rightarrow X_s \gamma$ spectrum
- predict $B \rightarrow X_u \ell \nu$ spectrum

![Graph showing $S(\omega, \mu)$ with different input scales $\mu_0 = 1$ GeV and $\mu_i = 1.5$ GeV]
The $B$-meson shape function (= $b$-quark pdf in HQET)

Approach 2:

- Calculate partonic matrix element: $\hat{S}_{\text{part.}}(\hat{\omega}, \mu_0) = \delta(\hat{\omega}) + \frac{\alpha_s(\mu_0)}{\pi} [\ldots]$
- Generate model shape function via convolution

\[
\hat{S}(\hat{\omega}, \mu_0) := \int dk \hat{S}_{\text{part.}}(\hat{\omega} - k, \mu_0) \hat{F}(k)
\]

- $\hat{F}(k)$ normalized to HQET parameters
- can be expanded in terms of suitable basis functions
- systematic studies of theoretical uncertainties in global fits [\ldots to be done]
Values of $|V_{ub}|$ determined at NLO and NNLO. In the columns labeled $|V_{ub}|$ the first error is experimental, the second is the sum of all theoretical and parametric errors except for that from $m_b^*$, and the third is that from $m_b^*$.

| Method              | $\Delta B^{\text{exp}} \times 10^{-4}$ | $|V_{ub}| \times 10^{-3}$ NLO | $|V_{ub}| \times 10^{-3}$ NNLO |
|---------------------|----------------------------------------|--------------------------------|--------------------------------|
| $E_l > 2.1 \text{ GeV}$ CLEO | 3.3 ± 0.2 ± 0.7  | 3.56 ± 0.40^{+0.48}_{-0.27}^{+0.31}_{-0.26} | 3.81 ± 0.43^{+0.33}_{-0.21}^{+0.31}_{-0.26} |
| $E_l > 2.0 \text{ GeV}$ BABAR | 5.7 ± 0.4 ± 0.5  | 3.97 ± 0.22^{+0.37}_{-0.23}^{+0.26}_{-0.25} | 4.30 ± 0.24^{+0.26}_{-0.20}^{+0.28}_{-0.27} |
| $E_l > 1.9 \text{ GeV}$ BELLE | 8.5 ± 0.4 ± 1.5  | 4.27 ± 0.39^{+0.32}_{-0.19}^{+0.25}_{-0.22} | 4.65 ± 0.43^{+0.27}_{-0.18}^{+0.27}_{-0.24} |
| $M_X < 1.7 \text{ GeV}$ BELLE | 12.3 ± 1.1 ± 1.2 | 3.55 ± 0.24^{+0.22}_{-0.13}^{+0.21}_{-0.19} | 3.87 ± 0.26^{+0.21}_{-0.13}^{+0.21}_{-0.19} |
| $M_X < 1.55 \text{ GeV}$ BABAR | 11.7 ± 0.9 ± 0.7 | 3.67 ± 0.18^{+0.29}_{-0.17}^{+0.26}_{-0.24} | 3.96 ± 0.19^{+0.20}_{-0.13}^{+0.26}_{-0.24} |
| $P_+ < 0.66 \text{ GeV}$ BELLE | 11.0 ± 1.0 ± 1.6 | 3.56 ± 0.31^{+0.30}_{-0.17}^{+0.27}_{-0.24} | 3.84 ± 0.33^{+0.21}_{-0.13}^{+0.26}_{-0.22} |
| $P_+ < 0.66 \text{ GeV}$ BABAR | 9.4 ± 1.0 ± 0.8  | 3.30 ± 0.23^{+0.27}_{-0.16}^{+0.25}_{-0.22} | 3.55 ± 0.24^{+0.19}_{-0.13}^{+0.24}_{-0.21} |

NNLO effects important ($\sim 10\%$ shift in $|V_{ub}|$)
2.) SCET in Exclusive $B$ Decays

Factorization Theorems for Decay Amplitudes

\[ A_i(B \to \gamma + \text{lept.}) = \xi_M(\mu) \cdot T_i^I(\mu) + T_i^{II}(\mu) \otimes \phi_B(\mu) \]

\[ A_i(B \to M + \text{lept.}) = \xi_M(\mu) \cdot T_i^I(\mu) + T_i^{II}(\mu) \otimes \phi_B(\mu) \otimes \phi_M(\mu) \]

\[ A_i(B \to MM') = \xi_M(\mu) \cdot T_i^I(\mu) \otimes \phi_{M'}(\mu) + T_i^{II}(\mu) \otimes \phi_B(\mu) \otimes \phi_M(\mu) \otimes \phi_{M'}(\mu) \]

- universal transition form factor $\xi_M$ (non-perturbative input)
- two-particle LCDAs for $B$-meson and light hadrons $M$ (non-perturbative input)
- perturbative coefficient functions: $T_i^I$ ("non-factorizable")
  \[ T_i^{II} = H_i \otimes J \] ("factorizable" in SCET$_I \to$ SCET$_{II}$)

$\Lambda/m_b$ Power-corrections lead to more factorizable and non-factorizable terms!
Recent Perturbative Results

- **NNLO vertex corrections in non-leptonic $B$ decays ($T^I_1$):**
  - imaginary part
  - real part
  
  [Bell 07]
  [Bell 09; Beneke/Huber/Li 09]

- **NLO Spectator scattering in non-leptonic $B$ decays ($T^II_1$):**
  - tree amplitudes
  - leading penguin amplitudes

  [Beneke/Jäger 05; Kivel 06; Pilipp 07]
  [Beneke/Jäger 06]

- **$O(\alpha_s^2)$ corrections in $B \to V\gamma$ decays:**
  - Contributions from $O^\gamma_7$ and $O^g_8$

  [Ali/Greub/Pecjak 07]
Collinear modes in exclusive final state have the same virtualities (i.e. same transverse momenta) as soft spectators in $B$-meson.

Dimensional regularization not sufficient to render integration over individual soft and collinear momentum regions IR-finite.

$\Rightarrow$ Soft and collinear dynamics entangled in a non-factorizable manner.

Still, in the endpoint region certain symmetry relations hold, similar to those known from the Isgur-Wise function in HQET.

$\Rightarrow$ Universal non-factorizable matrix elements in SCET$_1$, e.g.

$$
\langle \pi(p) | (\bar{\xi}_{hc} W_{hc}) \Gamma (Y_s h_\nu) | B(v) \rangle \propto \xi_\pi(n+p, \mu) \text{ tr} \left[ \frac{n+\hat{n}}{4} \Gamma \frac{1+\psi}{2} \right]
$$

Corrections to symmetry relations are factorizable (!) (or power-suppressed)

(similar statement for corrections to “naive” factorization in non-leptonic $B$ decays)
A toy integral:

- Consider (UV-finite) integral in \( D = 4 - 2\epsilon \) dimensions

\[
I = \int [\tilde{dk}] \frac{1}{[(k - l)^2][k^2 - m^2][(p - k)^2 - m^2]},
\]

with \( l^2 = m^2 \), \( p^2 = 0 \), and large momentum transfer \( p \cdot l \gg m^2 \)

- Integral decomposes into 3 momentum regions:

  - hard-collinear:

\[
I_{hc} = \int [\tilde{dk}] \frac{1}{[k^2 - (n_+ k)(n_- l)][k^2 - (n_- k)(n_+ p)]}
\]

\[
= -\frac{1}{(n_+ p)(n_- l)} \left\{ \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu^2}{(n_+ p)(n_- l)} + \frac{1}{2} \ln^2 \frac{\mu^2}{(n_+ p)(n_- l)} - \frac{\pi^2}{12} \right\},
\]

well-defined in dim-reg – can be reproduced by SCET\(_1\) Feynman rules.
A toy integral:

- Consider (UV-finite) integral in $D = 4 - 2\epsilon$ dimensions

\[ I = \int [\tilde{d}k] \frac{1}{[(k - l)^2][k^2 - m^2][(p - k)^2 - m^2]} , \]

with $l^2 = m^2$, $p^2 = 0$, and large momentum transfer $p \cdot l \gg m^2$

- Integral decomposes into 3 momentum regions:
  - **collinear:**

\[ I_c = \int [\tilde{d}k] \frac{[-\nu^2]^\delta}{[-(n_+ k)(n_- l)]^{1+\delta} [k^2 - m^2][k^2 - m^2 - (n_- k)(n_+ p)]} \]

\[ = -\frac{1}{(n_+ p)(n_- l)} \left( -\frac{1}{\delta} + \ln \frac{(n_+ p)(n_- l)}{\nu^2} \right) \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \right) , \]

not defined in dim-reg – requires additional IR regulator (here analytic reg.)
A toy integral:

Consider (UV-finite) integral in $D = 4 - 2\epsilon$ dimensions

$$I = \int [dk] \frac{1}{[(k-l)^2][k^2 - m^2][(p-k)^2 - m^2]} ,$$

with $l^2 = m^2$, $p^2 = 0$, and large momentum transfer $p \cdot l \gg m^2$

Integral decomposes into 3 momentum regions:

- **soft:**

$$I_s = \int [dk] \frac{[-\nu^2]^{\delta}}{[(k-l)^2]^{1+\delta}[k^2 - m^2][- (n_- k)(n_+ p)]}$$

$$= -\frac{1}{(n_+p)(n_- l)} \left( \left[ \frac{1}{\delta} - \ln \frac{m^2}{\nu^2} \right] \left[ \frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \right] - \frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \frac{\mu^2}{m^2} - \frac{1}{2} \ln^2 \frac{\mu^2}{m^2} + \frac{5\pi^2}{12} \right) .$$

not defined in dim-reg – requires additional IR regulator (here analytic reg.)
A toy integral:

- Consider (UV-finite) integral in $D = 4 - 2\epsilon$ dimensions

$$I = \int \frac{1}{[(k - l)^2][k^2 - m^2][(p - k)^2 - m^2]} \,,$$

with $l^2 = m^2$, $p^2 = 0$, and large momentum transfer $p \cdot l \gg m^2$

- Integral decomposes into 3 momentum regions: [see Beneke/Smirnov]

- Additional IR regulator drops out in sum, 

$$I = I_{hc} + (I_c + I_s) + \mathcal{O}(\frac{m^2}{p \cdot l})$$

non-perturbative / non-factorizable

Recent ideas to solve the issue by so-called “zero-bin” subtractions [Manohar/Stewart] have been shown not to work consistently in exclusive decays [Beneke/Vernazza]
Consider correlation function in SCET$_{1}$:
exclusive final state (e.g. pion) is replaced by (off-shell) interpolating current.

\[ \Rightarrow \text{Factorization theorem for correlation function (soft} \otimes \text{hard-collinear)} \]

Dispersion relation between
- (unphysical) region of large (hc) space-like momenta
- physical spectral function, containing the hadronic state

\[ \Rightarrow \text{Sum rule for non-factorizable form factor in SCET}_{1}: \]
\[ \xi_{\pi}(q^{2}, \mu) \text{ in terms of light-cone distribution amplitudes of } B \text{ meson} \]

- correlation function at tree level:
\[ \Pi_{0}(n{\cdot}p') = f_{B}m_{B} \int_{0}^{\infty} d\omega \frac{\phi_{B}^{B}(\omega)}{\omega - n{\cdot}p' - i\eta} \]

- $\phi^{B}_{-}(\omega)$: distribution amplitude for light-cone momentum of $B$-meson spectator.
Consider correlation function in SCET\(_1\): exclusive final state (e.g. pion) is replaced by (off-shell) interpolating current.

⇒ Factorization theorem for correlation function \((\text{soft} \otimes \text{hard-collinear})\)

- Dispersion relation between
  - (unphysical) region of large (hc) space-like momenta
  - physical spectral function, containing the hadronic state

⇒ Sum rule for non-factorizable form factor in SCET\(_1\):

\[\xi_\pi(q^2, \mu)\] in terms of light-cone distribution amplitudes of \(B\) meson

\[m_b f_\pi \xi_\pi = \frac{1}{\pi} \int_0^{\omega_s} d\omega' e^{-\omega' / \omega_M} \text{Im} \left[ \Pi_0(\omega') \right]\]

[\(\omega_s, \omega_M\): intrinsic sum-rule parameters to be optimized]
Radiative corrections

- Corrections involving $\phi^B_-(\omega)$:

  ![Diagram with various Feynman diagrams involving $J_\pi$, $J_0$, and $h_c$]

  - Explicit calculation: Factorization works (on the level of correlator) ✓
  - But $O(\alpha_s)$ result for *form factor* contains (non-resummed) **large logarithms** involving sum rule parameters. (!)

- Corrections involving 3-particle LCWF:

  (only tree-level available so far [Khodjamirian/Mannel/Offen])
Predictions for non-leptonic $B$ decays:

- Specify **hadronic input** (lattice, sum rules, ... or data).
- Guesstimate size of **power corrections**.
- Calculate corrections to naive factorization to sufficient **accuracy** (including renormalization running in SCET$_\text{II}$)
- Compare with experiment.

[here: for tree-dom. decays at NNLO, Beneke/Huber/Li 09]

<table>
<thead>
<tr>
<th></th>
<th>Theory I (lattice,SR)</th>
<th>Theory II (data)</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^- \rightarrow \pi^- \pi^0$</td>
<td>$5.43^{+0.06}<em>{-0.06} +0.12^{+0.15}</em>{-0.84}$</td>
<td>$5.82^{+0.07}<em>{-0.06} +0.14^{+0.12}</em>{-1.35}$</td>
<td>$5.59^{+0.41}_{-0.40}$</td>
</tr>
<tr>
<td>$\bar{B}_d^0 \rightarrow \pi^+ \pi^-$</td>
<td>$7.37^{+0.86}<em>{-0.61} +0.12^{+0.22}</em>{-0.97}$</td>
<td>$5.70^{+0.70}<em>{-0.55} +0.16^{+0.17}</em>{-0.97}$</td>
<td>$5.16 \pm 0.22$</td>
</tr>
<tr>
<td>$\bar{B}_d^0 \rightarrow \pi^0 \pi^0$</td>
<td>$0.33^{+0.10}<em>{-0.08} +0.12^{+0.17}</em>{-0.42}$</td>
<td>$0.63^{+0.12}<em>{-0.10} +0.14^{+0.16}</em>{-0.42}$</td>
<td>$1.55 \pm 0.19$</td>
</tr>
<tr>
<td>$B^- \rightarrow \pi^- \rho^0$</td>
<td>$8.68^{+0.42}<em>{-0.41} +0.21^{+0.15}</em>{-1.56}$</td>
<td>$9.84^{+0.41}<em>{-0.40} +2.54^{+0.85}</em>{-2.52}$</td>
<td>$8.3^{+1.2}_{-1.3}$</td>
</tr>
<tr>
<td>$B^- \rightarrow \pi^0 \rho^-$</td>
<td>$12.38^{+0.90}<em>{-0.77} +0.16^{+2.18}</em>{-1.41}$</td>
<td>$12.13^{+0.85}<em>{-0.73} +2.23^{+1.49}</em>{-2.17}$</td>
<td>$10.9^{+1.4}_{-1.5}$</td>
</tr>
<tr>
<td>$\bar{B}_d^0 \rightarrow \pi^+ \rho^-$</td>
<td>$17.80^{+0.62}<em>{-0.56} +1.76^{+2.10}</em>{-2.10}$</td>
<td>$13.76^{+0.49}<em>{-0.44} +1.77^{+0.34}</em>{-2.18}$</td>
<td>$15.7 \pm 1.8$</td>
</tr>
<tr>
<td>$\bar{B}_d^0 \rightarrow \pi^+ \rho^+$</td>
<td>$10.28^{+0.39}<em>{-0.39} +1.37^{+1.32}</em>{-1.42}$</td>
<td>$8.14^{+0.34}<em>{-0.33} +1.35^{+0.49}</em>{-1.49}$</td>
<td>$7.3 \pm 1.2$</td>
</tr>
<tr>
<td>$\bar{B}_d^0 \rightarrow \pi^0 \rho^+\pi^0$</td>
<td>$28.08^{+0.27}<em>{-0.19} +3.82^{+3.50}</em>{-1.20}$</td>
<td>$21.90^{+0.20}<em>{-0.12} +3.06^{+3.55}</em>{-3.55}$</td>
<td>$23.0 \pm 2.3$</td>
</tr>
<tr>
<td>$B^- \rightarrow \rho^- \rho^0$</td>
<td>$0.52^{+0.04}<em>{-0.03} +1.11^{+0.15}</em>{-0.43}$</td>
<td>$1.49^{+0.07}<em>{-0.07} +1.77^{+1.29}</em>{-1.29}$</td>
<td>$2.0 \pm 0.5$</td>
</tr>
<tr>
<td>$\bar{B}_d^0 \rightarrow \rho^+ \rho^-\rho^0\rho^0$</td>
<td>$18.42^{+0.23}<em>{-0.21} +3.92^{+2.55}</em>{-3.50}$</td>
<td>$19.06^{+0.24}<em>{-0.22} +4.59^{+4.22}</em>{-4.22}$</td>
<td>$22.8^{+1.8}_{-1.9}$</td>
</tr>
<tr>
<td>$\bar{B}_d^0 \rightarrow \rho^+ \rho^-\rho^0\rho^0$</td>
<td>$25.98^{+0.85}<em>{-0.77} +2.93^{+3.43}</em>{-3.43}$</td>
<td>$20.66^{+0.68}<em>{-0.62} +2.99^{+2.75}</em>{-3.75}$</td>
<td>$23.7^{+3.1}_{-3.2}$</td>
</tr>
<tr>
<td>$\bar{B}_d^0 \rightarrow \rho^+ \rho^-\rho^0\rho^0$</td>
<td>$0.39^{+0.03}<em>{-0.03} +0.83^{+0.36}</em>{-0.36}$</td>
<td>$1.05^{+0.05}<em>{-0.04} +1.62^{+1.04}</em>{-1.04}$</td>
<td>$0.55^{+0.22}_{-0.24}$</td>
</tr>
</tbody>
</table>

CP-averaged branching fractions in units of $10^{-6}$ of tree-dominated $B \rightarrow \pi \pi$, $\pi \rho$ and $\rho_L \rho_L$ decays. The first error on a quantity comes from the CKM parameters, while the second one stems from all other parameters added in quadrature. (⋆, ⋆⋆, †: additional overall uncertainty from form factors)

Th. Feldmann (TUM) SCET and B-decays Vienna, Nov 2009 24 / 31
SCET helps:

- to separate effects associated to different dynamical scales appearing in processes involving soft and energetic particles,
- to establish the corresponding factorization theorems,
- to define/identify process-independent non-perturbative input parameters/functions.
- to resum large logarithms in RG-improved perturbation theory.
Summary

SCET Applications:

- **Inclusive** $B$ decays:
  - Factorization theorems
  - Precise determination of $|V_{ub}|$ from $B \to X_u \ell \nu$ (→ $B$-meson shape function)
  - Not discussed: SM Precision Tests in $B \to X_s \gamma$ [Lee/Neubert/Paz 09]
  - Not discussed: Shape-function effects in $B \to X_s \ell^+\ell^-$ [Lee/Ligeti/Stewart/Tackmann 06]

- **Exclusive** $B$ decays:
  - Factorization theorems
  - Endpoint divergencies → non-factorizable dynamics (→ form factor, power corr.)
  - QCD corrections to non-leptonic $B$ decays ((N)NLO, depending on hadronic input)
  - SCET sum rules for (soft) form factors

- Collider Physics (QCD + EW):
  [see A. Hoang’s talk]
Backup Slides:
Modified plus distributions

\[
\int_{\leq 0}^{M} du F(u) \left( \frac{1}{u} \right)^{[m]} = \int_{0}^{M} du \frac{F(u) - F(0)}{u} + F(0) \ln \left( \frac{M}{m} \right),
\]

\[
\int_{\leq 0}^{M} du F(u) \left( \frac{\ln(u/m)}{u} \right)^{[m]} = \int_{0}^{M} du \frac{F(u) - F(0)}{u} \ln \frac{u}{m} + \frac{F(0)}{2} \ln^2 \left( \frac{M}{m} \right).
\]

satisfying

\[- \frac{1}{\pi} \text{Im} \left[ \ln \left( - \frac{u}{m} \right) \frac{1}{u} \right] = \left( \frac{1}{u} \right)^{[m]},\]

\[- \frac{1}{\pi} \text{Im} \left[ \ln^2 \left( - \frac{u}{m} \right) \frac{1}{u} \right] = 2 \left( \frac{\ln(u/m)}{u} \right)^{[m]} - \frac{\pi^2}{3} \delta(u),\]
Theoretical accuracy in $B \rightarrow X_s \gamma$ (large photon energy)

**Phenomenological Importance:**

- Good understanding of $d\Gamma/dE_\gamma$ important for $|V_{ub}|$ extraction (see above)
- $\Gamma(B \rightarrow X_s \gamma)$ sensitive to New Physics.

**Complication:**

Operators in weak effective Hamiltonian for $b \rightarrow s$ transitions contribute differently to hadronization process:

- electromagnetic operator $\mathcal{O}_7(b \rightarrow s\gamma)$
- chromomagnetic operator $\mathcal{O}_8(b \rightarrow sg)$
- 4-quark operators $\mathcal{O}_{1-6}(b \rightarrow s q\bar{q})$

New effects at sub-leading order in $1/m_b$ expansion:

- Photon does not couple directly to short-distance $b \rightarrow s$ transition.
  $\Rightarrow$ New Factorization Theorem

[Th. Feldmann (TUM)]

[SCET and B-decays]

[Vienna, Nov 2009 28 / 31]
Structure of New Factorization Formula

Features of “resolved” photon contribution:

- Involves new jet function $\bar{J}$ in opposite direction to $X_s$
- New soft functions from operators that are non-local in 2 light-cone directions
- Potential mechanism to observe CP Violation in $B \rightarrow X_s\gamma$
- Leading mechanism for Isospin Violation in $B \rightarrow X_s\gamma$
- Difficult to estimate – Vacuum Insertion Approximation $\sim 5\%$
Light-Cone Distribution Amplitudes

**B-mesons:**

- 2-particle LCDAs defined from HQET matrix elements:
  \[
  \langle 0 | \bar{q}(z) \beta [z, 0] h_v(0) \alpha | B(v) \rangle \quad \text{(with } z^2 = 0) \]

- 2 independent Dirac structures \( \rightarrow \phi_B^+(\omega), \phi_B^-(\omega), \)
  with light-cone momentum \( \omega \) of the light quark (after Fourier transform.)

**Properties:**

- \( \frac{1}{\omega} \) moment of \( \phi_B^+(\omega) \) relevant for leading contribution in factorization theorem.
- 1-loop evolution equation for \( \phi_B^+(\omega, \mu) \)
  \( \text{[Lange/Neubert 03]} \)
- Phenomenological parametrizations:
  - Sum rules: \( \langle \omega^{-1} \rangle_{\mu=1 \text{ GeV}} = (2.15 \pm 0.5) / \text{GeV} \)
  \( \text{[Braun/Ivanov/Korchemsky 03]} \)
  - Moment analysis: \( \langle \omega^{-1} \rangle_{\mu=1 \text{ GeV}} = (2.09 \pm 0.24) / \text{GeV} \)
  \( \text{[Lee/Neubert 05]} \)
Non-relativistic Toy-Model for $B$-meson LCDAs:

- Light constituent quark mass $m$
  \[ \Rightarrow \phi_B^{\pm}(\omega, \mu \sim m) \simeq \delta(\omega - m) \]

- Study evolution towards relativistic scales $\mu \gg m$:
  \[(\text{WW approx. for } \phi_B^{-}(\omega))\]

- Light quark mass $m$ (assumption: $m \gg \Lambda$)

- $\phi_B^{\pm}(\omega, \mu \sim m) \simeq \delta(\omega - m)$

- Study evolution towards relativistic scales $\mu \gg m$:
  \[(\text{WW approx. for } \phi_B^{-}(\omega))\]

- $\phi_B^{\pm}(\omega) \propto \omega$ for $\omega \to 0$
- $\phi_B^{-}(\omega) \propto \text{const.}$ for $\omega \to 0$
- Radiative tail for $\phi_B^{\pm}(\omega)$: positive moments do not exist (analogous to SF)

[Bell/Feldmann 08, Talk by G. Bell at SCET’08]