# Universal EFT for Strongly Interacting Quantum Systems


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**6th Vienna Central European Seminar on Particle Physics and QFT**

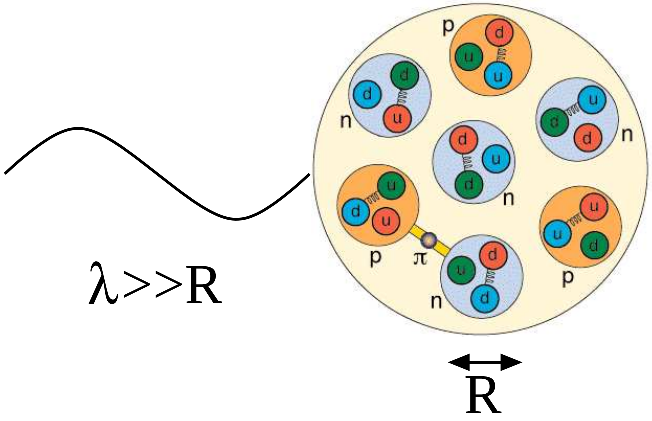
# Agenda

- Introduction
- Resonant Interactions and Weakly-Bound States
- Effective Field Theory for Large Scattering Length
- Applications
  - Ultracold atoms
  - Hadronic Molecules
  - Halo nuclei
- Summary and Outlook

**Collaborators:** E. Braaten, D. Canham, D. Kang, L. Platter, R. Springer, ...

**Review article:** Braaten, HWH, Phys. Rep. **428** (2006) 259

# Effective Theory




- Separation of scales:
  $1/k = \lambda \gg R$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of $kR$

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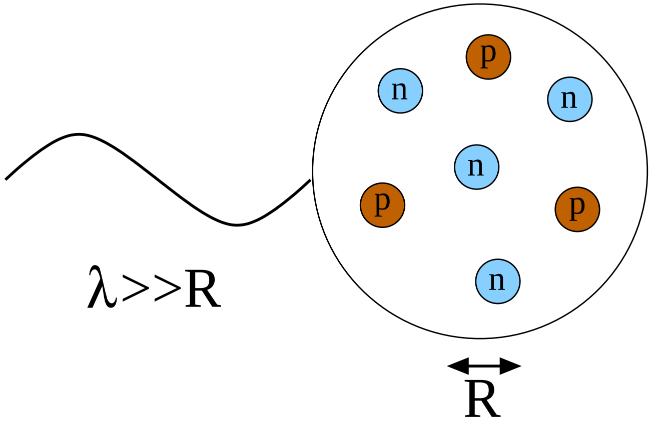
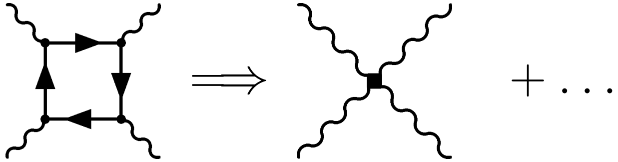



- Short-distance physics not resolved
  $\longrightarrow$ capture in low-energy constants using renormalization
  $\longrightarrow$ include long-range physics explicitly
- Systematic, model independent $\rightarrow$ universal properties
- Classic example: light-light-scattering (Euler, Heisenberg, 1936)

  Simpler theory for $\omega \ll m_e$:

  $\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \rightarrow \mathcal{L}_{eff}[A_\mu]$



# Resonant Interactions

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- Large scattering length: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots$
- Natural expansion parameter: $\ell/|a|$, $k\ell$,...

$$a > 0 \quad \Longrightarrow \quad B_d = \frac{1}{2\mu a^2} + \mathcal{O}(\ell/a)$$

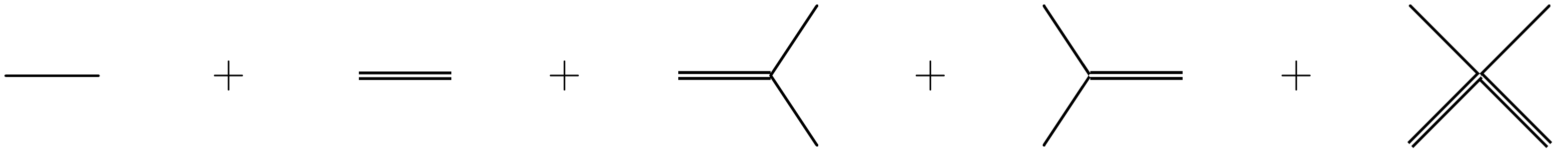
- Atomic physics:
  - $^4$He: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim l_{vdW} \longrightarrow B_d \approx 100$ neV
  - Feshbach resonances $\Longrightarrow$ variable scattering length
- Nuclear physics: $S$-wave $NN$-scattering, halo nuclei,...
  - $^1S_0$, $^3S_1$: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - $^6$He $\Rightarrow \alpha nn$: $2n$ separation energy $\approx 1$ MeV
- Particle physics:
  - $X(3872)$ as a $D^0\bar{D}^{0*}$ molecule? ($J^{PC} = 1^{++}$)

$$B_X = m_{D^0} + m_{D^{0*}} - m_X = (0.3 \pm 0.4) \text{ MeV}$$

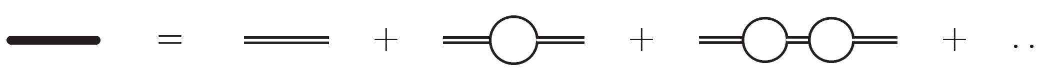
# Two-Body System in EFT

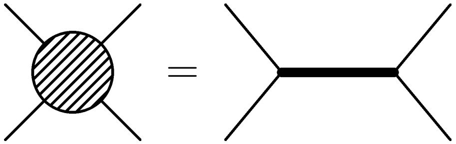
- Effective Lagrangian (Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

$$\mathcal{L}_d = \psi^\dagger\left(i\partial_t + \frac{\vec{\nabla}^2}{2m}\right)\psi + \frac{g_2}{4}d^\dagger d - \frac{g_2}{4}(d^\dagger\psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36}d^\dagger d\psi^\dagger\psi + ..$$



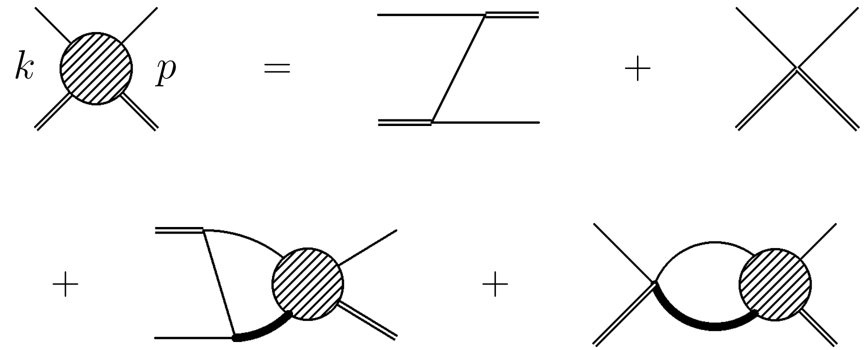
- Interacting dimeron propagator $\longrightarrow$ sum bubbles



- Two-body amplitude $\mathcal{T}_2(k,k)$:  $\propto \dfrac{1}{1/a - ik} + \ldots$
- Matching: $g_2 \longleftarrow a, B_d, \ldots$
- RG fixed points of $g_2$: $a = 0$ and $a = \infty$ (scale invariance)
- Higher order corrections $\Longrightarrow$ perturbation theory

# Three-Body System in EFT

- Three-body equation :

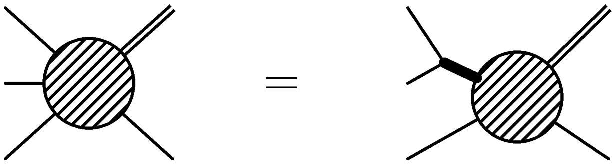



$$\mathcal{T}_3(k,p) = M(k,p) + \frac{4}{\pi}\int_0^{\Lambda} dq\, q^2\, M(q,p) D_d(q)\, \mathcal{T}_3(k,q)$$

with $\quad M(k,p) \quad = \quad \underbrace{F(k,p)}_{\text{1-atom exchange}} \quad \underbrace{-\frac{g_3}{9g_2^2}}_{H(\Lambda)/\Lambda^2}$

($g_3 = 0$, $\Lambda \to \infty \longrightarrow$ Skorniakov, Ter-Martirosian ’57)

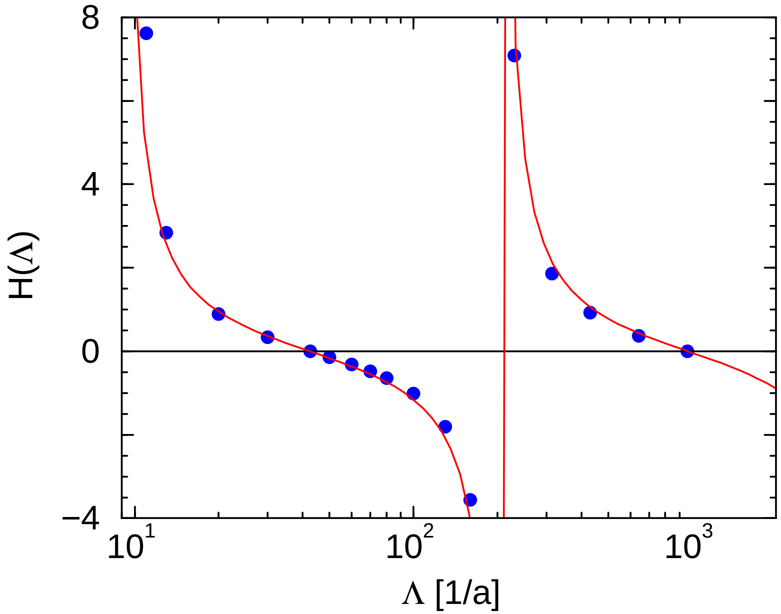
- Recombination, break-up:



# Renormalization

- Observables are independent of regulator/cutoff $\Lambda$

$\Rightarrow$ Running coupling $H(\Lambda)$




- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Full scale invariance broken to discrete subgroup

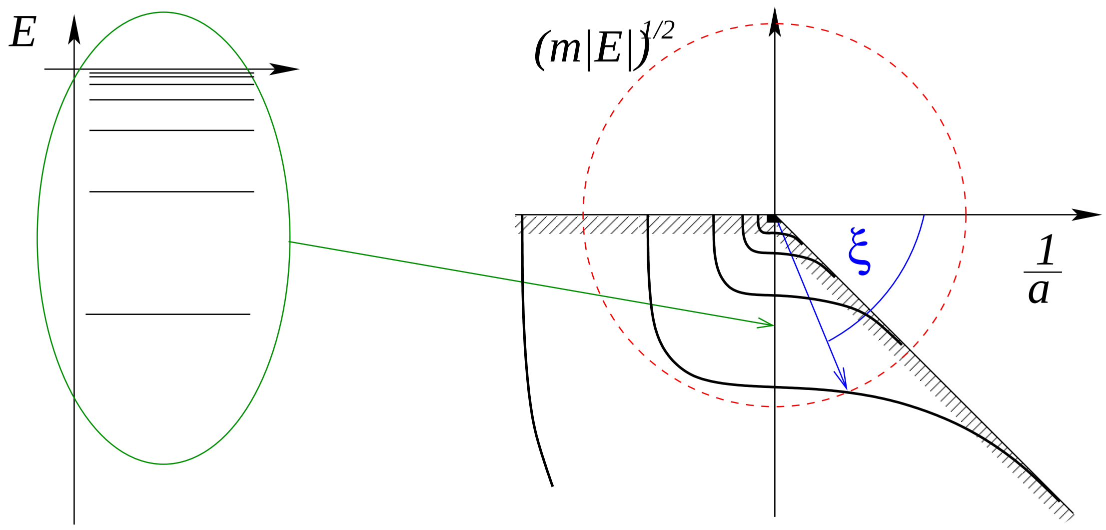
$$H(\Lambda) = \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance
- Matching: $\Lambda_* \longleftarrow B_t,\, K_3, \ldots \longrightarrow \kappa_*, a_*, a'_*$

# Limit Cycle: Efimov Effect

- Universal spectrum of three-body states

  (V. Efimov, Phys. Lett. **33B** (1970) 563)



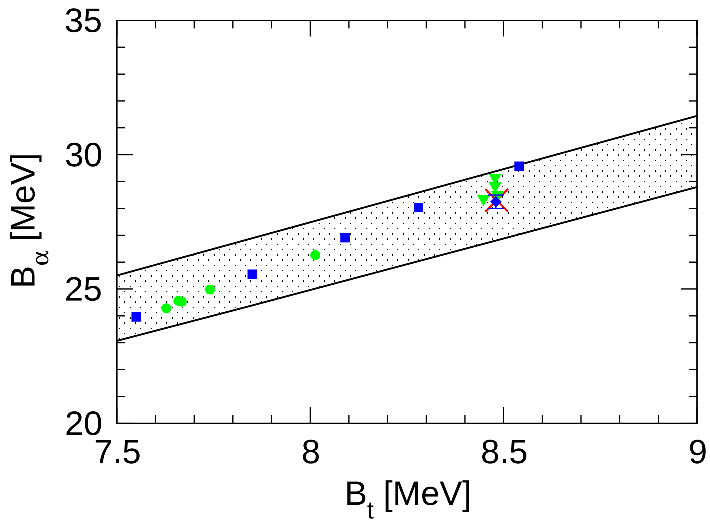

  - Discrete scale invariance for fixed angle $\xi$
  - Geometrical spectrum für $1/a \to 0$

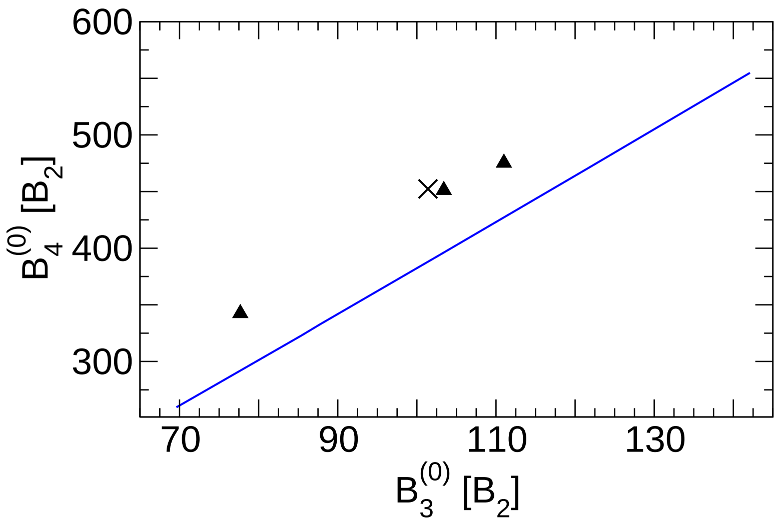
$$B_3^{(n)}/B_3^{(n+1)} \xrightarrow{1/a \to 0} 515.035\ldots$$

- Ultracold atoms $\Longrightarrow$ variable scattering length

# Universal Correlations

- Two parameters at LO
  $\Rightarrow$ universal correlations generated by 3-body parameter
- RG analysis (Platter, HWH, Meißner, 2004)
  $\Rightarrow$ No four-body parameter at LO
  $\Rightarrow$ 4-body observables are correlated $\Longrightarrow$ Tjon line




- Nuclear physics: $\Lambda$ dependence of $V_{low-k}$ (Bogner et al., 2004)
- Tjon line also at NLO (Kirscher et al., 2009)

# More on the 4-Body System

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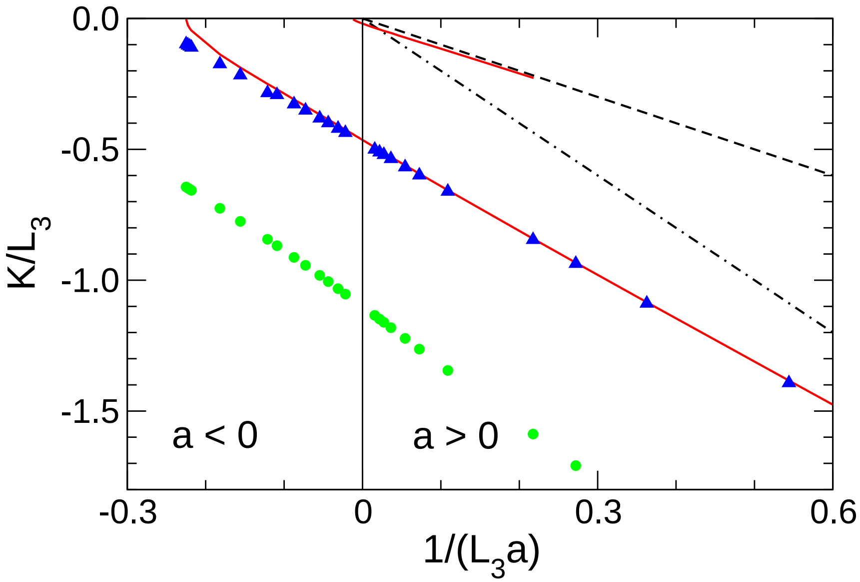
- Universal properties of 4-body system with large $a$
  - Bound state spectrum, scattering observables, ...
- “Efimov-plot”: 4-body bound state spectrum as function of $1/a$

$$K = \mathrm{sign}(E)\sqrt{m|E|}$$

$$B_4^{(0)} = 5B_3^{(0)} \qquad (1/a \equiv 0)$$

$$B_4^{(1)} = 1.01B_3^{(0)}$$

(Platter, HWH, EPJA **32** (2007) 113)



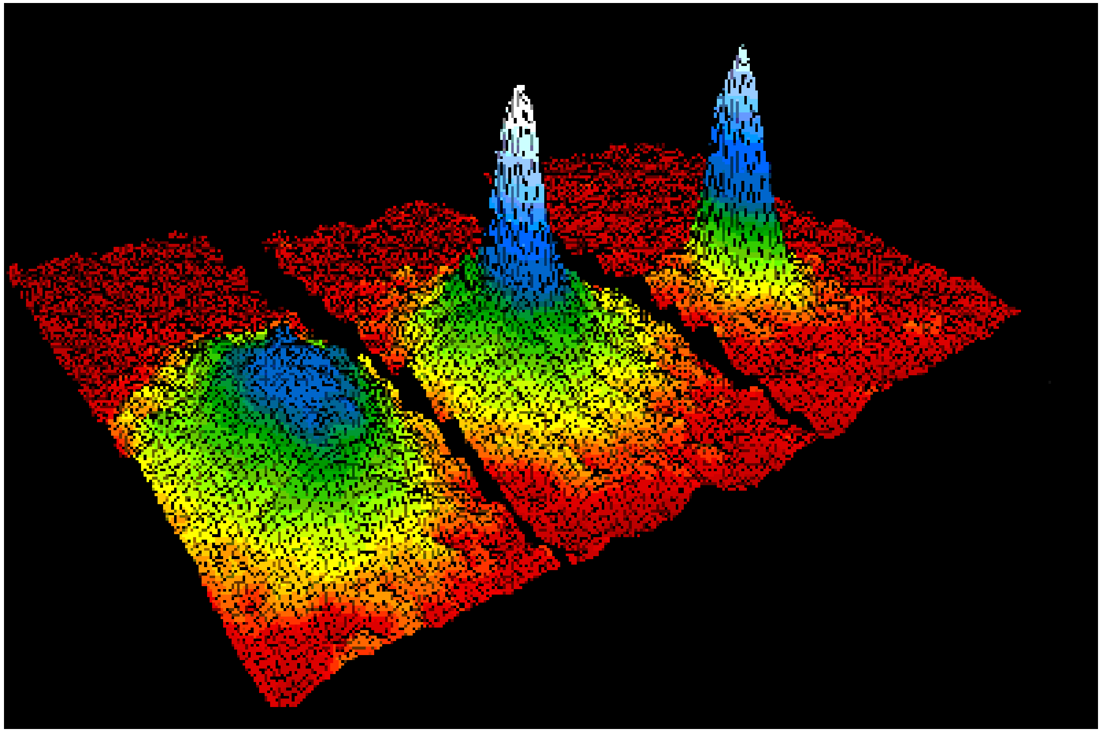

- Improved theoretical decription and observation in ultracold atoms

von Stecher, D’Incao, Greene, Nature Physics **5** (2009) 417

Ferlaino, Knoop, Berninger, Harm, D’Incao, Nägerl, Grimm, PRL **102** (2009) 140401

# Efimov Physics in Cold Atoms

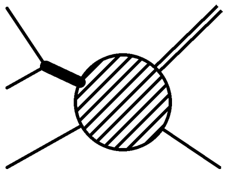
- Velocity distribution ($T =$ 400 nK, 200 nK, 50 nK)



(Source: http://jilawww.colorado.edu/bec/)

- Few-body loss rates provide window on Efimov physics
- Variable scattering length via Feshbach resonances

# Three-Body Recombination



- Three-body recombination:

  3 atoms $\rightarrow$ dimer + atom $\Rightarrow$ loss of atoms

- Recombination constant: $\dot{n}_A = -K_3 \, n_A^3$
- $K_3$ has log-periodic dependence on scattering length

  (Nielsen, Macek, 1999; Esry, Greene, Burke, 1999; Bedaque, Braaten, HWH, 2000)

- Resonant enhancement for $a < 0$
- Universal line shape of recombination resonance

  (Braaten, HWH, 2004)

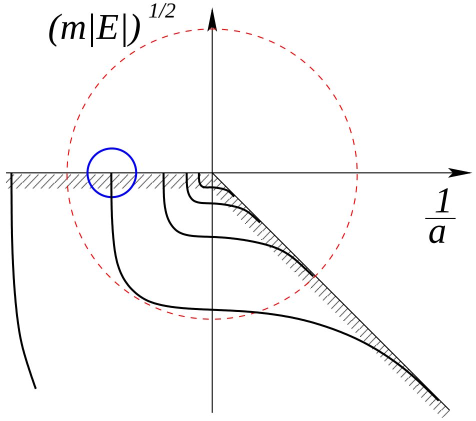
$$K_3^{deep} = \frac{(4677 \pm 2)\ \sinh 2\eta_*}{\sin^2 [s_0 \ln(a/a'_*)] + \sinh^2 \eta_*} \frac{\hbar\, a^4}{m}\,, \qquad s_0 \approx 1.00624..$$

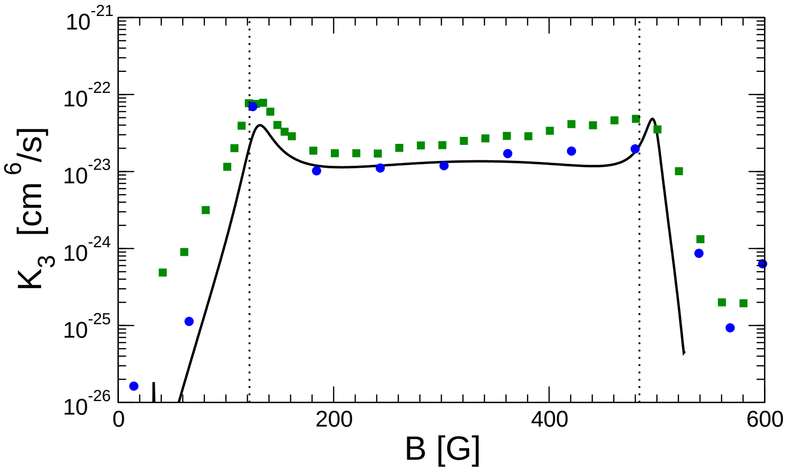
- Evidence for Efimov trimers in $^{133}$Cs

  (Kraemer et al. (Innsbruck), Nature **440** (2006) 315 )

# Efimov Physics with Fermions

- Efimov effect for fermions $\Rightarrow \geq 3$ spin states ($|1\rangle, |2\rangle, |3\rangle, ..$)
- Experimental evidence for Efimov states in $^6$Li
  - ■ Ottenstein et al. (Heidelberg), Phys. Rev. Lett. **101** (2008) 203202
  - ● Huckans et al. (Penn State), Phys. Rev. Lett. **102** (2009) 165302



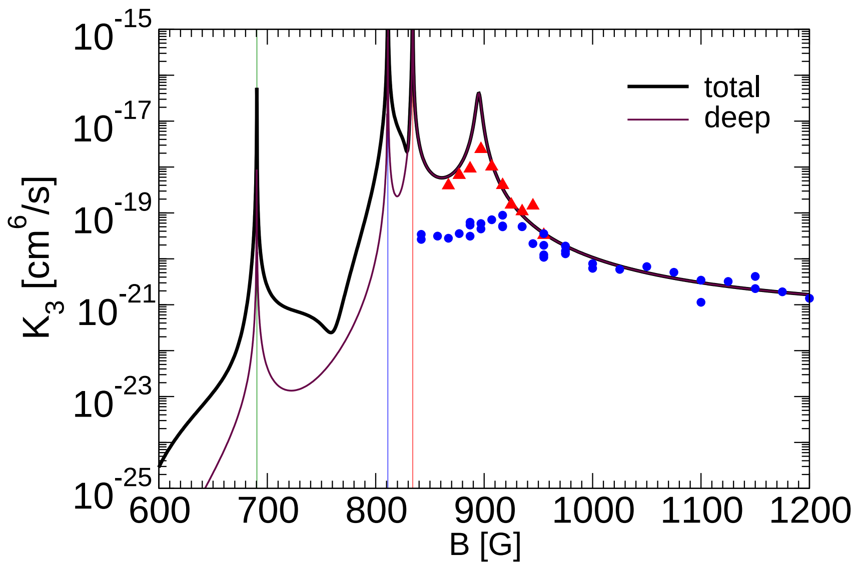



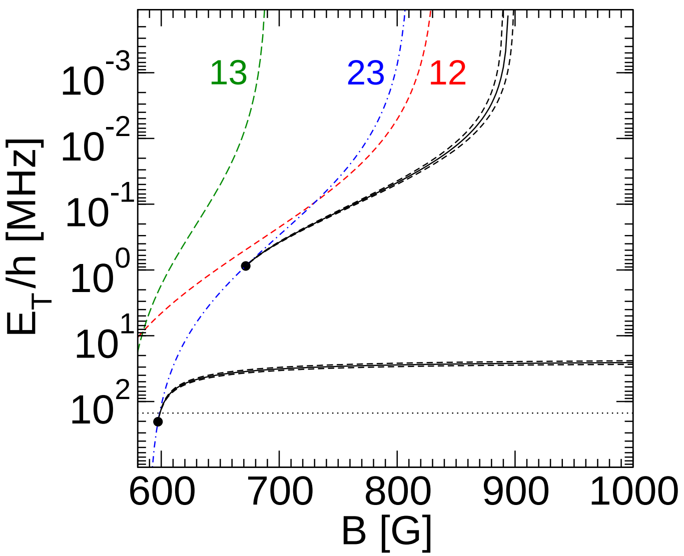

Braaten, HWH, Kang, Platter, Phys. Rev. Lett. **103** (2009) 073202

- Systematic normalization error: 70-90%
- Related work: Naidon, Ueda; Schmidt, Floerchinger, Wetterich (2009)

# Efimov Physics in $^6$Li

- Recombination resonances in high field region ($|a| \gtrsim 30\,\ell_{vdW}$)
  Williams et al. (Penn State), arXiv:0908.0789
- Recombination and bound state spectrum




Braaten, HWH, Kang, Platter, arXiv:0908.4046

- Predictions for:
  - Two trimer states and widths
  - Atom-dimer relaxation resonance ($1-23$)

# Efimov Physics in Other Atoms

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- Atom-dimer resonance in $^{133}$Cs

  (Knoop et al. (Innsbruck), Nature Physics **5** (2009) 227)

  (cf. Helfrich, HWH, EPL **86** (2009) 53003)

- Heteronuclear resonances in a mixture of $^{41}$K and $^{87}$Rb atoms

  (Barontini et al. (Florence), Phys. Rev. Lett. **103** (2009) 043201)

  $\Rightarrow$ Connected K-Rb-Rb resonances for $a > 0$ and $a < 0$

- Efimov spectrum in ultracold $^{39}$K atoms

  (Zaccanti et al. (Florence), Nature Physics **5** (2009) 586)

  $\Rightarrow$ Observation of first two states of an Efimov spectrum

- Observation of three- and four-body resonances in $^{7}$Li

  (Gross et al. (Ramat-Gan), Phys. Rev. Lett. **103** (2009) 163202)

  (Pollack, Dries, Hulet (Rice), arXiv:0911.0893)

# Exotic Charmonium Mesons

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- Many new $c\bar{c}$-mesons at B-factories: $X$, $Y$, $Z$
  - Challenge for understanding of QCD
  - Large scattering length physics important
- Example: $X(3872)$ (Belle, CDF, BaBar, D0)

$$m_X = (3871.55 \pm 0.20)\ \text{MeV} \qquad \Gamma < 2.3\ \text{MeV} \qquad J^{PC} = 1^{++}$$

- No ordinary $c\bar{c}$-state
  - Decays violate isospin
  - Measured mass depends on decay channel
- Nature of $X(3872)$ ?
  - $D^0 D^{0*}$-molecule? (cf. Tornquist, 1991)
  - Tetraquark
  - Charmonium Hybrid
  - ...

# Nature of $X(3872)$

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- Nature of $X(3872)$ not finally resolved
- Assumption: $X(3872)$ is weakly-bound $D^0$-$\bar{D}^{0*}$-molecule

$\Longrightarrow |X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0 D^{0*}\rangle)/\sqrt{2}\,, \qquad B_X = (0.26 \pm 0.41)$ MeV

$\Longrightarrow$ universal properties (cf. Braaten et al., 2003-2008, ...)

  - Explains isospin violation in decays of $X(3872) \Rightarrow$ superposition of $I = 1$ and $I = 0$
  - Different masses due to different line shapes in decay channels

- EFT with explicit pions: short distance contributions dominate (Fleming, Kusunoki, Mehen, van Kolck, 2007)

$\Longrightarrow$ EFT for large scattering length is applicable

- Large scattering length determines interaction of $X(3872)$ with $D^0$ and $D^{0*}$

# Interactions of $X(3872)$



- Large scattering length determines interaction of $X(3872)$ with $D^0$ and $D^{0*}$
- Efimov effect?

  $\Rightarrow$ occurs if 2 out of 3 pairs have resonant interactions
- $X(3872)$: only 3 out of 6 pairs have resonant interactions

  $\Rightarrow$ no Efimov effect (Braaten, Kusunoki, 2003)

  $\Rightarrow$ no $X$-$D^0$- and $X$-$D^{0*}$-molecules

  $\Rightarrow$ no three-body interaction at leading order

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  $\Rightarrow$ no $X$-$D^0$- and $X$-$D^{0*}$-molecules

  $\Rightarrow$ no three-body interaction at leading order
- But: parameter-free prediction of $X$-$D^0$-, $X$-$D^{0*}$-scattering
- Low-energy parameters: $B_X = (0.26 \pm 0.41)$ MeV

  $\Rightarrow$ Scattering length in the $X$ channel: $a = (8.8^{+\infty}_{-3.3})$ fm

# $X$-$D$- and $X$-$D^*$-Scattering

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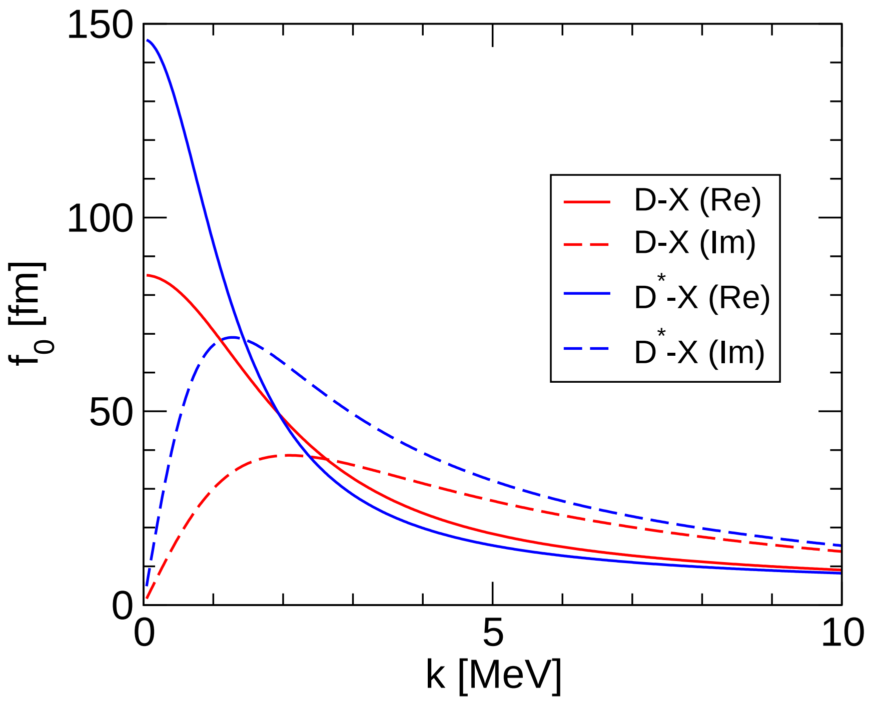
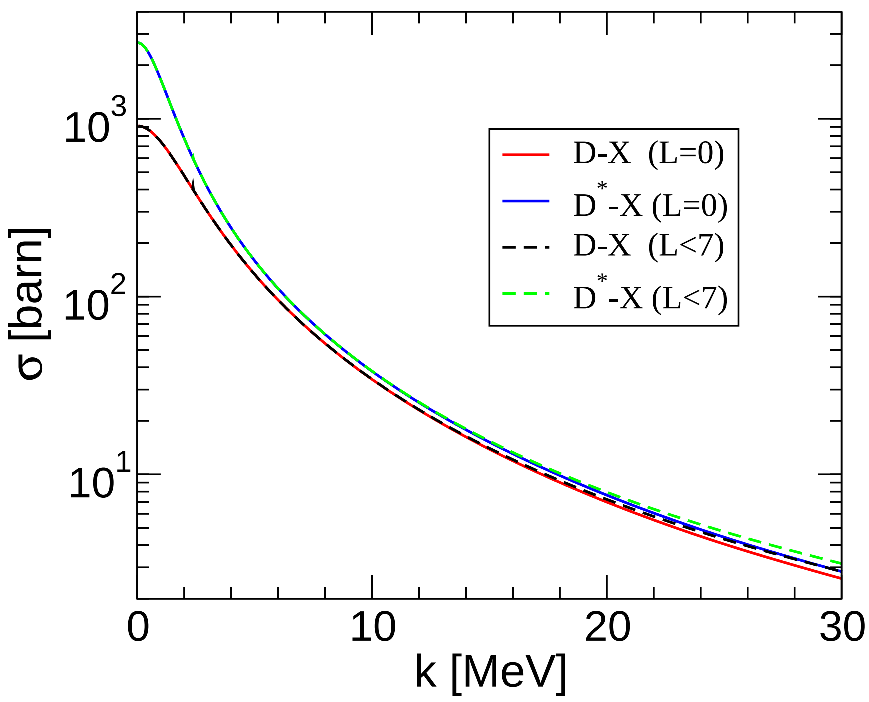
- Predictions for scattering amplitude/cross section




Canham, HWH, Springer, Phys. Rev. D **80**, 014009 (2009)

- Three-body scattering lengths

$$a_{D^0 X} = a_{\bar{D}^0 X} = -9.7 a\,, \quad \text{and} \quad a_{D^{*0} X} = a_{\bar{D}^{*0} X} = -16.6\, a$$

# Experimental Observation ?

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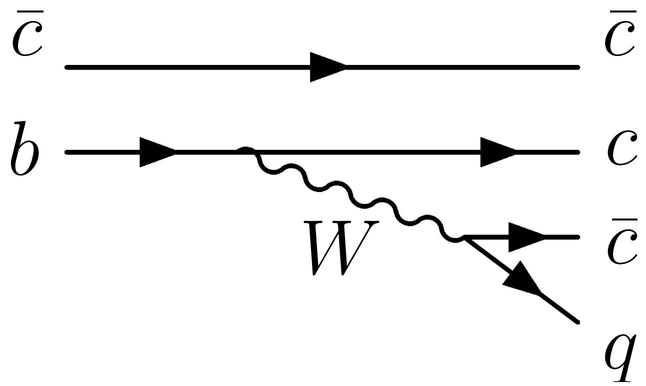
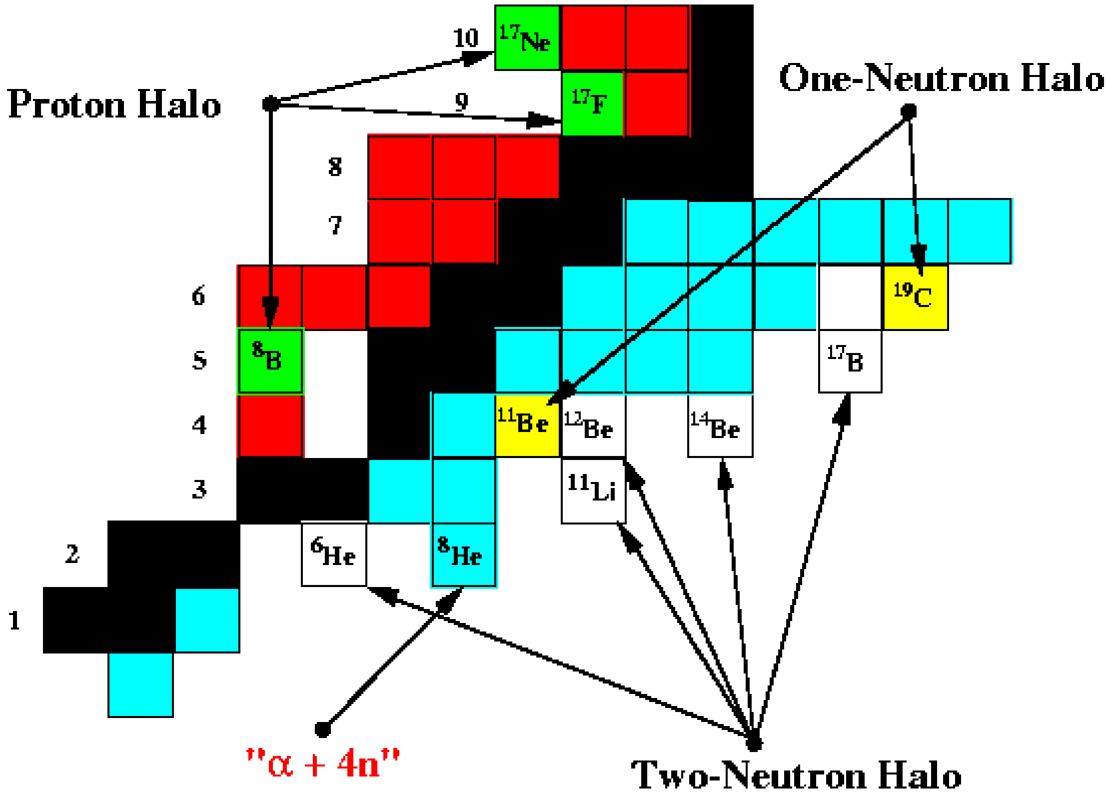
- Behavior of $X(3872)$ produced in isolation should be distinguishable from its behavior when in the presence of $D^0, D^{*0}, \bar{D}^0, \bar{D}^{*0}$
- Rare events in $B\bar{B}$ production ($B \to X$, $\bar{B} \to D, D^*$)
- Final state interaction of $D$, $D^*$ mesons in $B_c$-decays
- Example: quark-level $B_c$ decay yielding three charmed/anticharmed quarks in final state




- Process may be accessible at the LHC

# Halo Nuclei

- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$
  $\longrightarrow$ close to "nucleon drip line" $\longrightarrow$ scale separation $\longrightarrow$ EFT




http://www.nupecc.org

- EFT for halo nuclei $\iff$ cluster models

# 3-Body Halos

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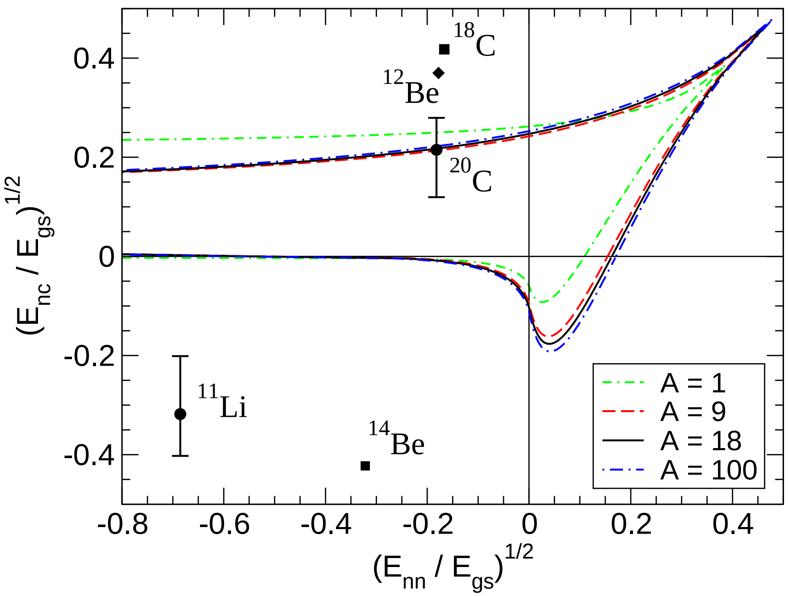
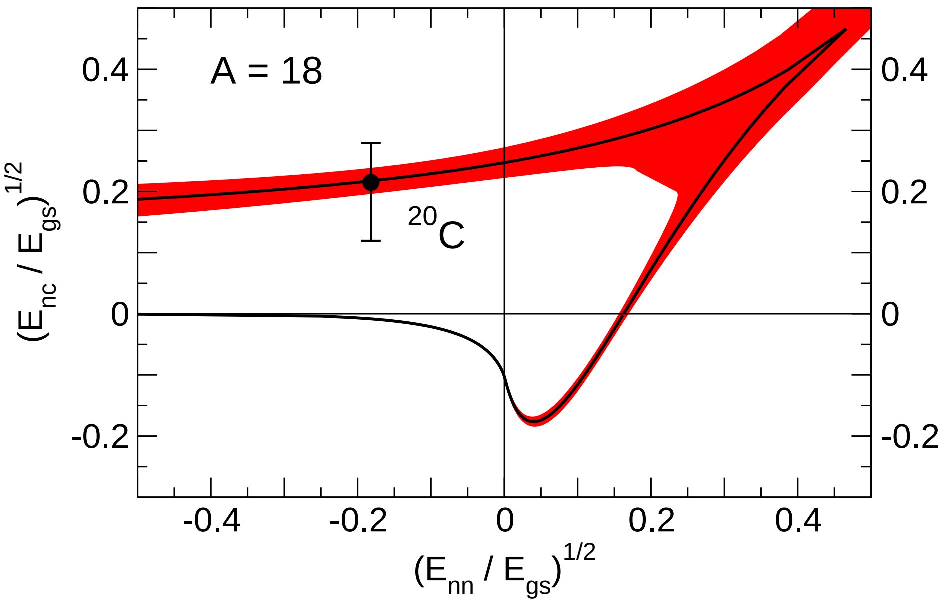
- Examples: $^{14}\text{Be} \longleftrightarrow {}^{12}\text{Be} + n + n$, $\quad {}^{20}\text{C} \longleftrightarrow {}^{18}\text{C} + n + n$
- "Effective" 3-body system: separation energy of valence nucleons small compared to binding energy of "core"
- Efimov effect in halo nuclei? $\Rightarrow$ excited states




Canham, HWH, Eur. Phys. J. A **37** (2008) 367 (cf. Amorim, Frederico, Tomio, 1997)

- Unchanged by NLO range corrections (Canham, HWH, arXiv:0911.3238)

# Form Factors and Radii (NLO)

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- Range corrections: $r_e \approx 1/m_\pi = 1.4$ fm
- Structure of halo nuclei $\rightarrow$ matter form factors, radii

| nucleus | $B_{nnc}$ [keV] | $B_{nc}$ [keV] | $\sqrt{\langle r_{nn}^2 \rangle}$ [fm] | $\sqrt{\langle r_{nc}^2 \rangle}$ [fm] |
|---|---|---|---|---|
| $^{14}$Be | 1120 | -200.0 | $3.9 \pm 0.1$ | $3.3 \pm 0.1$ |
| $^{20}$C | 3506 | 162 | $3.0 \pm 0.1$ | $2.5 \pm 0.1$ |
| | 3506 | 60 | $2.8 \pm 0.1$ | $2.4 \pm 0.1$ |
| $^{20}$C$^*$ | $65 \pm 1.0$ | 60 | $43.2 \pm 0.5$ | $38.7 \pm 0.4$ |

Canham, HWH, arXiv:0911.3238

- Input: TUNL Nuclear data evaluation project, ...
- Experiment: $^{14}$Be $\rightarrow \sqrt{\langle r_{nn}^2 \rangle} = (5.4 \pm 1.0)$ fm
  (Marques et al., Phys. Rev. C **64** (2001) 061301)

# Summary and Outlook

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- Effective field theory for large scattering length
  - Discrete scale invariance, universal correlations,...
- Applications in atomic, nuclear, and particle physics
  - Cold atoms close to Feshbach resonance
  - Scattering properties of the $X(3872)$
  - Halo nuclei
- Future directions:
  - Hadronic molecules: universal properties, three-body molecules? (e.g. $Y(4660) \leftrightarrow \psi' f_0(980) \leftrightarrow \psi' K\bar{K}$)
  - Three-nucleon system on the lattice: finite volume corrections, limit cycle in “deformed” QCD?
  - Halo nuclei: reactions, external currents, ...
  - Cold atoms: heteronuclear systems, $N \geq 4$, 2d-systems, ...

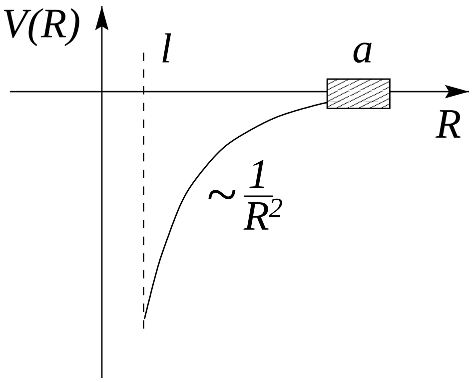
# Additional Slides

# Efimov Physics

(V. Efimov, Phys. Lett. **33B** (1970) 563)

- Three-body system with large scattering length $a$
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2\kappa^2}{m}}_{E} f(R)$$




- Singular Potential: renormalization required
- Boundary condition at small $R$: breaks scale invariance

$\Longrightarrow$ dependence of observables on 3-body parameter (and $a$)

- EFT formulation: boundary condition $\Rightarrow$ 3-body interaction

# Universal Correlations

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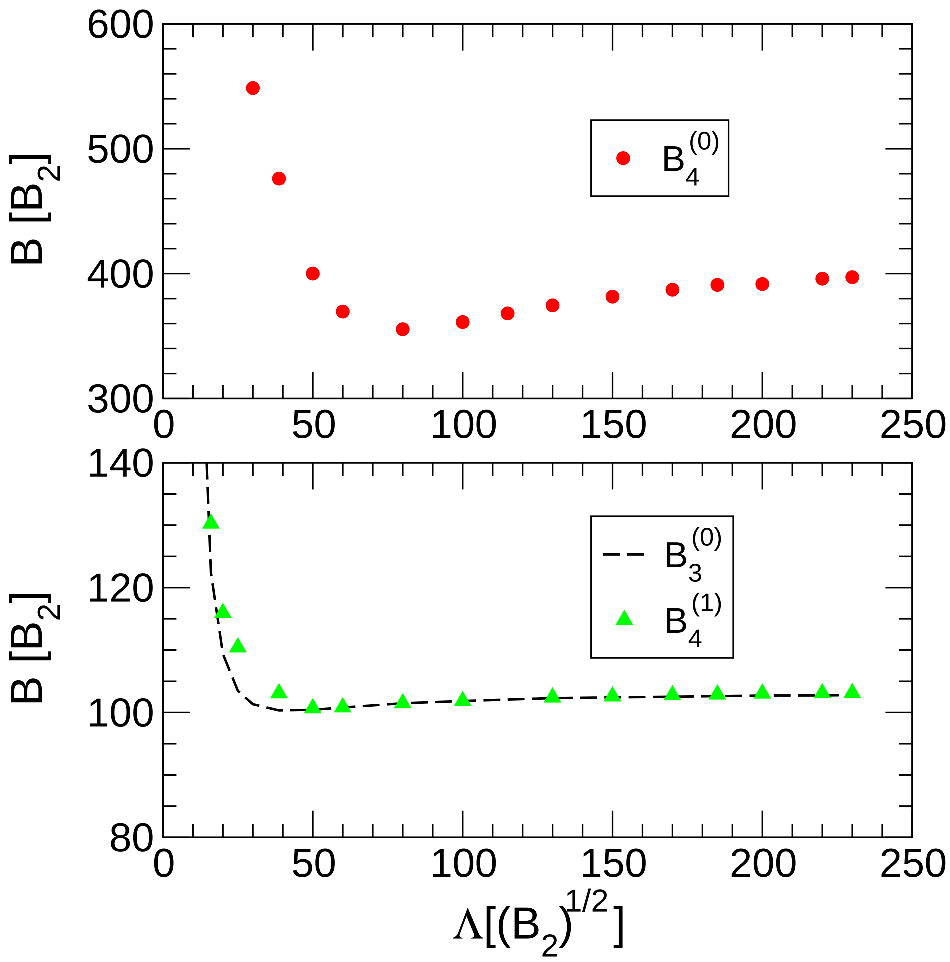
- 2 Parameters at LO ⇒ 3-body observables are correlated
  ⟹ Phillips line (Phillips, 1968)
- No four-body parameter at LO (Platter, HWH, Meißner, 2004)



# Form Factors and Radii

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- Structure of halo nuclei → matter form factors, radii

| nucleus | $B_{nnc}$ [keV] | $B_{nc}$ [keV] | $\sqrt{\langle r_{nn}^2 \rangle}$ [fm] | $\sqrt{\langle r_{nc}^2 \rangle}$ [fm] |
|---|---|---|---|---|
| $^{14}$Be | 1120 | -200.0 | $4.1 \pm 0.5$ | $3.5 \pm 0.5$ |
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| | 3506 | 60 | $2.8 \pm 0.2$ | $2.3 \pm 0.2$ |
| $^{20}$C$^*$ | $65 \pm 6.8$ | 60 | $42 \pm 3$ | $38 \pm 3$ |

Canham, HWH, Eur. Phys. J. A **37** (2008) 367

(cf. Yamashita, Tomio, Frederico, 2004)

- Input: TUNL Nuclear data evaluation project, ...
- Experiment: $^{14}$Be → $\sqrt{\langle r_{nn}^2 \rangle} = (5.4 \pm 1.0)$ fm

  (Marques et al., Phys. Rev. C **64** (2001) 061301)